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Sensitivity Analysis on Stochastic Equilibrium Transportation Networks using Genetic Algorithm

Halim Ceylan
Michael G. H. Bell

This study deals with the sensitivity analysis of an equilibrium transportation networks using genetic algorithm approach and uses the bi-level iterative sensitivity algorithm. Therefore, integrated Genetic Algorithm-TRANSYT and Path Flow Estimator (GATPFE) is developed for signalized road networks for various level of perceived travel time in order to test the sensitivity of perceived travel time error in an urban stochastic road networks. Level of information provided to drivers correspondingly affects the signal timing parameters and hence the Stochastic User Equilibrium (SUE) link flows. When the information on road system is increased, the road users try to avoid conflicting links. Therefore, the stochastic equilibrium assignment concept tends to be user equilibrium. The GATPFE is used to solve the bi-level problem, where the Area Traffic Control (ATC) is the upper-level and the SUE assignment is the lower-level. The GATPFE is tested for six-junction network taken from literature. The results show that the integrated GATPFE can be applied to carry out sensitivity analysis at the equilibrium network design problems for various level of information and it simultaneously optimize the signal timings (i.e. network common cycle time, signal stage and offsets between junctions).

Keywords: Genetic Algorithm, Stochastic User Equilibrium Assignment, Sensitivity Analysis, Dispersion parameter

Introduction

Mutual interaction of Traffic Assignment (TA) and Area Traffic Control (ATC) can explicitly be considered, producing the so-called combined control and assignment problem. Allsop (1974)

Halim Ceylan is with the Department of Civil Engineering, Engineering Faculty, Pamukkale University, Denizli, Turkey.
Michael G. H. Bell is with the Department of Civil and Environmental Engineering, Imperial College, London, U.K.

stated that "when all or part of the network is subject to traffic control, the relationship between travel cost and traffic flow on some or all of the links in the network depends on the control parameters, and these can therefore be used to influence the number of journeys made through the network and the routes taken". Thus, there exists a strong interdependence between traffic control and traffic assignment.
The solution of the combined TA and ATC can widely be obtained in literature (Yang and Yagar, 1995, Chiou, 1998; Chiou, 2003; Ceylan and Bell, 2003) based on various approaches such as mutually consistent, bi-level or genetic algorithm. The literature mainly deals with the solution of the problem based on Wardropian User Equilibrium principle. A few studies deals with the Stochastic User Equilibrium Approach [Ceylan and Bell, 2003] for the combined ATC and TA problem.

The allocation of the dispersion parameter, α, on a signal-controlled road networks affects the drivers' perception of travel time to any changes in a network such as signal timing or road closure. So that reason, it is logical to consider a level of information provided to the road users about road system. Any changes in travel time perception will consequently affect the signal timing at a signal-controlled junction and the assignment process onto a network. So far no assumption is made to allow stochastic variations of perceived travel time on a stochastic equilibrium network. Clark and Watling [2002] carried out the SA for probit-based SUE networks, but no consideration is given to the SA for the combined ATC and TA. In this study, the stochastic errors on perceived travel time are relaxed and re-optimization for the signal timing parameters are made under perturbed equilibrium flows.

Signal setting parameters for a random variation of perceived travel time, and corresponding equilibrium flows that optimize some measure of performance are sought. Due to the complexity of finding the optimum signal settings with different values of perceived travel time, Genetic Algorithms (GA) [Goldberg, 1989] approach has been applied to combined ATC and TA to solve the problem. One of the current limitations in available solution techniques in ATC problem is that no method so far allows stochastic user equilibrium assignment and no method allows re-optimization of signal timings when the level of information is changed for road users.

Sensitivity analysis (SA) in an ATC with equilibrium transportation networks may be carried out to meet the following objectives.

It is to explore the uncertainties during the mathematical modeling of the combined TA and ATC due to the uncertainties in the link performance functions,
It is to obtain the affect of randomness in route choice due to difference in the perceived travel time by road users.
It is to find the perturbed-equilibrium link flows to the signal settings whether signal settings require re-setting when the level of information is changed.

SA for the combined TA and ATC is basically not dealt with in literature due to the difficulties in formulating in the link cost function by including the common cycle time and the offset variable, especially for the SUE case. Ying and Miyagi [2001] carried out SA for SUE networks and proposed a dual approach to formulate the problem. Although they included sensitivity parameters in the link cost function, but no consideration is given to the inclusion of signal control parameter for the combined ATC and TA problem. Furthermore, there is no consideration about the signal timing re-optimization when SUE equilibrium flows are perturbed. This difficulty comes from complexity of combined signal setting and traffic assignment problem. This complexity is basically the non-convex nature of the problem. When the problem is non-convex, we first need to solve the combined assignment and traffic control problem, then we need to carry out the sensitivity analysis. Therefore, GATPFE is developed to solve the combined assignment traffic control problem, and then sensitivity analysis is carried out on the stochastic equilibrium link flows and the signal settings.

In urban networks, the most significant component of delay arises at junctions because there capacity is least. One of the important components of delay occurred due to cyclic variations of signals and tends to increase with flow. Thus many traffic simulation models concentrate on queuing behavior and make simplifying assumptions [Yin, 2000; Yang and Yagar, 1995] about link travel times. The TRANSYT network study tool [Robertson, 1969; Vincent et al, 1980] assumes that queues occupy no physical space (the "vertical queuing" assumption) and that travel time from stop-line to stop-line have a geometric distribution. The appropriate representation of a link travel time function in a coordinated signalized road networks is a difficult tusk due to finding an mathematical expressions for an offset variables. Hence for different values of degree of saturation, the approximate expressions are used [Chiou, 1998; Chiou, 2003; Ceylan, 2002].

For the purpose of solving the problem, a bi-level iterative sensitivity algorithm has been used. The upper level problem is signal setting while the lower level problem is finding SUE equilibrium link flows. It is, however, known [Sheffi and Powell, 1983] that there are local optima. It is not certain that the local solution obtained is also the global optimum because equilibrium network design is generally a non-convex optimization problem. Hence, a GA approach is used to globally optimize signal setting at the upper level by calling TRANSYT traffic model to evaluate the fitness function for different values of perceived travel time.

Stochastic User Equilibrium (SUE) and Genetic Algorithm Formulation

Notation

$N = \{1,2,3,\ldots,N_j\}$ be a set of N_j nodes each of which represents a signal-controlled junction;
$L = \{1,2,3,\ldots,N_L\}$ be a set of N_L links where each traffic stream approaching any junction is represented by its own link;
$\boldsymbol{W} = \{w = (o,s)\}$ be the set of origin-destination pairs;
P_w = Set of paths between each origin-destination pair w, $\forall w \in \boldsymbol{W}$;
$\mathbf{t} = [t_w; \forall w \in \boldsymbol{W}]$ be the vector of travel demand between each origin-destination pair;
C_{min} = minimum specified common cycle time;
C_{max} = the maximum specified common cycle time;
C = Common cycle time at a signalized road network
$\boldsymbol{\theta} = [\theta_{1n}; \forall n \in \boldsymbol{N}]$ be the vector of start of green for stage 1 at each junction relative to an arbitrary time origin for the network as a whole;
$\boldsymbol{\phi} = [\phi_{in}; \ \forall \ i \in S_n, \ \forall \ n \in \boldsymbol{N}]$ be the vector of green times, where element ϕ_{in} is the duration of green for stage i at junction n;
$\boldsymbol{\psi} = (c, \ \boldsymbol{\theta}, \ \boldsymbol{\phi})$ be the vector of feasible signal timings;
$\boldsymbol{\theta}$ = Vector of feasible range of offset variables
$\boldsymbol{\phi}$ = Vector of duration of green times
$\boldsymbol{\psi} = (c, \boldsymbol{\theta}, \boldsymbol{\phi})$ whole vector of feasible set of signal timings
$\boldsymbol{\Omega}_0$ =Vector of feasible region for signal timings

$\mathbf{q}=[q_a\,;\forall a \in \boldsymbol{L}]$ be the vector of the average flow q_a on link a;
$\mathbf{h}=[h_h; \quad \forall h \in \boldsymbol{P}_w, \ \forall w \in \boldsymbol{W}]$ be the vector of all path flows, where element h_h is traffic flow on path h;
$\boldsymbol{\delta}=[\delta_{ah}; \ \forall a \in \boldsymbol{L}, \ \forall h \in \boldsymbol{P}_w, \ \forall w \in \boldsymbol{W}]$ be the link/path incidence matrix, where $\delta_{ah}=1$ if link a is on path h, and $\delta_{ah}=0$ otherwise;
$\mathbf{q}^*(\psi)$ =Vector of stochastic user equilibrium link flows
Λ = OD-path incidence matrix
y= Vector of expected minimum origin-destination cost
$\mathbf{g}(\mathbf{q},\psi)$ = Vector of path travel times,
$\mathbf{c}^0$ = Vector of free-flow link travel times,
$\mathbf{c}(\mathbf{q},\psi)$ = Vector of all link travel times,
d_a^U = Uniform delay at a signal-controlled junction
d_a^{ro} = Random plus over saturation delay at a signalised junction
P =Matrix of link choice probabilities.
X_{tt} =Potential solution matrix of dimension [pz$\times$ l] for the GA random search space
pz= Population size
l = Total number of binary bits in the string (i.e. chromosome)
tt = Generation number
PI= network performance index to be minimized

SUE formulation

According to multinomial logit model the probability that path p is chosen for a trip from i to j is given by

$$P_p^W(g)=\frac{\exp(-\alpha C_p)}{\sum\limits_{k\in \boldsymbol{P}_W}\exp(-\alpha C_k)}, \quad \forall k \neq p \in \boldsymbol{P}_u \ \forall w \in \boldsymbol{W} \tag{1}$$

At stochastic user equilibrium, the path flows are

$$h_p = t_W P_p^W(\mathbf{g}(\mathbf{h})), \quad \forall p \in \boldsymbol{P}_W, \qquad \forall w \in \boldsymbol{W} \tag{2}$$

Then the following conversion of link flows q to path flows h, path travel times $\mathbf{g(q)}$ to link travel times $\mathbf{c(q)}$, and path flows h to origin-destination demand t is calculated using the incidence relationships as follows:

$$\mathbf{q} = \boldsymbol{\delta}\mathbf{h} \tag{3a}$$

$$\mathbf{g(q)} = \boldsymbol{\delta}^{\mathrm{T}}\mathbf{c(q)} \tag{3b}$$

$$\mathbf{t} = \boldsymbol{\Lambda}\mathbf{h} \tag{3c}$$

where T is the transpose of the link-path incidence matrix.

Sheffi (1985) has formulated the SUE equivalent minimization problem based on the expected minimum origin-destination costs. Following Bell and Iida (1997), the objective function has the form

$$\underset{q}{Minimise}\, Z(\mathbf{q}) = -\mathbf{t}^{\mathrm{T}}\mathbf{y(q)} + \mathbf{q}^{\mathrm{T}}\mathbf{c(q)} - \sum_{a\in L}\int_0^{q_a} c_a(x)dx \tag{4}$$

Subject to t=Λh, q=δh, h≥0

where

$$\mathbf{y(q)} = \frac{1}{\alpha}\ln[\sum_{p\in \boldsymbol{P}_w} \exp(-\alpha c_p)],\ \forall w \in \boldsymbol{W}$$

where α is the sensitivity parameter that governs the level of information to road users

Assuming separable link cost functions which are monotonically increasing with flows, link cost functions may be inverted. Integration by parts yields

$$\sum_{a\in \boldsymbol{L}}\int_{c_{\min}}^{c_a} q_a(x)dx = \mathbf{q}^{\mathrm{T}}\mathbf{c(q)} - \sum_{a\in \boldsymbol{L}}\int_0^{q_a} c_a(x)dx \tag{5}$$

When the link flows are represented as a function of link costs, the expression (5) reduces to

$$Z(\mathbf{q}) = -\mathbf{t}^{\mathrm{T}}\mathbf{y}(\mathbf{c}) + \sum_{a \in L} \int_{c_{\min}}^{c_a} q_a(x)dx \tag{6}$$

The derivative of the objective function (6) with respect to link cost is

$$\Delta Z(\mathbf{q}) = -\mathbf{t}^{\mathrm{T}}(\partial \mathbf{y} / \partial \mathbf{c}) + \mathbf{q}^{\mathrm{T}} \tag{7}$$

the Jacobian of the expected minimum origin-destination costs with respect to link costs is equal to the matrix of link choice proportions, namely

$$\partial \mathbf{y} / \partial \mathbf{c} = \mathbf{P}^{\mathrm{T}} \tag{8}$$

The first-order necessary conditions for the minimization problem requires that the derivative of the objective function vanish at the minimum point, namely

$$\Delta Z(\mathbf{q}) = -\mathbf{t}^{\mathrm{T}}(\partial \mathbf{y} / \partial \mathbf{c}) + \mathbf{q}^{\mathrm{T}} = 0 \tag{9}$$

The minimisation of equation (6), subject to flow conservation and non-negativity constraints, leads to the SUE point. As a result, the SUE flow pattern can be obtained by solving equivalent mathematical program.

As far as the uniqueness is concerned, it is sufficient to show the Hessian matrix of the objective function is positive definite anywhere, implying that the program is strictly convex.

The Hessian of the equation (6) is

$$\nabla^2 Z(\mathbf{q}) = \sum_{w \in W} - t_w(\partial y_w / \partial c \partial c) + \mathbf{J}^{-1} \tag{10}$$

where the Jacobean $\mathbf{J} = \partial \mathbf{c} / \partial \mathbf{q}$ (10a)

The Jacobean of the link cost function is positive definite if the link cost function are monotonically increasing, in which case the Jacobean matrix is also invertible.

The rate of change in expected minimum origin-destination costs with respect to increase in link costs will be zero or negative, as the probability of link choice decrease with increasing link costs. Hence the Hessian of the expected cost for origin destination pair w in W with respect to link costs, $\partial y_w / \partial \mathbf{c}\partial \mathbf{c}$ is negative semi-definite. The Hessian of (6) is therefore a positive definite matrix, since the sum of a series of positive semi-definite matrices and one positive matrix itself positive definite. This in turn establishes the objective function (6) to be convex with unique optimum. The optimum where

$$\mathbf{t}^{\mathrm{T}}(\partial \mathbf{y}/\partial \mathbf{c}) = \mathbf{t}^{\mathrm{T}}\mathbf{P} = \mathbf{q}^{\mathrm{T}} \tag{11}$$

or when transposed

$$\mathrm{q=Pt} \tag{12}$$

The first-order necessary condition (9) and the SUE condition (12) coincide. Hence, the optimum of Z(q) defines a stochastic user equilibrium.

Path Flow Estimator (PFE)

The underlying theory of the PFE is the logit SUE model based on the notion that perceived cost determines driver route choice. It is a flexible traffic assignment software tool that has been developed by Transport Operations Research Group (TORG), University of Newcastle upon Tyne, UK, to support both on-line urban traffic management and off-line transportation planning. The basic idea is to find the path flows and hence links flows, which satisfy equilibrium condition where all travelers' perceive the shortest path (allowing for delays due to congestion) according to their own perception of travel time. The PFE gives an estimation of average flows and travel times in a network that are consistent with the assignment of flows to paths according to the logit model. The PFE algorithm assigns flows to paths according to the logit-path choice model.

Consider the following convex optimization problem

$$Min\, Z(\mathbf{h}) = \mathbf{h}^{\mathrm{T}}(\ln(\mathbf{h}) - \mathbf{1}) + \alpha \sum_{a \in \boldsymbol{L}} \int_{x=0}^{x=q_i} c_a(x)dx \tag{13}$$

subject to t=Λh, h≥0

is strictly convex in h, because the Hessian of Z(h) is positive definite. The convex optimization problem generates the required gradient vector

$$\nabla Z(\mathbf{h}) = \ln(\mathbf{h}) + \alpha g(\mathbf{h}) \tag{13a}$$

For practical purposes, the non-negativity conditions can be neglected as the objective function is not defined for non-positive path flows. This optimization problem was originally proposed by Fisk (1980).

One of the practical solution methods for (13), proposed by Powell and Sheffi [1982], made use of the method of successive averages. It follows the following steps [see for details Bell and Iida, 1997]:

Step 1 (initialization)
q←0
n←1

Step 2 (find a logit assignment for fixed link costs)
c←c(q)
q*←logit assignment for c

Step 3 (method of successive averages)
q←q(1-1/n)+q*(1/n)
if convergence insufficient then
return to Step 2
else stop.

The PFE uses an iterative balancing algorithm [Bell et al., 1997] combined with the Method of Successive Averages (MSA) to solve the logit-based stochastic user equilibrium assignment by providing the network topology, the O-D matrix, the network links and their corresponding link cost functions. The PFE algorithm is given in Figure 1.

Genetic algorithm

Genetic Algorithm (GA) is based on nature's theory of evolution, survival of the fittest. GA's have been developed by Holland [1975] as reported by Goldberg [1989] and Gen and Cheng [1997]. The GA is an iterative process that involves reproduction, crossover and mutation. In the last few years, GA has been successfully used to achieve the optimal design of signal timings. The first appearance of GAs for traffic signal optimization was due to Foy et al., [1992], in which the green timings and common cycle time were the explicit decisional variables and the offset variables were the implicit decisional variable in a four-junction network when flows remain fixed. In the optimization process, a simple microscopic simulation model was used to evaluate alternative solutions based on minimizing delay. The results showed an improvement in the system performance when GA was used and suggested that GAs have the potential to optimize signal timing. The results, however, were not compared with what could be achieved using existing optimization tools. In addition, the GA model is applied to a very simple system with two-stage operation and no explicit offsets between intersections. More typical real-world applications are needed to prove its effectiveness. It was also concluded that the GA model may be able to solve more difficult problems than traditional control strategies and search methods in terms of convergence and that good convergence were reported in that study.

A different approach (i.e. hybrid use of GA and TRANSYT hill-climbing) has been followed by Hadi et al., [1993] in which the GA model is expanded to optimize signal staging and timing together with TRANSYT-7F. A GA was used to optimize stage sequence by way of look-up table and network common cycle time. The green timings were calculated based on TRANSYT optimization routine. The objective function was the TRANSYT performance index. The results were compared with the existing network signal timings for the example network. It was concluded that GA model has a potential to optimize signal settings in conjunction with TRANSYT hill-climbing. However, their approach was the hybrid use of GA and TRANSYT optimizing the signal timings, where the green timings were calculated using TRANSYT hill-climbing, but no allowance is given to the stochastic route choice.

Briefly, the steps involved in the GA are:

Represent all the variables in the optimization problem in binary form. Hence all variables will be represented by a string of bits. All variables will be in one block. For example, if we have four variables, $\psi_1, \psi_2, \psi_3, \psi_4$; where ψ_1 can have a value between 0 and 120, ψ_2, ψ_3 and ψ_4 can have a value between 5 and 120, and then the following string will represent all four variables. In other words, one binary number of lengths 20 can represent all the four variables in (14).

$$\underbrace{1\ 0\ 0\ 1\ 1}_{\psi_1}\ \underbrace{0\ 1\ 1\ 0\ 1}_{\psi_2}\ \underbrace{1\ 0\ 1\ 0\ 1}_{\psi_3}\ \underbrace{0\ 0\ 1\ 1\ 1}_{\psi_4} \tag{14}$$

Randomly selects two numbers of 20 bits length parents. Those two numbers needs to satisfy all the constraints.
Perform what is called crossover on the two selected numbers. Crossover between two variables in a simple process where the bits of the two variables, after a randomly chosen point of crossover, interchange values with the corresponding bits in the other variable. For example, if we have two parents as in (15a), crossover will yield the variables as in (15b). In this example, bits 5 to 20 exchange values with corresponding bits in the other number

Parent A 1 1 0 1 0 0 1 1 0 1 1 1 1 1 1 0 0 0 1 0
Parent B 0 0 1 1 1 1 0 0 0 0 0 1 1 1 0 0 1 1 0 1 (15a)
Point of crossover
Parent A 1 1 0 1 0 0 1 0 0 0 0 1 1 1 0 0 1 1 0 1
Parent B 0 0 1 1 1 1 0 1 0 1 1 1 1 1 1 0 0 0 1 0 (15b)

Perform mutation on the new offsprings. Mutation is simply to change the value in a randomly chosen bit from 0 to 1or 1 to 0, as shown in (15c) and (15d), where bold letters indicate the mutated bits.

Offspring A 1 1 0 1 0 0 1 1 0 1 1 1 1 1 1 0 0 0 1 0

(15c)

Offspring A 1 1 0 1 0 0 1 1 0 1 1 **0** 1 1 1 0 0 0 1 0

(15d)

Repeat steps 3 through to 4 for the pre-defined maximum number of generations. If any stage of mutation any of the variables violate the problem constraints, then such variables will be discarded and a new number chosen randomly.

The reproduction is a process that selects the most fit individual strings according to some selection operators, such as tournament [Goldberg and Deb, 1991] selection with or without fitness scaling applied. It is responsible for choosing the members that will be allowed to reproduce during the current generation. These members are selected on the basis of their fitness, F, values and the most fit individuals are passed on to future generations. The best solution will be achieved at the end of the generations. Moreover, elitism operator is used to ensure that the chromosome of the best parent generated to date is carried forward into the next generation. After the population is generated, the GA checks to see if the best parent has been replicated; if not, then a random individual is chosen and the chromosome set of the best parent is mapped into that individual.

Problem Formulation

The bi-level formulation for the ATC is presented as an interaction between decision-takers and a decision maker in following way:

$$\underset{\boldsymbol{\psi}}{Min}\, PI(\mathbf{q}^*(\boldsymbol{\psi}), \boldsymbol{\psi}) = \sum_{a \in L} (Ww_a D_a(q, \psi) + Kk_a S_a(q, \psi)) \tag{16}$$

subject to subject to $\boldsymbol{\psi}(C, \boldsymbol{\theta}, \boldsymbol{\varphi}) \in \boldsymbol{\Omega}_0$ (16a) where $\mathbf{q}^*(\psi)$ can be obtained in the following way [Bell and Iida, 1997].

$$\text{Min } Z(\mathbf{q},\ \psi) = \mathbf{q} - \mathbf{P}(\mathbf{c}(\mathbf{q},\psi))\mathbf{t} \tag{17}$$

At equilibrium, this function has value zero, so

$$Z(\mathbf{q}^*, \psi) = \mathbf{0} \tag{18}$$

It is possible to infer how equilibrium link flows change with the signal setting parameters. Note that

$$\mathbf{dq}/\mathbf{d}\psi = (\partial Z(\mathbf{q}^*,\psi)/\partial \mathbf{q})^{-1}(\partial Z(\mathbf{q}^*,\psi)/\partial \psi) \tag{19}$$

provided that the matrix $(\partial Z(\mathbf{q}^*,\psi)/\partial \mathbf{q})$ is invertible. Regarding the first term of the right hand side of the (19)

$$(\partial Z(\mathbf{q}^*,\psi)/\partial \psi) = \mathbf{I} - \sum_{all\ j} t_j - (\partial \boldsymbol{p}_j / \partial \mathbf{c})\mathbf{J} \tag{20}$$

where p_j is the j^{th} column of the matrix of link choice proportions P.

In the case of logit model as in PFE with sensitivity parameter it may be readily verified that

$$\partial p_{ij} / \partial c_{i'} = \begin{cases} -\alpha p_{ij} + \alpha p_{ij}{}^2 & \text{if } i = i' \\ -\alpha p_{ii'j} + \alpha p_{ij} p_{i'j} & \text{otherwise} \end{cases} \tag{20a}$$

where $p_{ii'j}$ is the proportion of traffic from j^{th} trip table element using both links i and i'

Concerning the second term on the right hand side of (19),

$$(\partial Z(\mathbf{q}^*,\psi)/\partial\psi) = -(\sum_{all\ j} t_j(\partial \mathbf{p}_j/\partial \mathbf{c})(\partial \mathbf{c}/\partial \psi) \tag{21}$$

Eqn. (19) can also be written as:

$$(\partial Z(\mathbf{q}^*,\psi)/\partial\psi) = -(\sum_{all\ j} t_j(\partial \mathbf{p}_j/\partial \mathbf{c}) - \mathbf{P}(\partial \mathbf{t}/\partial \mathbf{y})\mathbf{P}^T \tag{22}$$

where y is a function of c(q, Ψ) and associated with minimum O-D cost.

The evaluation of p_{ij} and $p_{ii'j}$ can be obtained in Bell and Iida [1997a].

Parameter α determines the sensitivity of assignment to path cost. As α increases, the importance of the second part of the objective function increases in solving the lower-level problem. In the limit, the assignment tends to UE. As α tends to zero, the driver preference for lesser-cost paths disappears. Hence the assignment becomes completely random. As α increases $t^T q$ reduces monotonically [Bell et al.; 1997]. For given different values of the α, the GATPFE model can be run until convergence can be found for each value of deterrence parameter. Therefore, the effect on system performance of any change in the perception of travel time (i.e. α) is searched for by way of GATPFE model. As perception parameter, α, becomes smaller, the perception error of travel times increase.

The GA works with the expression operation that is performed based on fitness evaluation. The fitness indicates the goodness of design, and therefore, the objective function is a logical choice for the fitness measure. The fitness function selected in this study is:

$$MaxF(x) = 1/PI(\mathbf{q}^*(\psi),\psi) \tag{23}$$

$$\text{subject to } \psi(C,\boldsymbol{\theta},\boldsymbol{\varphi}) \in \boldsymbol{\Omega}_0 \tag{23a}$$

where F(x) is the fitness function for the GA

Each signal-timing variable is transformed for use in the GA process as follows:

1. *For cycle time*

$$C = C_{\min} + \beta_i \Delta C_i \qquad \text{i=1} \tag{24}$$

where ΔC_i is the precision of cycle time and can be calculated as: $\Delta C_i = \dfrac{C_{i,\max} - C_{i,\min}}{2^{l_i} - 1}$, l_i is the required number of binary digits and β_i is the integer resulting from binary representation of the cycle time. Although a higher degree of precision can be obtained by increasing the string length, it is not always desirable because computational cost of GAs also increases, as the binary string gets longer.

2. *For offsets*

$$\theta = \beta_i \frac{C}{2^{l_i} - 1} \qquad i=2,3\ldots,N_j \tag{25}$$

Mapping the vector of offset values to a corresponding signal stage change time at every junction is carried out as follows:

$$\theta_i = S_{i,j} \qquad i=1,2,\ldots\ldots,N_j;\ j=1,2,..,m$$

3. *For stage green timings*

Let $r_1, r_2, \ldots, r_m$ be the numbers representing by the genetic strings for m stages of a particular junction, and $I_1, I_2, \ldots, I_m$ be the length of the intergreen times between the stages.

The binary bit strings (i.e. $r_1, r_2, \ldots, r_m$) can be encoded as follows first;

$$r_i = r_{\min} + \beta_i \frac{(r_{\max} - r_{\min})}{2^{l_i} - 1} \qquad i=1,2,\ldots,m$$

where r_{min} and r_{max} are set as C_{min} and C_{max} respectively.

Then, using the following relation the green timings can be distributed to the all signal stages as follows second:

$$\phi_i = \phi_{\min,i} + \frac{r_i}{\sum_{k=1}^{m} r_i}\left(C - \sum_{k=1}^{m} I_k - \sum_{k=1}^{m} \phi_{\min,k}\right) \qquad i=1,2,\ldots,m \tag{26}$$

Solution for Combined Assignment and Signal Control

The following bi-level iterative sensitivity algorithm is used to solve the combined TA and ATC problem:

Upper level: Solve the upper level problem (16) for Ψ^* given q, and proceed to lower level.

Lower level: Given Ψ^* fined new SUE link flows, q, and $\mathbf{y}^{\mathrm{T}}(\partial\mathbf{t}/\partial\mathbf{y})(\partial\mathbf{y}/\partial\mathbf{c})(\partial\mathbf{c}/\partial\mathbf{s}) + (\mathbf{c}^T - \mathbf{q}^{\mathrm{T}}\mathbf{J})(d\mathbf{q}/d\psi)$. And proceed to the upper level.

The solution of integrated GA, TRANSYT and PFE, referred to GATPFE, can be seen in Figure 2. As can be seen from Figure 2, the GA provides a feasible set of signal timings both for the traffic assignment process and traffic control. If any given signal timings by the GA violates the constraints, the GATPFE will automatically discards those chromosomes from the pool due to Equations (24)-(26). The equilibrium link flows are decided by the signal timings from the GA and the resulting network performance index is calculated from the resulting equilibrium flows and signal timings.

The algorithmic steps of the iterative sensitivity algorithm can be outlined in the following way.

Step 0. Initialization. Set the user-specified GA parameters that are the population size, probability of crossover and mutation, generation number, number of possibilities per decision variable and reproduction operator. Set the decision variables Ψ as binary strings to form a chromosome x by giving the minimum

ψ_{min} and maximum ψ_{max} specified lengths for decision variables.

Step 1. Generate the initial random population of signal timings X_{tt}; set tt=1

Step 2. Decode all signal timings of X_{tt} and transfer them into feasible signal timings [see for details, Ceylan, 2002], and to map the chromosomes to the corresponding real numbers by using (24)-(26).

Step 3. Solve the lower level problem by way of the PFE. This gives an stochastic equilibrium link flows and update the link travel time function for each link a in L. The link cost function is the sum of free-flow travel time under prevailing traffic condition (i.e. c_a^0) and average delay to a vehicle at the stop-line at a signal-controlled junction over the time slice t (i.e. d_a (t)). By assuming a separable function of q_a, the link cost function is:

$$c_a(q_a, \psi) = c_a^0 + d_a(t)$$

since $d_a(t) = d_a^U + d_a^{ro}(t)$,, the link cost function is expressed as

$$c_a(q_a, \psi) = c_a^0 + d_a^U + d_a^{ro}(t)$$

Note that TRANSYT traffic model provides network performance index for given signal timings. For different initial signal timings, there is a problem of being trapped at bad local optimum due to the non-convex nature of the problem. On the other hand, GA starts with large population base in order not being trapped at bad local optimum (i.e. GA searches globally).

Step 4. Get the network performance index for resulting signal timing at Step 1 and the corresponding equilibrium link flows resulting in Step 3 by running TRANSYT.

Step 5. Calculate the fitness functions for each chromosome x_j

Step 6. Reproduce the population X_{tt} according to the distribution of the fitness function values.

Step 7. Carry out the crossover operator by a random choice with probability P_c.

Crossover probability (denoted by p_c) is defined as the ratio of the number of offspring produced in each generation to the population size. This ratio controls the expected number p_c*pz of chromosomes to undergo the crossover operation. A higher crossover rate allows exploration of more of the solution space and reduces the chances of settling for a bad local optimum, but the higher the crossover rate, the longer the computation time. After the new population has been filled with crossed over members, mutation can take place. Based on previous studies Goldberg [1989] and Carroll [996] set the probability of crossover (p_c) between 0.5 and 0.8. Hence, p_c is selected as 0.5 in this study.

Step 8. Carry out the mutation operator by a random choice with probability P_m, then we have a new population X_{tt+1}.

Mutation probability (denoted by p_m) is a parameter that controls the probability with which a given string position alters its value. The p_m controls the rate at which new genes are introduced into the population for trial; if it is too low, many genes that would have been useful are never tried out; but if it is too high, there will be much random perturbation, the offspring will start losing their resemblance to the parents, and the algorithm will lose the ability to learn from the history of the search. p_m can be set to 1/pz [Carroll, 1996].

Step 9. If the difference between the population average fitness and population best fitness index is less than 5%, re-start population and go to the Step 1. Else go to Step 10.

Step 10. If tt=maximal generation number, the chromosome with the highest fitness is adopted as the optimal solution of the problem. Else set tt=tt+1 and return to Step 2

Numerical Example

The test network is illustrated based upon the one used by Allsop and Charlesworth [1977] and Chiou [1998]. Basic layouts of the network for use in signal settings and in traffic assignment are given in Figure 3a and 3b, where Figure 3a is adapted from Chiou [1998] and Figure 3b and 3c is adapted from Charlesworth [1977]. Allocation for signal stages at each junction is given in Figure 3c. Travel demands for each origin and destination are those used by Charlesworth [1977] and also given in Table 1. Fixed set of input data for undelayed travel time and saturation flow for each link entering each junction are also taken from Charlesworth [1977] and given in Table 2, where we assume that travel times on the non-signal controlled exit links (i.e. turning links) are one second because these values are constant throughout the computation process, and for those links that have greens over the whole cycle the travel times are constant throughout this computation.

The given level of information provided to road users can be identified in Table 3, where different values of α and their corresponding performance index in terms of £/h and veh-h/h are presented in the test network. The application of iterative sensitivity algorithm by way of the GATPFE can be seen in Table 3. It provides the network cycle time, and network performance index when the level of information is provided to road usres.

Generally, there is a tendency of better system performance with less computational demand when α becomes bigger. Road users try to avoid conflicting links when α is bigger. This tendency can be seen in Table 2a. For example, the flows on link 1 are increasing, whilst the flows in link 2 are decreasing when α is bigger. For example, when the level of information increases, drivers avoid junctions 5 and 6 in the test network in South-North (S-N) direction.

During the application of the GATPFE, all the user-specified GA parameters are kept fixed, and if there is no improvement on the last 20 generations, then the GATPFE is assumed to be converged [see for convergence details, Ceylan 2002]. The GA parameters used during analysis are: pz=40, p_m=0.25, uniform crossover p_c=0.5, number of possibilities per parameter 256 (l_i=8).

In Table 4a and Table 4b, the final value of equilibrium link flows and the corresponding final value of degree of saturation are given for various α. Note that the degrees of saturation is greater than 90% for none of the links.

Table 5 shows the different values of dispersion parameter against the start of stage times and corresponding green times for each stage. As can be seen from Table 5, for each value for the α the signal settings are changed.

Table 1 Origin-Destination demand for the test network in vehicles/hours

Origin/Destination	A	B	D	E	F	Origin totals
A	--	250	700	30	200	1180
C	40	20	200	130	900	1290
D	400	250	--	50*	100	800
E	300	130	30*	--	20	480
G	550	450	170	60	20	1250
Destination totals	1290	1100	1100	270	1240	5000

* where the travel demand between O-D pair D and E are not included in this numerical test which can be allocated directly via links 12 and 13

Conclusions

This study investigates the sensitivity of the perceived travel time error and its corresponding affects to the signal timings. Traffic is assigned to paths according to a logit path choice model on the basis of the path travel times and the dispersion parameter. For set of links, network performance index (PI) is calculated so as to achieve equilibrium between demand and capacity. The dispersion parameter governs the degree of dispersion across alternative paths. There is a preference for the cheaper path but some traffic would opt for the more expensive path, perhaps out of ignorance at SUE. The problem is formulated as a bi-level problem and iterative sensitivity algorithm is used to solve the problem by way of GATPFE. The following conclusions can be drawn from this study.

It was found that when the sensitivity parameter α is bigger, the performance index is lower and there is less computation demand. Hence, system performance is improved. The GATPFE model showed good progress in terms of convergence when α is less than 1. Because the

Table 2 Fixed data for the test network

Junction	Link	c^0	s	Junction	Link	c^0	s
1	1	1	2000	4	5	20	1800
	2	1	1600		6	20	1850
	16	10	2900		10	10	2200
	19	10	1500		11	1	2000
					12	1	1800
					13	1	2200
2	3	10	3200	5	8	15	1850
	15	15	2600		9	15	1700
	23	15	3200		17	10	1700
					21	15	3200
3	4	15	3200	6	7	10	1800
	14	20	3200		18	15	1700
	20	1	2800		22	1	3600

where c^0, s are the free-flow travel time in seconds and saturation flows in vehicles/hour, and for entry links to the network one second default travel time given

α becomes smaller the perception error on travel time becomes in effect higher and the assignment becomes more stochastic. When α is bigger, the SUE approaches the UE case.

When the level of information is increased, the link flows on non-conflicting paths (hence links) tends to increase, whilst the link flows on conflicting paths are decreased. For example, the path flow on South-

North direction using junction 5-6 decreases whilst the flow on the path, which uses junction 2 and 3 increases. When α varies between 0.01 and 1.0, there is not much change in the link flows.

Table 3 Sensitivity parameter changes on drivers' perception of travel time

Sensitivity Parameter (α)	Generation Number	Cycle time seconds	Performance Index £/h	veh-h/h
0.01	120	76	719.0	75.9
0.1	120	73	719.3	75.8
1.0	75	77	712.5	75.4
10.0	75	66	709.0	74.6
25.0	60	75	700.1	73.8
50.0	50	64	689.4	71.7

Table 4a Final values of equilibrium link flows resulting from the GATPFE for different values of the sensitivity parameter in veh/h

α	q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_9	q_{10}	q_{11}	q_{12}
0.01	714	465	714	578	635	173	464	479	120	480	499	250
0.1	714	465	714	578	635	173	464	479	120	480	499	250
1.0	716	463	716	579	636	173	462	478	120	479	499	250
10.0	743	436	743	591	672	169	436	450	139	451	482	267
25.0	783	396	783	621	687	166	396	424	132	435	485	264
50.0	811	368	811	648	723	167	368	397	133	398	488	261

α	q_{13}	q_{14}	q_{15}	q_{16}	q_{17}	q_{18}	q_{19}	q_{20}	q_{21}	q_{22}	q_{23}
0.01	450	788	789	664	410	351	626	1290	1058	1250	840
0.1	450	788	789	664	410	351	626	1290	1058	1250	840
1.0	450	789	790	663	409	350	625	1290	1057	1250	837
10.0	450	778	798	626	421	341	661	1290	1020	1250	777
25.0	450	787	803	602	411	336	686	1290	1040	1250	737

50.0	450	799	821	605	399	317	682	1290	1024	1250	719

Table 4b Final value of degree of saturation for different values of α (%)

α	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
0.01	4	53	42	53	57	36	63	50	38	79	79	22
0.1	4	53	43	53	57	36	63	51	40	80	79	22
1.0	4	54	44	54	57	34	62	51	39	80	80	22
10.0	4	47	45	55	60	38	67	46	41	80	72	23
25.0	4	41	44	56	60	32	66	45	39	78	76	24
50.0	4	40	46	65	61	30	59	42	41	83	78	25

α	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}
0.01	78	72	58	67	87	51	76	83	87	71	71
0.1	78	72	58	67	88	50	76	84	86	72	71
1.0	75	73	60	63	88	50	78	82	85	72	67
10.0	84	73	60	71	91	55	77	84	91	67	67
25.0	73	71	55	71	86	59	76	84	87	62	69
50.0	69	80	58	70	88	54	79	82	89	65	68

The signal timings are changed for different values of α. This shows that the level of information provided also affects the signal timing optimization on a signalized road network that the signal settings on equilibrium network design problem should be re-optimized.

The effect on small changes on the O-D demand is not taken into account in this study. This is out of the scope of this study. Future work should be on the affect of the small changes of the demand to an SUE equilibrium link flows as well as signal timings.

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Table 5 Start of green timings for various values of α changes in seconds

	Junction 1 Start of green		Junction 2 Start of green		Junction 3 Start of green		Junction 4 Start of green		
	Stage 1	Stage 2	Stage 1	Stage 2	Stage 1	Stage 2	Stage 1	Stage 2	Stage 3
0.01	39	69	18	62	52	22	5	33	58
0.1	40	69	20	62	53	24	10	37	61
1.0	0	32	59	25	13	60	44	72	20
10.0	32	56	10	48	41	15	0	26	47
25.0	34	60	5	51	40	10	68	21	44
50.0	19	42	15	54	45	21	5	29	47

	Junction 5 Start of green			Junction 6 Start of green	
	Stage 1	Stage 2	Stage 3	Stage 1	Stage 2
0.01	26	44	69	0	35
0.1	30	47	71	0	34
1.0	64	5	30	47	6
10.0	20	37	59	2	30
25.0	15	33	58	62	16
50.0	20	36	57	58	20

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Initialisation and input

OD $\leftarrow$ user supplied $\forall w \in \boldsymbol{W}$

$q_a \leftarrow 0, \ \forall a \in \boldsymbol{L}$

$c_a \leftarrow c_a(q_a), \ \forall a \in \boldsymbol{L}$

n $\leftarrow$ 0, iteration counter

Repeat

Update link costs

$$n \leftarrow n+1$$

$$c_{an+1} = (1/n)c_a(q_a) + (1-1/n)c_a \quad \forall a \in \boldsymbol{L}$$

Build fastest paths and store new paths in link/path incidence matrix

$$\delta_{ap} \leftarrow 1 \text{ or } 0, \ \forall a \in \boldsymbol{L} \text{ and } \forall p \in \boldsymbol{P}_w$$

Calculate new path costs

$$g_p = \sum_{a \in L} \delta_{ap} c_a, \ \forall p \in \boldsymbol{P}_w$$

Calculate new path flows

$$h_p \leftarrow t_w \frac{\exp(-\alpha g_p)}{\sum_{p \in \mathbf{P}_w} \exp(-\alpha g_p)} \quad \forall p \in \boldsymbol{P}_w, \ \forall w \in \boldsymbol{W}$$

Calculate new link flows

$$q_a \leftarrow \sum_{p \in \mathbf{P}_w} \delta_{ap} h_p \qquad \forall a \in \boldsymbol{L}$$

Until no new paths and link flows converged

Output

Path flows	$h_p, \ \forall p \in \boldsymbol{P}_w$
Path travel times	$g_p, \ \forall p \in \boldsymbol{P}_w$
Link flows	$q_a, \ \forall a \in \boldsymbol{L}$
Link travel times	$c_a, \ \forall a \in \boldsymbol{L}$

Fig. 1 The algorithm steps of the PFE

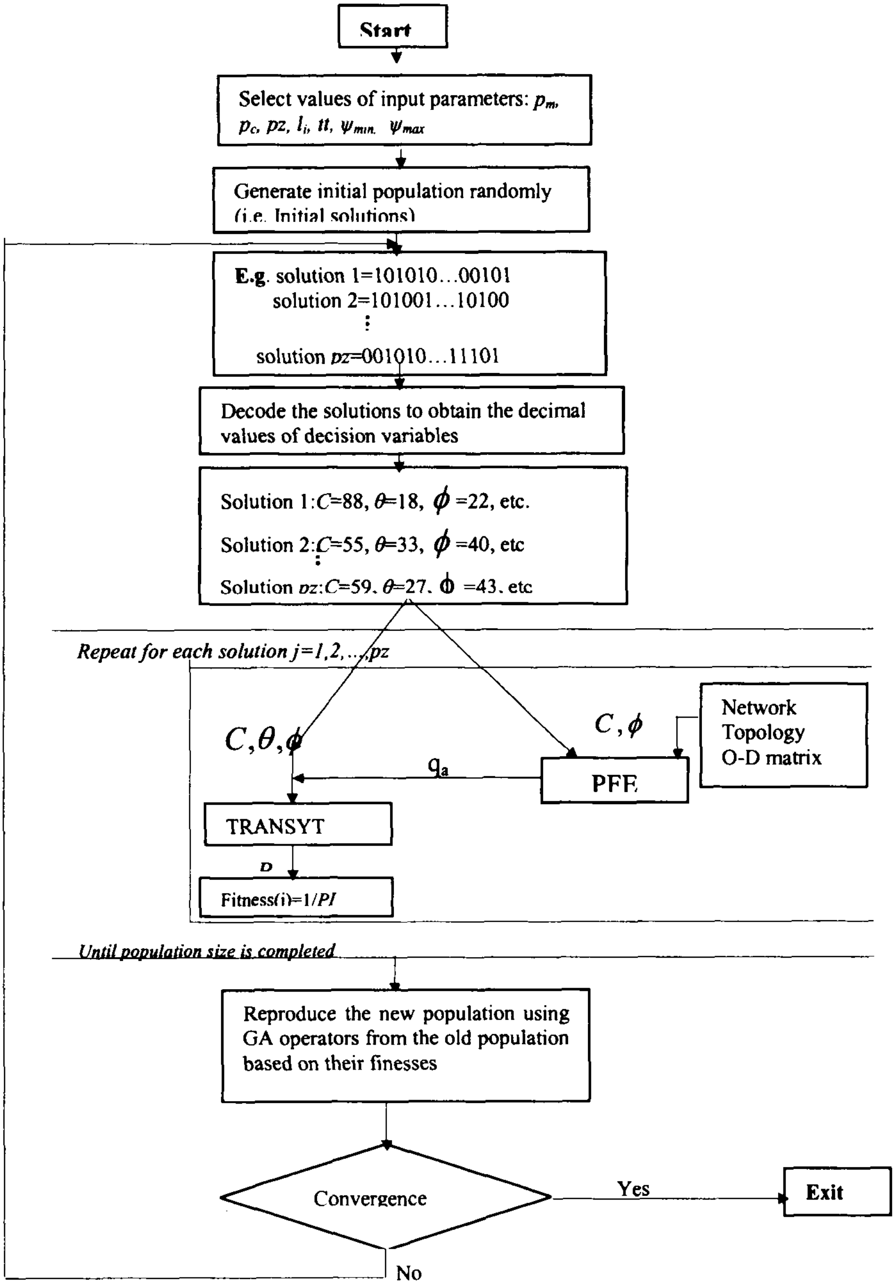

Fig. 2. The GATPFE model

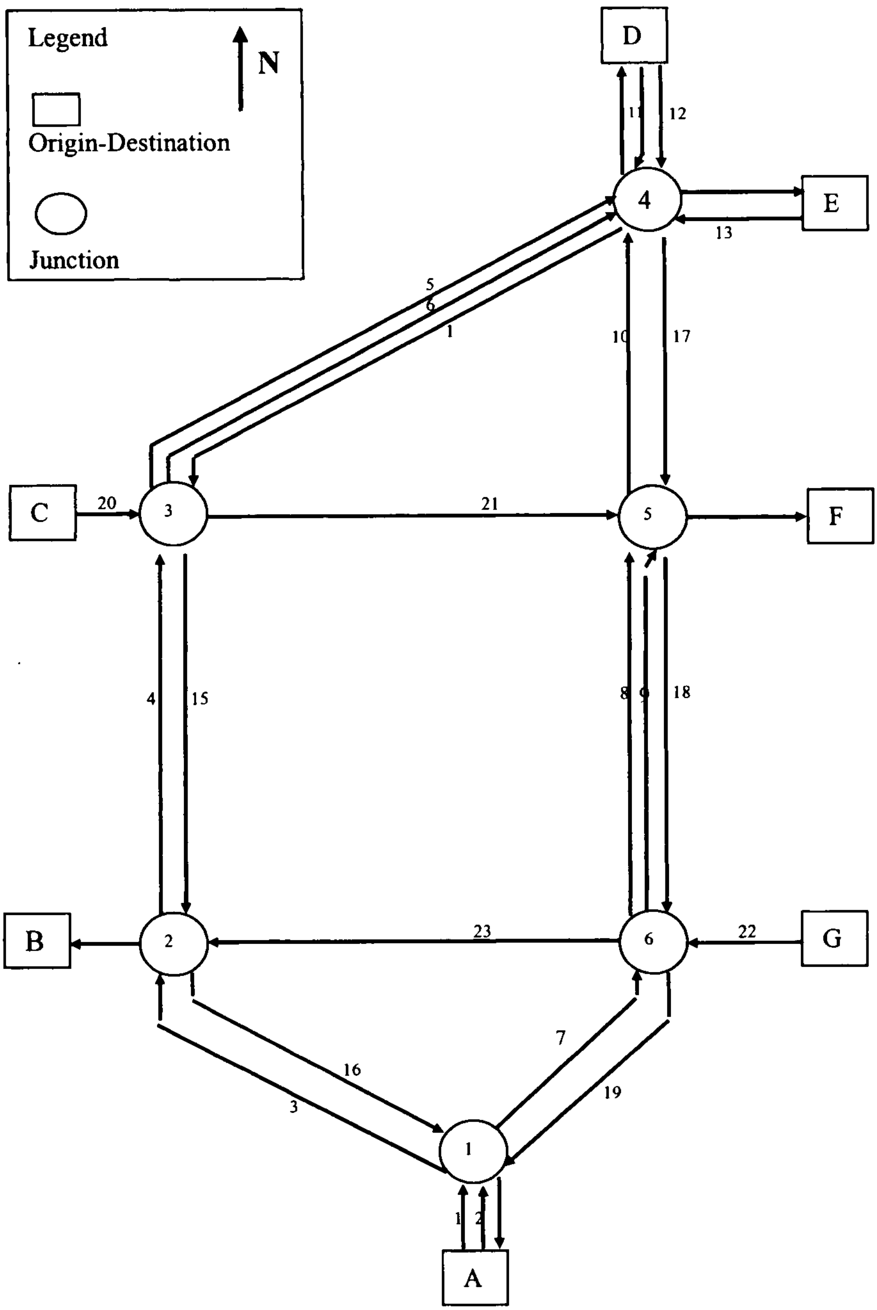

Fig. 3a Layout for the test network

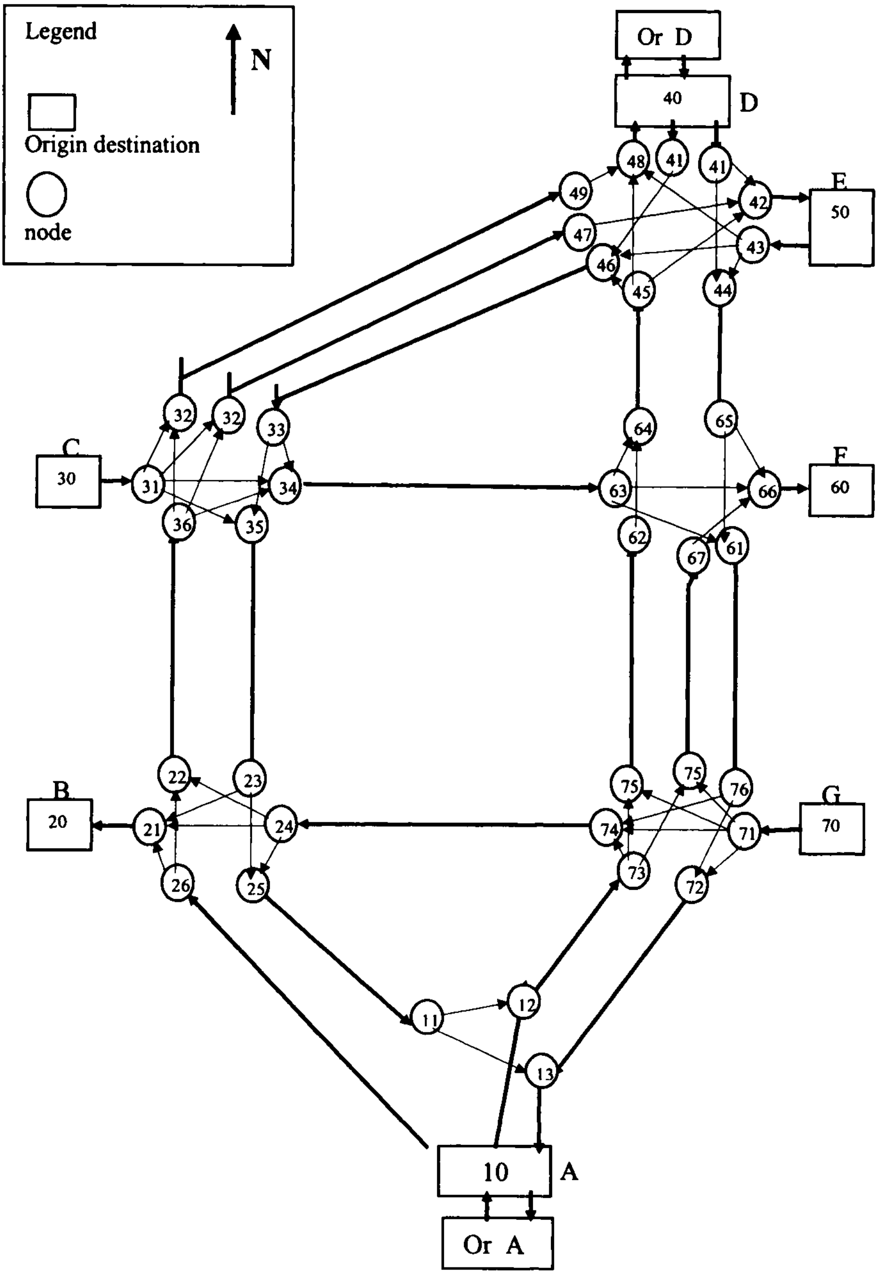

Fig. 3b Representation for traffic assignment use of nodes and links for test network

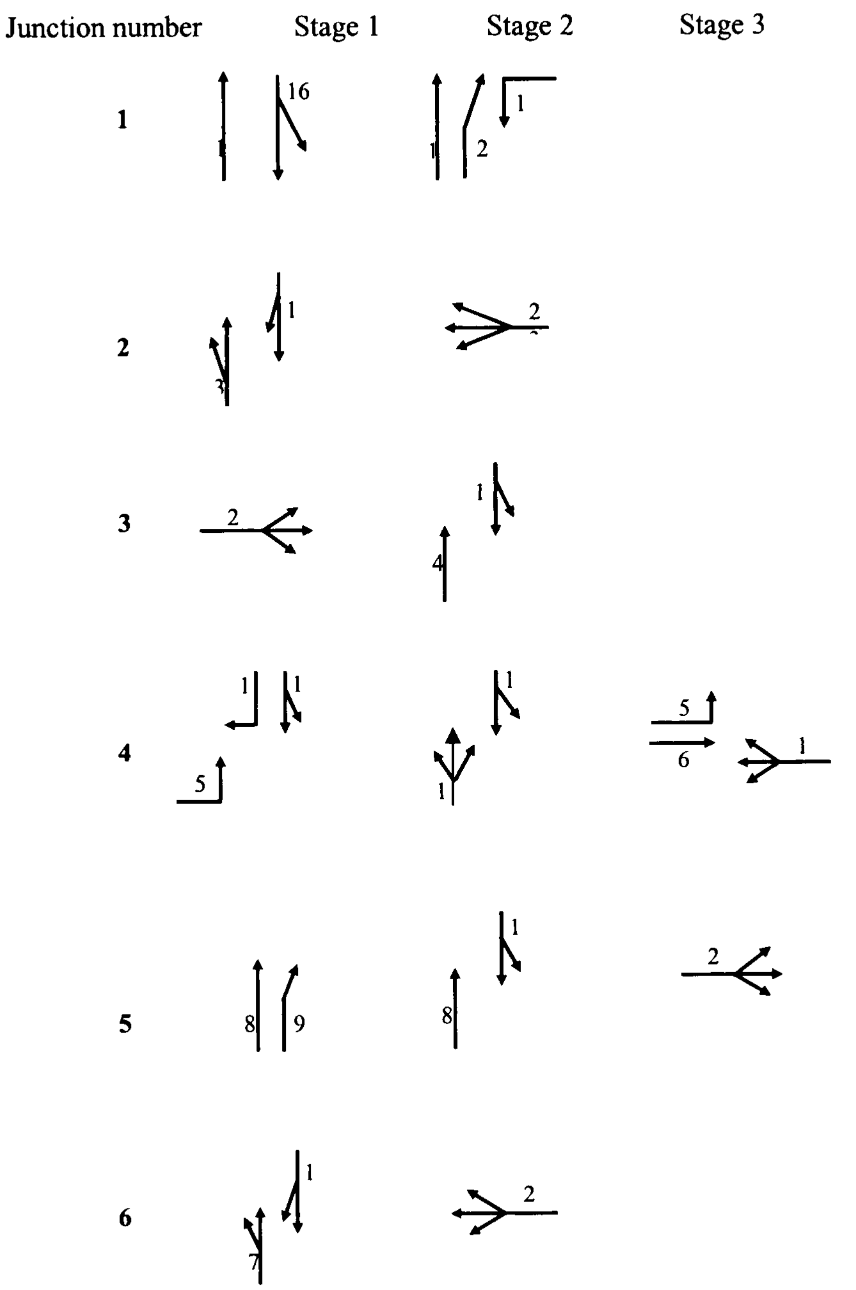

Fig. 3c Stage configurations at six junctions of the test network