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Combined solution of capacity expansion and signal setting problems for signalized road networks

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Abstract

Traffic congestion has been growing at an unsustainable rate and decreasing the quality of life of people living in many countries especially in last few decades. At the same time, congestion causes decreasing accessibility and mobility whereas it leads to increase travel time and air pollution. Although various optimization techniques in determining signal timings or optimal capacity expansion have been discussed separately in the literature, few studies have been considered for solving the both problems simultaneously. Thus, it can be emphasized that the majority of literature fails to highlight an indispensable relationship between these two problems. To fill this gap, a bi-level solution methodology based on Differential Evolution (DE) algorithm is proposed in this study. The upper level deals with minimizing total system travel cost under given budget and signal timing plan while the User Equilibrium link flows are determined by VISUM at the lower level. In this study, the DE based solution algorithm is coded in VBA which is combined with VISUM for solving the problem. In order to illustrate the efficiency of the proposed algorithm, it is applied to real data of Sioux-Falls city network which has 76 links, 24 nodes and 552 OD trips. In this network, 7 nodes are considered as signalized junction, and 16 links which connect these nodes are chosen as candidate for capacity expansion. Results indicated that the proposed algorithm shows significant performance in solving the combined problem for signalized road networks.

Keywords: Capacity expansion; signal optimization; differential evolution; VISUM

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1. Introduction

Since most of transportation problems are considerably complex and their solutions depend on large number of parameters, it is generally preferred to separate such problems into sequence of sub-problems. However, due to high interdependence of capacity expansion and signal setting problems, an indispensable relationship between these two problems should be taken into consideration. Since a Signalized Road Network Design Problem (SRNDP) with link capacity expansion has multiple objectives, it is common to formulate it using a bilevel model. As known, road users make route choice decisions based on the link capacities and signal settings in the process of traffic assignment at the lower level and planners try to redesign the capacity expansion plan and the signal settings based on road users' behaviour and the state of the road network at the upper level. Therefore, both problems should be solved simultaneously and thus more reliable results can be provided to decision makers/planners to design the signalized road networks. From a capacity expansion viewpoint, the first formulation for the Continuous Network Design Problem (CNDP) was proposed by Abdulaal and LeBlanc (1979). After this study, Suwansirikul et al. (1987) proposed equilibrium decomposition optimization heuristic for solving the CNDP and tested this heuristic on several networks. Following, a number of heuristic algorithms were developed for the CNDP (Friesz et al., 1990; Friesz et al., 1992; Friesz et al., 1993; Yang, 1995; Yang, 1997). Moreover, Chiou (2005) used gradient-based methods to solve the CNDP, and numerical applications are conducted in several test networks. Karoonsoontawong and Waller (2006) presented several heuristic methods for solving the resulting problem. Li et al. (2012) presented a global optimization method and converted the CNDP into a sequence of single level problems. Baskan (2013, 2014) attempted to solve the bilevel formulation of the CNDP using Cuckoo Search and Harmony Search algorithms, respectively. On the other hand, from the point of signal setting problem, Allsop and Charlesworth (1977) proposed a mutually consistent approach for signal setting problem. Heydecker and Khoo (1990) proposed a linear constraint approximation to the equilibrium flows and solved signal setting problem as a constrained optimization problem. Yang and Yagar (1995) proposed a sensitivity analysis based algorithm to solve the SRNDP. Moreover, Cascetta et al. (2006) discussed models and algorithms for the signal setting problem with stochastic traffic assignment where numerical tests are reported on a small-scale real network. Teklu et al. (2007) considered the problem of signal timing optimization in a road network and GA based algorithm is proposed for solving the resulting problem. Dell'Orco et al. (2013) presented a bi-level formulation for finding optimal signal settings under Stochastic User Equilibrium (SUE) conditions.

An overview to the literature reveals that finding of optimal signal timings or capacity expansion plan has been discussed separately in most cases. However, considering both problems together, few studies have been carried out in the literature. That is, Ziyou and Yifan (2002) raised that solving signal setting problem with link capacity expansions provides more realistic information for planners. Chiou (2008a) considered a signalized road network where the set of link capacity expansions and signal settings are simultaneously determined. Promising results are obtained after numerical calculations performed. Similarly, Chiou (2008b) presented a solution model for a signalized road network with link capacity expansions by considering the maximum possible increase in travel demands. Karoonsoontawong and Waller (2010) developed a robust optimization formulation that simultaneously solves capacity expansion, signal optimization and dynamic traffic assignment problems.

The organization of this paper is as follows. In next section, problem formulation is given. The solution algorithm is presented in Section 3. Numerical applications are explained with the results of the proposed model in Section 4. In last section, conclusions and future studies are remarked.

2. Problem formulation

The notations used in this paper are given as follows:

A	the set of links in the network
K_{rs}	the set of paths between O-D pair $rs \, \forall r \in R,\ s \in S$
R	the set of origins in the network
S	the set of destinations in the network

M the set of signal stages, $\forall m \in M$
N the set of candidate links for capacity expansion, $\forall n \in N$
L the set of signalized junctions, $\forall l \in L$
$\boldsymbol{\psi}(\phi, \zeta)$ the set of signal timing variables
$\mathbf{D}$ the vector of O-D pair demands, $\mathbf{D} = [D_{rs}] \quad \forall r \in R, \quad s \in S$
$\mathbf{f}$ the vector of path flows, $\mathbf{f} = [f_k^{rs}]$, $\forall r \in R, \quad s \in S, \quad k \in K_{rs}$
$\mathbf{t}$ the vector of link travel times, $\mathbf{t} = [t_a(x_a, y_a)] \quad \forall a \in A$
$\mathbf{x}$ the vector of equilibrium link flows, $\mathbf{x} = [x_a]$, $\forall a \in A$
$\mathbf{y}$ the vector of link capacity expansions, $\mathbf{y} = [y_a]$, $\forall a \in A$
$\boldsymbol{\phi}$ the vector of duration of green timings
d_a the cost coefficient, $\forall a \in A$
θ_a the link capacity, $\forall a \in A$
Z upper level objective function
z lower level objective function
I intergreen time between signal stages
ρ conversion factor from investment cost to travel times
$g_a(y_a)$ investment function, $\forall a \in A$
$\delta_{a,k}^{rs}$ the link/path incidence matrix variables, $\forall r \in R, \quad s \in S, \quad k \in K_{rs}, a \in A$. $\delta_{a,k}^{rs} = 1$ if route k between O-D pair rs uses link a, and $\delta_{a,k}^{rs} = 0$ otherwise
α_a, β_a the parameters of link cost function, $\forall a \in A$
$\phi^{\min}$ minimum duration of green timings
$\phi^{\max}$ maximum duration of green timings
ζ_l cycle time, $\forall l \in L$
h total number of upper level decision variables
F mutation factor
CR crossover rate
u number of generation
$\boldsymbol{\lambda}^u$ mutant vector at generation u
$\boldsymbol{\omega}^u$ trial vector at generation u
p number of the population

2.1. Upper level formulation

Since the multiple objectives exist in the SRNDP with link capacity expansions, it is common to solve it using bilevel programming. Although the bi-level formulation is not only way of defining this problem, we have preferred it in order to avoid computational difficulties experienced with other solution techniques. The upper level deals with finding optimal signal timings and capacity expansion plan for a given signalized road network by minimizing the total system and construction costs. On the other hand, in the lower level, User Equilibrium (UE) link flows are determined by considering capacity expansion plan and signal timings given in the upper level. Therefore, the SRNDP with link capacity expansions can be recognized within the framework of a leader-follower, where the planner is the leader and the road user is the follower (Fisk, 1984). The objective function of the upper level can be formulated as follows:

$$\min_{y} \quad Z(x,y,\psi) = \sum_{a\in A} (t_a(x_a, y_a)x_a + \rho g_a(y_a)) \tag{1}$$

$$\text{s.t.} \qquad 0 \le y_a \le y_a^{\max} \quad , \qquad \forall a \in A \tag{2}$$

$$\phi_m^{\min} \le \phi_m \le \phi_m^{\max}, \quad \forall m \in M$$

where $y_a^{\max}$ is the upper-bound capacity expansion of link $a \in A$ which ensures that the investment cost of link a will not exceed the related budget. It represents also the non-negativity constraint of the upper level decision variables.

2.2. Lower level formulation

In the lower level, the user's route choice behavior can be characterized by the UE assignment which distributes the demand according to the Wardrop's first principle. It states that the travel times of all used paths between the same Origin-Destination (O-D) pair are equal and less than any unused paths (Wardrop, 1952). As known, static UE problem is convex because the link travel time functions are monotonically increasing function. Additionally, if the cost functions are also strictly increasing, then the link flow solution is unique (Smith, 1979). On the other hand, this hypothesis underlies the unrealistic assumption that every road user is fully informed about the state of the road network. However, it is approved due to fundamental advantage which guarantees the existence and uniqueness of assignment result. Therefore, this approach is commonly preferred to find the equilibrium link flows in road network design problems. The static UE assignment problem can be formulated as follows:

$$\min_{x} \quad z = \sum_{a\in A} \int_0^{x_a} t_a(w, y_a, \psi_a)dw \tag{3}$$

$$\text{s.t.} \qquad \sum_{k\in K} f_k^{rs} = D_{rs} \qquad \forall r \in R, \quad s \in S, \quad k \in K_{rs} \tag{4}$$

$$x_a = \sum_{rs} \sum_{k\in K_{rs}} f_k^{rs} \delta_{a,k}^{rs} \qquad \forall r \in R, \quad s \in S, \quad a \in A, k \in K_{rs} \tag{5}$$

$$f_k^{rs} \ge 0 \qquad \forall r \in R, \quad s \in S, \quad k \in K_{rs} \tag{6}$$

$$\sum_{i=1}^{m} \phi_i + I_i = \zeta_l \qquad \forall l \in L \tag{7}$$

where the definition $t_a(w, y_a, \psi_a)$ in the Eq. (3) represents the cost of link a $\forall a \in A$, which is usually expressed by the following function called the Bureau of Public Roads (BPR) function as:

$$t_a(x_a, y_a, \psi_a) = \alpha_a + \beta_a \left(\frac{x_a}{\theta_a + y_a}\right)^4 \tag{8}$$

where α, β are the parameters and θ is the link capacity for link a $\forall a \in A$. Additionally, Eq. (4) represents that the total volume of all routes have to be equal the demand from origin r to destination s; Eq. (5) shows volume of link a results from the sum of volumes of all routes which contain this link; Eq. (6) describes that all route volumes have to be positive and Eq. (7) states that the sum of duration of green timings and intergreen times of signal stages of an intersection equals to its cycle time. Since UE assignment is a convex problem, it can be numerically solved by various methods. In this study, VISUM (PTV AG, 2014) software is used for finding equilibrium link flows at the lower level by means of given capacity expansion plan and signal timings at the upper level. As known, VISUM offers a number of UE assignment models such as Incremental assignment, Equilibrium assignment, LUCE and ICA in order to serve different objectives. In this paper, we have used Equilibrium assignment module which distributes the demand according to the Wardrop's first principle. Since the procedure of Equilibrium assignment is only terminated when all routes of any OD pair are in the balanced state, it provides more realistic results.

3. Solution algorithm

In order to solve the SRNDP with link capacity expansions we developed a heuristic solution algorithm, called **D**ifferential **E**volution for **S**ignalized **RO**ad **N**etworks (DESRON), using bilevel programming model. In the DESRON, the decision variables of a signalized road network are generated in the upper level while the UE link flows are determined by VISUM in the lower level. The solution algorithm is coded in VBA programming language due to outstanding advantages of VISUM that allows to import and export data using VBA. In the upper level of the proposed algorithm, capacity expansion and signal setting variables are generated by using Differential Evolution (DE) algorithm because of its recently successful applications for solving complex optimization problems. The DE is a simple and powerful heuristic algorithm in which the initial population is improved to reach to vicinity of the global or near-global optimum for a given optimization problem through repeated cycle of mutation, crossover and selection procedures (Liu et al., 2010). In the DE, two fundamental control parameters are used to manage the optimization process. The first one is the mutation factor (F), which is used to obtain mutant vector from selected three solution vectors in the population and the second one is the crossover rate (CR) that is the probability of consideration of the mutant vector (see for details Storn and Price, 1995). In the DESRON, the upper level decision variables are generated as following way:

link capacity expansions						duration of green timings					
y_{11}	y_{12}	..	..	..	y_{1n}	ϕ_{11}	ϕ_{12}	..	..	..	ϕ_{1m}
y_{21}	y_{22}	..	..	..	y_{2n}	ϕ_{21}	ϕ_{22}	..	..	..	ϕ_{2m}
↓	↓				↓	↓	↓				↓
y_{p1}	y_{p2}	..	..	..	y_{pn}	ϕ_{p1}	ϕ_{p2}	..	..	..	ϕ_{pm}

Fig. 1 Population structure of the DESRON

where n is the number of candidate links for capacity expansion in the network, m is the set of signal stages and p is the number of population. In the DESRON, similarly to other heuristic algorithms, each row of population is filled with randomly generated capacity expansions for a set of selected links and duration of green timings by considering upper and lower boundaries and so called target vectors are created. Moreover, cycle times of signalized junctions in the network can be simply calculated using generated green timings and predefined intergreen times. Following, the upper level decision variables are input to VISUM and equilibrium link flows are determined after executed traffic assignment process. Thus, the objective function values are found for target vectors located in each row of the population in the DESRON. The algorithm steps are given in Fig. 2. After then, each element of a mutant vector is obtained from randomly selected three solution vectors using Eq. (9).

$$\lambda_i^{j,u} = \delta_1^{j,u} + F(\delta_2^{j,u} - \delta_3^{j,u}) \ , \quad i = 1,2,....p \ , \quad j = 1,2,....h \tag{9}$$

where $\delta_1^{j,u}$, $\delta_2^{j,u}$ and $\delta_3^{j,u}$ are randomly selected decision variables within the range [0, p] at generation u, h is the total number of upper level decision variables and $\delta_1^{j,u} \neq \delta_2^{j,u} \neq \delta_3^{j,u}$. After creating the mutant vector for each target vector, the other improvement process of the DE is performed using crossover operator to obtain trial vector. At this step, each member of the trial vector, $\omega_i^{j,u}$, is chosen from the mutant vector with the probability of CR or from the target vector with the probability of (1-CR) as given in Eq. (10).

$$\omega_i^{j,u} = \begin{cases} \lambda_i^{j,u}, & \text{if rand}(0,1) \le CR \quad \text{or } i = i_{rand} \\ y_i^{j,u} \wedge \phi_i^{j,u}, & \text{otherwise} \end{cases} \tag{10}$$

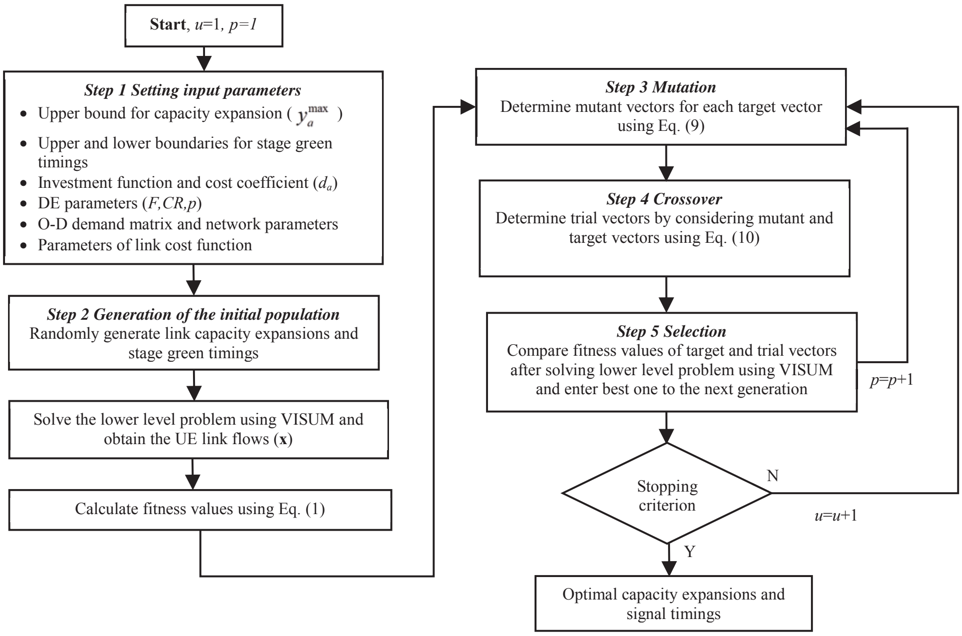

Fig. 2 Steps of the DESRON algorithm

As can be seen from Eq. (10), the value of CR is compared with the output of a uniform random number generator, rand $(0,1)$, to determine either mutant vector or target vector will provide the member of the trial vector. If the random generated number is less than or equal to CR at generation u, the trial parameter is chosen from the mutant vector, $\lambda^{j,u}$; otherwise the parameter is chosen from the members of the target vector, $y_i^{j,u} \wedge \phi_i^{j,u}$. Additionally, the constraint, $i = i_{rand}$, where i_{rand} is the uniformly distributed random number in the range [1, p], ensures that at least one member of the trial vector is taken from the mutant vector. At this step, the trial vector $\boldsymbol{\omega}^u$ is compared with the target vector by way of determining the objective function values. In order to determine the fitness value of trial vector, the capacity expansion and signal timing variables located in the trial vector are input to VISUM and traffic assignment process is executed. After then the fitness value of trial vector is calculated using Eq. (1). Finally, after comparing the objective function values of trial and target vectors, best one enters to the generation u+1 as shown in Eq. (11). Mutation, crossover and selection steps of the DESRON are repeated until a predetermined stopping criterion is met or maximum number of generations is reached.

$$\mathbf{y}^{u+1} = \begin{cases} \boldsymbol{\omega}^u, & \text{if } f(\boldsymbol{\omega}^u) \le f(\mathbf{y}^u) \\ \mathbf{y}^u \wedge \boldsymbol{\phi}^u, & \text{otherwise} \end{cases} \tag{11}$$

4. Numerical application

In order to show the capability of the DESRON in solving the SRNDP with capacity expansions, the city of Sioux Falls is used which is probably the most used network in the literature. The network has 24 nodes and 76 links. The link parameters of the network and travel demands between 552 O-D pairs are taken from Baskan (2014).

The link travel time function is used as given in Eq. (8). The dashed links are candidates for capacity expansion as shown in Figure 3.

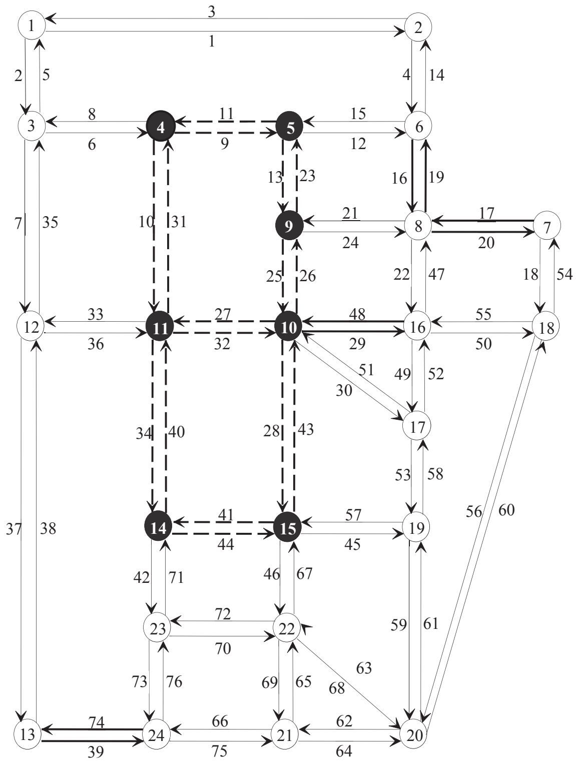

Fig. 3 Sioux Falls network

Additionally, 7 nodes marked as bold in the network are considered as signalized junctions. It is assumed that nodes numbered with 4, 5, 9 and 14 are three stage junctions and the rest are four stages. The upper level objective function for the Sioux Falls network is formulated as in Eq. (12). Lower and upper bounds for stage green timings are selected 7 and 40 sec, respectively. Intergreen times for all stages are taken 5 sec. In addition, maximum capacity expansion $y_a^{max}, \forall a \in A$ is considered as 20 while parameters of the DE algorithm *F*, *CR* and *p* are selected 0.8, 0.8 and 10, respectively in keeping with the literature.

$$\min_{y} \quad Z(x, y, \psi) = \sum_{a \in \boldsymbol{A}} (t_a(x_a, y_a)x_a + 0.001 d_a y_a^2) \tag{12}$$

$$\text{s.t.} \qquad 0 \le y_a \le y_a^{max} \quad , \qquad \forall a \in A$$

$$\phi_m^{min} \le \phi_m \le \phi_m^{max}, \qquad \forall m \in M$$

At the lower level, UE traffic assignment is performed by way of VISUM. It is assumed that the DESRON has found optimum or near-optimum solution when the relative error between average and best objective function values is less than the value of 0.001 for Sioux Falls network. The convergence graph of the DESRON can be seen in Figure 4.

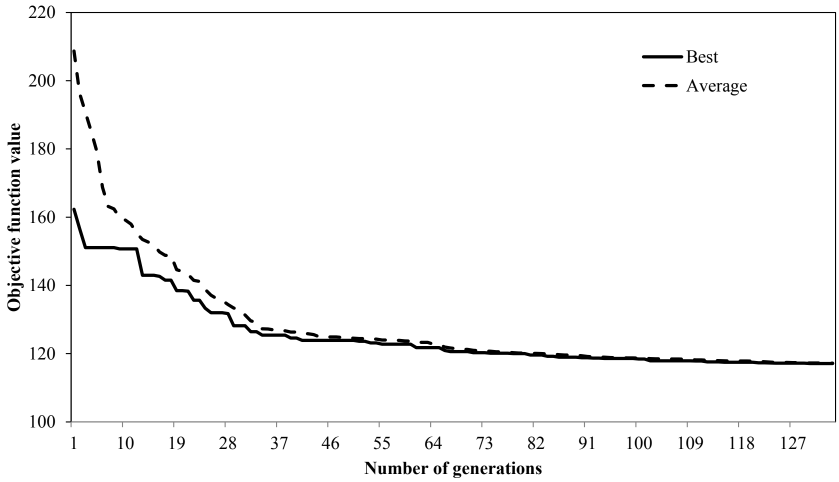

Fig. 4 Convergence graph of the DESRON for the Sioux Falls network

Maximum generation number is selected 200 and the algorithm is terminated after 134 generations since predefined stopping criterion is met. In the initialization process, the DESRON randomly generates capacity expansion and signal timing variables in terms of their selected upper and lower bounds and thus the initial population is created. The initial value of objective function is found as 162 considering current population. After then mutation, crossover and selection operators are applied to initial population and the final objective function value is obtained as about 117 after rapidly decreasing of its initial value especially during first 50 generations. Resulting equilibrium link flows are given in Table 1.

Table 1 Resulting equilibrium link flows

Link number	Volume (veh/h)	Link number	Volume (veh/h)	Link number	Volume (veh/h)
1	7.04	***27***	**19.70**	*53*	10.40
2	12.00	***28***	**24.40**	*54*	16.20
3	8.12	*29*	11.10	*55*	21.20
4	6.60	*30*	8.91	*56*	21.90
5	11.00	***31***	**9.98**	*57*	20.30
6	18.20	***32***	**13.90**	*58*	10.60
7	15.80	*33*	9.34	*59*	9.85
8	17.50	***34***	**12.20**	*60*	21.80
9	**27.20**	*35*	15.40	*61*	10.00
10	**2.25**	*36*	10.00	*62*	7.67
11	**18.90**	*37*	16.50	*63*	7.70
12	8.19	*38*	16.80	*64*	7.70
13	**24.40**	*39*	11.70	*65*	9.31
14	7.68	***40***	**13.60**	*66*	10.20
15	7.59	***41***	**12.40**	*67*	21.40
16	13.90	*42*	8.10	*68*	7.70
17	12.00	***43***	**22.90**	*69*	8.17
18	20.30	***44***	**7.60**	*70*	10.80
19	14.40	*45*	20.20	*71*	4.58
20	16.20	*46*	18.30	*72*	8.88
21	1.97	*47*	8.25	*73*	8.94
22	9.85	*48*	16.70	*74*	12.00
23	**16.70**	*49*	12.50	*75*	11.40
24	8.23	*50*	17.00	*76*	7.40
25	**23.40**	*51*	9.28		
26	**22.20**	*52*	12.20		

The maximum equilibrium link flow in the network is observed from link 9 while link 21 has least volume as shown in Table 1. Equilibrium link flows on the candidate links for capacity expansion, which are marked in bold, can be also seen in Table 1. It should be emphasized that links 25 and 28 have more traffic volume than most of links and thus more capacity expansion is performed to these links than other candidate links in the network as shown in Table 2.

Table 2 Optimal link capacity expansions

Link number	y_a	Link number	y_a
9	5.18	*27*	2.69
11	2.74	*32*	1.07
13	1.18	*34*	7.15
23	0.18	*40*	8.81
25	10.91	*28*	13.10
26	2.22	*43*	3.50
10	0.00	*44*	1.05
31	0.46	*41*	3.79

Additionally, optimal signal timings are obtained after running the DESRON and given in Table 3. Minimum and maximum cycle times for the Sioux Falls network are found as 47 and 135 for junctions 14 and 11, respectively. It should be noted that these results depend on the selected stage configurations since different stage plans may affect equilibrium link flows and optimal signal timings in the network. In addition, completely different results can be obtained in case coordinated signalized network is considered. It should not overlook that link volumes are in accordance with the value of stage green timings in which these links have the right of way as can be seen in Tables 1 and 3. This result may be evaluated as evidence that the DESRON can be used in order to solve the SRNDP with link capacity expansions.

Table 3 Resulting optimal signal timings

Junction number	Cycle time (sec)	Stage number	Start of green (sec)	End of green (sec)	Junction number	Cycle time (sec)	Stage number	Start of green (sec)	End of green (sec)
4	114	1	0	31	*10*	100	1	0	15
		2	36	64			2	20	34
		3	69	109			3	39	70
5	84	1	0	22			4	75	95
		2	27	47	*11*	135	1	0	35
		3	52	79			2	40	47
9	74	1	0	37			3	52	86
		2	42	49			4	91	130
		3	54	69	*15*	89	1	0	15
14	47	1	0	12			2	20	39
		2	17	30			3	44	60
		3	35	42			4	65	84

5. Conclusions

In this study, the DESRON algorithm is proposed for solving the SRNDP with link capacity expansions using bi-level programming model. The upper level aims to minimize total travel cost under given budget and signal timing plan while the UE link flows are found by VISUM at the lower level. The DE based heuristic solution algorithm is combined with VISUM for solving the problem. In order to test the proposed algorithm, it is applied to Sioux Falls network in which 7 junctions are considered as signalized junction, and 16 links which connect these nodes are chosen as candidate for capacity expansion. Results show that the DESRON is capable to find optimal capacity expansions and signal timings simultaneously. In future, the proposed algorithm will be applied to large scaled road networks in order to see how it reacts to such conditions. Using the proposed algorithm on large scaled road networks with possible good results may help planners to use sources effectively.

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