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Preservice Mathematics Teachers' Understandings of The Class Inclusion Between Kite and Square

Emine Gaye ÇONTAY*, Asuman DUATEPE PAKSU

[a]Pamukkale University, Faculty of Education, Denizli, 20070, Turkey

Abstract

The aim of this study is to examine the preservice mathematics teachers' understanding of class inclusion between kites and squares with the framework of Van Hiele levels. This descriptive study was conducted with 5 sophomore preservice teachers in Turkey. When we look at all the responses to the questions in terms of Van Hiele geometry thinking levels, it can be said that only one preservice teacher understood class inclusion relations and most of the preservice teachers were not at the expected level.

Keywords: *definition, class inclusion, kite, preservice teachers, Van Hiele,*

1. Introduction

Understanding class inclusion is the ability to have an overview of the possible relationships that exist among figures and this ability contributes students' deductive reasonings (Currie & Pegg, 1998). Hierarchical class inclusion is economical for example if a condition is true for parallelograms; it is also true for rhombuses, squares and rectangles (Fujita & Jones, 2007). It also helps to define the concepts more economically, helps to formulate the theorems to form more special concepts and contributes to problem solving processes and serves a global perspective (de Villers, 1994).

2. Related Literature

According to Fuys and others (1988), at Van Hiele Level 0, students perceive the shape of a kite as a whole, they can identify, name and compare it by its appearance. At this stage they solve the problems by operating on shapes visually rather than by using properties but they can not analyze figures in terms of their components and they can not make generalizations. At Level 1; students know components of a kite and they can analyze the relationships between these components and describe kites in terms of their

* Emine Gaye ÇONTAY. Tel.: +902582961165 fax: +902582961200.
E-mail address:gayeermec@gmail.com.

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characteristics. However they can not explain how certain properties of a figure are related and they can not understand class inclusions. At Van Hiele Level 2, students can identify minimum sets of the properties of a kite. They understand class inclusions as well as they can formulate a definition of a kite. At this stage they can give informal deductive arguments but they can not understand the meaning of these deductions in axiomatic sense. At Van Hiele Level 3, students can understand the properties of a formal definition and prove the axiomatic relations which were informally explained at Level 2 and they can give formal deductive arguments. At Van Hiele Level 4 students can rigorously establish theorems in axiomatic systems and they can compare these systems.

When we look over the Van Hiele Theory, it is clear that the development of the relationships between the characteristics of the figures, class inclusion, and formal definition understanding occur at Van Hiele Level 2 (Currie & Pegg, 1998; de Villers, 1998).

Many studies say that students have problems in classifying the quadrilaterals (Currie ve Pegg, 1998; de Villers, 1994; Erez & Yerushalmy, 2006; Fujita & Jones, 2006, 2007; Monaghan, 2000).

Viglietti (2011) who conducted a study with mathematics teachers, found that teachers had imperfect knowledge of basic plain figures and kite was one of the subjects which they had many problems with. He found that most of the responses of teachers were based on physical appearances of the figures. Guiterrez and others (1991) examined Van Hiele geometric thinking levels of students and found that a student could have reasoning at two sequential levels at the same time.

The aim of this study is to examine the preservice mathematics teachers' understanding of class inclusion between kites and squares with the framework of Van Hiele levels.

3. Methods

This study was conducted with 5 sophomore preservice teachers from Elementary Mathematics Education Department at a state university in Aegean Region in Turkey. All of the participants got highest grade in geometry course (A1).

The instrument used in the clinical interview was formed by getting inspired from the study of Fuys and others (1988). The clinical interview has two parts. For the first part, the cardboards on which "kite" and "non-kite" was written were placed on the table in front of the preservice teachers and different quadrilaterals were given to them one by one at the same sequence. Then they were asked whether it was a kite or not. The quadrilaterals that were given to preservice teachers were; 1) unspecific quadrilateral, 2) kite, 3) trapezium, 4) square, 5) parallelogram, 6) rectangle, 7) unspecific quadrilateral Mainly in this part, preservice teachers were faced with the following questions:

1) Which of these are kites? (They were provided 7 different quadrilaterals and they were asked to put these into kites group or non-kites group)

1a) Why did you put square into the kites group?

1b) Why did you put square into non-kites group?

1c) Why did you put rectangle into the kites group?

1d) Why did you put rectangle into non-kites group?

The quadrilaterals which were given to preservice teachers and the true groupings are shown in Figure 1.

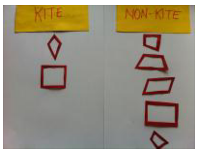

Fig. 1. Quadrilaterals and correct groupings

In the second part of the interview, the preservice teachers were given name cards for square, quadrilateral and kite and "is a special" arrows and they were asked if they could arrange these name

cards correctly. Then, the preservice teachers were asked if a square was a kite and they were asked to explain the reason. Particularly, the following questions were asked:
2i) Can you put arrows between these cards to show some relationships?
2ii) Is a square kite, Why?
The example of the relationship is shown below:

Fig. 2. The correct arrangement of square, quadrilateral and kite

Before the main study, a pilot study was conducted with 3 preservice teachers. All of the grades of preservice teachers were high in the geometry course (A2). After the clinical interviews of the pilot study, minor changes were done.
After the transcription of the clinical interview, the answers of all preservice teachers were read question by question and the common expressions were grouped. Then, the answers were analyzed with the framework of Van Hiele levels. For the confidentiality, preservice teachers are referred as PT1, PT2, PT3, PT4, PT5 instead of their real names.

4. Findings and Discussion

4.1. Findings and Discussion of the responses of the first question

For the first question of the clinical interview, the preservice teachers were given different kinds of quadrilaterals and they were asked to put these into kites or non-kites group. Four of the preservice teachers (PT2, PT3, PT4 and PT5) grouped all of the quadrilaterals correctly while one of them (PT1) made mistakes. PT1 said that the square (number 4) was not a kite while the quadrilateral (number 7) was a kite. PT1 grouped quadrilaterals as kites and trapezoids. She also said that, it was enough to claim a quadrilateral as a trapezoid if at least two sides of it were parallel and added that this was the reason why she grouped square as a trapezoid.
When PT4 was asked why she put square into kites, she changed her mind and put square into non-kites group. As a result, the number of preservice teachers who answered correctly decreased to 3 (PT2, PT3, PT5) while it was 4 (PT2, PT3, PT4, PT5) at first.
When the answers of the preservice teachers were analyzed in a detailed manner, the preservice teachers who gave correct answers (PT2, PT3, PT5) gave responses which can be classified as Van Hiele Level 1.
Only one preservice teacher (PT3) who grouped square correctly didn't give visual expressions. This preservice teacher didn't give explanations on appearance of the shapes for her other responses, either.
PT3: "Because these sides are equal namely two consecutive sides are equal, in here the same thing, too, that's the reason"
When the explanations of the preservice teacher [(PT4) who made groupings correctly at first and then said that a square wasn't a kite] were examined in a detailed manner, it was seen that the reason for the first decision of her depended on the idea of "*square is a special form of a kite*"
PT4: "Square…..I know it as being a kite's special form. Square and rhombus….so I put it into kites group"
When PT4 was asked about the reason for doing like that, she made correct explanations while thinking visually and put square into wrong group. The reason for having a true explanation (*square is a special form of kite*) at first and for having a wrong answer with having a true explanation (*a figure with two*

isosceles triangles combining at bases…) can be stemmed from rote memorization and she might not think deeply because of the visual memory in her mind.

In addition to the grouping error she made throughout the interview, PT1 gave visual expressions and besides she said that a square wasn't a kite. According to her it was a trapezoid because kite's two sides weren't parallel to each other.

PT1: "T*wo isosceles triangles coinciding in the bases first when we separate them like this* (shows on kite) *this these reciprocal sides aren't parallel so I say that square is not a kite it is a trapezoid because of parallelism.*

The other preservice teacher who made incorrect groupings (PT4) gave visual explanations, besides she cut kite into pieces (instead of cutting square into pieces) and told that angles of kite weren't 90° and angles had to be perpendicular and so. This preservice teacher also explained kite in terms of a square like the other who had incorrect groupings.

PT4: When all side lengths of kite are equal….the measure of these angles doesn't change… …again it is formed of two isosceles triangles coinciding in the bases. But these isosceles triangles then become congruent triangles but the angles of these don't become 90° any more. All the angles must be 90° for the square. This property must exist in square…..this figure (square) doesn't convert into here (kite)…."

PT1 and PT2 used visual expressions in their explanations (indicator of being at Van Hiele Level 0). Besides, they mentioned about the properties as they were in Van Hiele Level 1 in addition to their grouping errors. However, the properties they stated were not correct. So, it can be said that these preservice teacher s were in Van Hiele Level 0 to 1.

As it is seen, both of the preservice teachers who put square into the incorrect group, perceived kite as being a figure which is formed of a combination of two isosceles triangles, and also tried to explain kite in terms of a square.

From this point of view, it can be said that the preservice teachers gave definitions visually as at Van Hiele Level 0 while having insufficient explanations. These visual explanations weren't enough for them and so they were in need of reflection with additional features. It was seen that these preservice teachers made mistakes in making class inclusions.

A prompt then were given to PT4 about the difference between a kite and a square, the preservice teacher arranged the characteristics of the two figures in order after thinking in terms of differences and found the true answer by putting the square into kites group with saying that a square was a special form of a kite (with using class inclusion).

PT4: "Hmmm..the difference between a kite and a square…. all the sides are equal like we see here….there is no difference…also side is longer…in square angles of side must be 90°. In kite, there is no such provision. In other words we can say that a square is a special form of a kite…but I've thought in terms of angles of 90° shortly before. It is true when we think of a triangle but when we go to quadrilateral……hmmmm…the angles must be 360°..It could be 90° then I've forgotten the detail. A short time ago I thought of that angles…..I thought these angles as wide angles….these base angels combining angles…but these…hmmmm aren't wide angles they can be 90°"

Interviewer: How did you know that?

PT4: I knew.….when we look…from…the figure misguide when we look at the figure we think that these can be wide angles but when we divide 90° into two when we think here like 45-45 when we think about a square when we divide 90° into two precisely because diagonal lenth of this ….from here….I concluded this from the square"

As seen when the preservice teacher examined the two figures in terms of angles and sides besides their appearances she gave the correct answer. The preservice teacher deduced that square was a special form of a kite when she examined the difference between a square and a kite in terms of sides and angles. She justified her deduction with visual explanation by dividing the square into two isosceles triangles and by thinking the square visually. Thus, it can be said that when preservice teachers think in terms of class incusion characteristics besides visualization, they can change their inaccurate deductions to correct ones.

Besides, both of the preservice teachers who gave wrong answers to the question of "Is a square a kite?" (PT1 and PT4), mentioned that they had doubts about their explanations:

PT4: "Hmmm…this was an isosceles triangle….from isosceles triangle… I remember like that…I can remember wrong….When we equalize the lengths kite resolved to square as far as I remember…but…can't rightly say but….I know like that

Interview: Are you sure or not?

PT4:"I am not sure. I am not sure. But I remember like that.

Hence, it can be deduced that the explanations of these preservice teachers which are at Van Hiele Level 0 to 1 are hearsay, not contemplated and rote. Furthermore it can be said that these explanations doesn't canalize them to reflect on. The explanations of the preservice teachers were about the kite's form of two isosceles triangles and made wrong statements and they said that they had never thought on kite with this respect before.

All of the preservice teachers placed the rectangle into non-kites group correctly. One of the preservice teachers (PT1) had a wrong judgment and said that rectangle was not a kite because of being in the kites group. PT1 grouped quadrilaterals in two: trapezoids and non trapezoids and put rectangle into non kites group because of being in the trapezoid group and because it had at least two parallel sides.

PT1: "Because again it has two parallel sides like square, it resembles to trapezoid. In any case I put square, rectangle, rhombus in subclass of trapezoid …. Because all of them are trapezoids I say that it is not a kite"

Although PT1 could not make correct grouping, she answered the question correctly. Her explanations were rote and at Van Hiele Level 0 to 1.

All of the other preservice teachers stated that since consecutive sides are not equal rectangle is not a kite. They also mentioned about triangles forming the kite.

When we think that the inequality of consecutive sides is a sufficient explanation to claim the rectangle as not a kite, it can be said that preservice teachers gave unnecessary visual explanations. Hence it can be mentioned that preservice teachers had a tendency to deduce relying on visual explanation instead of considering the relationships between the characteristics.

When class inclusions about square and rectangle are discussed together, it can be said that preservice teachers who made mistakes in grouping square (PT1 and PT4) thought and deducted visually besides their wrong explanations and the preservice teachers (PT2, PT3, PT4, PT5) needed visual deduction when grouping the rectangle

4.2. Findings and Discussion of the responses of the second question

When the preservice teachers were given name cards of square, quadrilateral and kite and "is a special" arrows, only one preservice teacher (PT5) could arrange the name cards correctly at once. One of the remaining four (PT1) made a wrong arrangement, the remaining three (PT2, PT3, PT4) could reach the true answer after a two or more attempts (PT2 after 3 attempts, PT3 and PT4 after 4 attempts) with the help of the prompts of the interviewer.

Although PT2, PT3 and PT4 put square into kites group before, they couldn't group square as a special kite or a special quadrilateral at baptism. The reason for three of four preservice teachers who put the square into kites group making wrong arrangements or making the true arrangements after two or more attempts at baptism can be an indication of the notion that thinkings of these preservice teachers about class inclusion may be conventional.

When the preservice teachers were asked if every square was a kite or not, except for the only preservice teacher who didn't put square into kites group (PT1), all of the preservice teachers were in the same idea of that every square was a kite.

PT1 made incorrect groupings as putting square into trapezoids group and therefore said that a square wasn't a kite. The only preservice teacher who couldn't arrange the name cards correctly (PT1) was the only preservice teacher who answered the question at Van Hiele Level 0 to 1. So, it can be said that preservice teachers at visual to analysis level are not capable in understanding class inclusion relationships.

PT1: "This (shows square which is figure of number 4) is in trapezoid group of quadrilaterals. This (shows kite which is figure of number 2)is in the group of kites. Every square can't be a kite.

When the responses of the preservice teachers who said that every square was a kite examined, it was seen that three preservice teachers (PT2, PT3, PT4) were at Van Hiele Level 1, one preservice teacher was at Van Hiele Level 2 (PT5).

Three preservice teachers who gave explanations in terms of sides (PT2, PT4 and PT5) besides these characteristics, used the characteristics of the equality of angles and visual expressions as two isosceles triangle combining a square to explain that every square was a kite. In other words, these three preservice teachers needed to talk about the relations of angles and sides in addition to visual expressions in order to explain that every square was a kite

PT2: "We usually draw kite. See… there are two isosceles triangle… we draw like this but if these triangles are equal to each other… if its sides, angles are equal, hmmmm if everything is equal I mean if it is composed of right isosceles triangle thus and if I coincide them in bases then I can obtain a square"

PT4: The important thing for us was that two isosceles triangle combining in bases in kite. Square always provides this condition because every side length is equal every time it is equal I mean we can get two isosceles triangles when we combine its diagonals. Hmmm the base angles are then equal every time because of the angles because it divides the base angles in 45-45. Thus square can always be a kite.

Two preservice teachers who said that every square was a kite (PT2 and PT5) had visual expressions besides angle-side expressions and said that every kite wasn't a square but every square was a kite.

5. Conclusion

When we look at all the responses to the questions in terms of Van Hiele geometry thinking levels, it can be said that two preservice teachers (PT1 and PT4) were at Van Hiele Level 0 to 1, two preservice teachers were at Van Hiele Level 1 (PT1 and PT3), one preservice teacher was at Van Hiele Level 2 (PT5). Thus, congruent with (Fujita & Jones, 2007) it can be mentioned that preservice teachers in the study are good at class inclusion relations of kite and square and have problems about class inclusion processes of quadrilaterals.

The preservice teachers who put square into kites group were at Van Hiele Level 1 and 2, while the others who put square into non-kites group were at Van Hiele Level 0 to 1. Two of the preservice teachers who grouped square in the wrong way thought visually and cut the kite into pieces instead of doing this for square and sought its characteristics in the square.

The reason for doing this may be that they couldn't think of the characteristics of the figures at Van Hiele Level 0 to 1, despite they know the figure and they couldn't analyze the figure in terms of its components and they couldn't make generalizations about the figures (Fuys and others, 1988).

It can be said that the preservice teachers who gave correct answer to the question of classifying of the square didn't understand class inclusion relations and couldn't define kite in terms of its components.

It was seen that the preservice teachers' responses can be labeled between Van Hiele Level 0 and 2 in a fluxinal manner. We can say that congruent with the suggestions of Guiterrez and others (1991), responses of the same people can be varied according to task.

It can be mentioned that only one preservice teacher (PT5) understood class inclusion relations and most of the preservice teachers didn't reach the expected level.

When we think of the preservice teachers who got highest grade (A1) in geometry course, the unsufficiency of their responses may come from the relatively complex nature of kite. It could be harder for them to think about kite than square and rectangle. Besides, congruent with Viglietti (2011)'s study the responses of most preservice teachers were based on physical appearances of the figures

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