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Investigating Acceptable Level of Travel Demand before Capacity Enhancement for Signalized Urban Road Networks

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ABSTRACT

Increasing travel demand in urban areas triggers traffic congestion and increases delay in road networks. In this context, local authorities that are responsible for traffic operations seek to strike a balance between traffic volume and capacity to reduce total travel time on road networks. Since signalized intersections are the most critical components of road networks in terms of safety and operational issues, adjusting intersection signal timings becomes an effective method for authorities. When this tool remains incapable of overcoming traffic congestions, authorities take expensive measures such as increasing link capacities, lane additions or applying grade-separated junctions. However, it may be more useful to handle road networks as a whole by investigating the effects of optimizing signal timings of all intersections in the network. Therefore, it would be useful to investigate the right time for capacity enhancement on urban road networks to avoid premature investments considering limited resources of local authorities. In this study, effects of increasing travel demand on Total Travel Cost (TTC) is investigated by developing a bi-level programming model, called TRAvel COst Minimizer (TRACOM), in which the upper level minimizes the TTC subject to the stochastic user equilibrium link flows determined at the lower level. The TRACOM is applied to Allsop and Charlesworths' network for different common origin-destination demand multipliers. Results revealed that TTC values showed an approximate linear increase while the travel demand is increased up to 16%. After this value, TTC showed a sudden spike although the travel demand was linearly increased that means optimizing signal timings must be supported by applying capacity enhancement countermeasures.

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1. INTRODUCTION

Travel demand has substantially increased during recent years as a consequence of fast growing population and correspondingly generated mobility need especially in developing countries. Since a developing country can be defined as a country with less developed industrial base and less gross domestic product per capita relative to developed countries, urban road network management is a lot more essential for these countries. As developing countries have limited financial resources, they are required to efficiently use their resources especially in urban road network management in which investments made to improve road networks performance are quite expensive. A general view in urban road network management is that the local authorities generally try to take some countermeasures such as increasing link capacities, improving junctions, applying grade-separated junctions etc. on a budget basis to cope with increasing travel demand over time. However, many authorities apply these measures after they came across serious problems in terms of traffic congestion and related issues. Since authorities cannot properly predict the influence of increasing travel demand on the performance (e.g. total travel cost) of road networks, investments to overcome such problems cannot be made timely. Moreover, from the point of steering investment decisions, some investments made to increase network's performance may be unnecessary because authorities are already able to overcome such problems by applying some cost-efficient measures (e.g. setting signal timings) even in case of increasing travel demand. At this point, local authorities need to have information in order to decide whether they make expensive infrastructure investments or use suitable signal timings to overcome problems revealed with increasing travel demand. In other words, authorities can make their investment decisions more accurately if they know how much longer transport supply will respond to the increase in travel demand by applying suitable signal timings. As known, signalized intersections are designed in order to reduce overall cost in a road network and to ensure intersection safety as well. However, using inappropriate signal timings leads to increasing total travel cost in the network and to reducing its capacity. Thus, the problem of finding appropriate signal timings is an important issue for local authorities who is responsible for traffic management. Many researchers have investigated this problem with regards to different perspectives as given in Table 1 starting with almost last half century.

As can be seen in Table 1, studies on traffic signal timing optimization have started with well-known work presented by Webster [1]. Until the mid-1990s signal timing optimization had been carried out by using mathematical methods generally aiming to minimize delay. However, from the beginning of the 2000s, heuristic algorithms started to be used in signal optimization due to their successful applications especially in optimization field and to eliminate negative effects of derivatives of complex mathematical expressions used for optimizing signal timings. Since that time, various objective functions have been used instead of minimizing delay. In recent years, one of the most important issues is the selection of objective function to optimize signal timings in a road network, more properly. In this study, total travel cost (TTC) as an objective function has been selected in order to optimize signal timings. It is obvious that link cost function should be firstly defined to find the TTC in a given road network. For this purpose, the link cost function is presented as the sum of free-flow travel time, average uniform delay, and average random plus oversaturation delay per vehicle in order to define a network travel cost closer to the reality. Another reason for the use of the TTC is that the network-level solution of traffic congestion problem may be more effective in comparison with junction-level solution in which authorities try to solve the problem only for considered single junctions. This approach may increase the network congestion level instead of reducing.

Table 1 - Studies on traffic signal timing optimization

Study	Method	Objective function
Webster [1]	Mathematical methods	Minimizing delay
Allsop [2]	Convex programming	Minimizing delay
Allsop [3]	Linear programming	Maximizing capacity
Wong [4]	Approximate expressions	Minimizing delay
Heydecker [5]	Decomposition approach	Minimizing performance index
Wong [6]	Non-linear programming	Minimizing performance index
Wong [7]	Parallel computing	Minimizing performance index
Wong et al. [8]	Heuristic algorithm	Minimizing performance index
Girianna and Benekohal [9]	Heuristic algorithm	Maximizing number of vehicles
Ceylan and Bell [10]	Heuristic algorithm	Minimizing performance index
Ceylan [11]	Heuristic algorithm	Minimizing performance index
Chen and Xu [12]	Particle swarm optimization	Minimizing performance index
Dan and Xiaohong [13]	Genetic algorithm	Minimizing delay
Li [14]	Cell transmission model	Maximizing number of vehicles
Liu and Chang [15]	Genetic algorithm	Minimizing travel time
Ceylan and Ceylan [16]	Hybrid heuristic algorithm	Minimizing performance index
Dell'Orco et al. [17]	Harmony Search algorithm	Minimizing performance index
Dell'Orco et al. [18]	Bee colony algorithm	Minimizing performance index
Ozan et al. [19]	Reinforcement learning	Minimizing performance index
Christofa et al. [20]	Mixed integer programming	Minimizing person based delay
Srivastava and Sahana [21]	Evolutionary algorithm	Minimizing total wait time
Abdul Aziz et al. [22]	Reinforcement learning	Minimizing average delay

The main objective of this paper is to investigate the acceptable level of travel demand before capacity enhancement for signalized road networks. In this study, increasing travel demand, which is a result of growing population, increasing mobility requirement and changing land use pattern, is represented by a common Origin-Destination (O-D) demand multiplier. The base O-D demand matrix of a road network can be increased by this multiplier in order to find acceptable level of travel demand, which can be stated as a critical border for capacity enhancement on a road network. In this context, TTC as an objective function is defined as the sum of the multiplication of each link equilibrium flow and corresponding travel cost on a given road network. To minimize the TTC, a bi-level programming model, called **TRA**vel **CO**st **M**inimizer (TRACOM), is presented in which the upper level minimizes the TTC by using equilibrium link flows determined at the lower level. As known, equilibrium link flows can be found depending on either deterministic or stochastic approaches. In this study, Stochastic User Equilibrium (SUE) link flows are determined by using Path Flow Estimator (PFE) developed by Bell et al. [23] in order to represent the users' behaviors against signal timing changes performed at the upper level. The TRACOM model is developed based on Differential Evolution (DE) algorithm framework that is a widely used meta-heuristic method for solving complex optimization problems. Finally, the TRACOM is applied to Allsop and Charlesworths' network for different common O-D demand multipliers in order to reveal the effect of increasing travel demand on TTC.

The paper is organized as follows. Section 2 introduces problem formulation. A bi-level model is given in the third section. Section 4 presents a numerical application. Finally, last section is about discussions and conclusions.

2. PROBLEM FORMULATION

Local authorities are responsible to manage networks in urban roads, and they try to minimize the TTC even in case of increasing travel demand conditions to create robust road networks. On the other hand, minimizing TTC provides benefits for road users by reducing their travel time because they aim to complete their travels within minimum travel time. As known, travel demand continues to increase over the last years due to growing population, and thus increasing travel demand leads to an increase in the TTC in urban road networks. At this point, the problem is that how long transport supply will respond to increasing travel demand by optimizing signal timings. By this way, local authorities will not have to make expensive investments, and they can manage robust road networks with cost-efficient methods.

For this purpose, considering a road network with a set of O-D pairs, *K*, a set of directed links, *A*, a set of paths, *R*, and a set of nodes, *N*, link cost function on link *a* can be expressed as given in Eq. (1).

$$c_a(q_a) = c_a^0 + d_a^U + d_a^{ro}(t) \tag{1}$$

where q_a is flow on link *a*, $a \in A$, c_a^0 is free flow travel time on link *a*, d_a^U is average uniform delay to a vehicle arriving on link *a* (i.e. uniform component of total delay), $d_a^{ro}(t)$ is average random plus over saturation delay to a vehicle arriving on link *a* at time slice *t* (i.e. random component of total delay).

2.1. Random Plus Oversaturation Delay Component

Assuming that time slice *t* equals time period *T*, $d_a^{ro}(T)$ can be expressed by using well-known traffic simulation software TRANSYT delay formula proposed by Vincent et al. [24] as follows:

$$D_a^{ro}(T) = \frac{T}{4}[((q_a - \mu_a)^2 + \frac{4q_a}{T})^{0.5} + (q_a - \mu_a)] \tag{2}$$

$$d_a^{ro}(T) = \frac{D_a^{ro}(T)}{q_a} \tag{3}$$

where $D_a^{ro}(T)$ is random plus oversaturation delay on link *a* for time period *T*, $d_a^{ro}(T)$ average random plus oversaturation delay to a vehicle arriving on link *a* for time period *T*, and μ_a is the capacity for link *a*.

2.2. Uniform Delay Component

The calculation for uniform component of total delay for each link can be carried out on the basis of whole cycles for uniform arrivals and departures. On the other hand, delay and number of queues which are calculated over the time slice *t* is based on whole cycles which begin and end at the starts of effective red times. Thus, the uniform component of delay for each link *a* can be defined in two ways according to the degree of saturation of each link in the network as follows:

(*i*) For oversaturated links with $x_a \geq 1$;

(*ii*) For undersaturated links with accumulated queues $x_a < 1$ which can be identified as those with $L_a^{ro}(t) > L_a^s$.

where $L_a^{ro}(t)$ is random plus oversaturation component for the number of queueing vehicles at time slice *t*, L_a^s is number of queueing vehicles in steady state, and x_a is the degree of saturation on link *a*.

For oversaturated links: Let L_a^U be uniform queue, D_a^U be uniform delay, d_a^U be delay to a vehicle, Λ_a be proportion of green to cycle time, and *C* be the cycle time, the effect of cyclic variation in the cumulative arrivals, q_a on L_a^U and D_a^U is assumed as zero [25]. Thus, L_a^U and D_a^U are calculated on the basis of the difference between the cyclic cumulative departure graph from TRANSYT and uniform departure rate (i.e. capacity), μ_a for each link *a* in the time period *T* according to following expressions (see for details [26]).

$$L_a^U = \frac{C\mu_a(1-\Lambda_a)}{2} \tag{4}$$

$$D_a^U = L_a^U \tag{5}$$

$$d_a^U = \frac{C(1-\Lambda_a)}{2} \tag{6}$$

For undersaturated links with accumulated queues: Considering that there may be queue in undersaturated links, if the initial queue length, $L_a^{ro}(0)$ (i.e. *t*=0) is higher than the steady state queue length, L_a^s for time slice *t*, time is required for the accumulated queue of link *a* at the start of *t* to be dissipated. Let τ_a be the time needed, the following expression can be used [27].

$$\tau_a = \frac{L_a^{ro}(0) - L_a^s}{\mu_a(\hat{x}_a - x_a)} \tag{7}$$

where $\hat{x}_a$ is the degree of saturation for which the equilibrium queue is equal to the initial queue. $L_a^{ro}(0)$ and $\hat{x}_a$ can be iteratively obtained as proposed by Kimber and Hollis [27] as follows:

$$L_a^{ro}(0) = \frac{\lambda \hat{x}_a^2}{1-\hat{x}_a} + \hat{x}_a \tag{8}$$

$$\hat{x}_a = \frac{L_a^{ro}(0) + 1 - \sqrt{(L_a^{ro}(0)+1)^2 - 4(1-\lambda)L_a^{ro}(0)}}{2(1-\lambda)} \tag{9}$$

where λ is a constant between 0.4 and 0.6 [27].

It is assumed that the accumulated queue length decreases linearly at a rate which is the difference between the arrival rates corresponding to the degrees of saturation $\hat{x}_a$ and x_a, as it approaches the equilibrium queue length for time slice *t*. Although the Eq. (9) calculates the dissipation time for the accumulated queue length, it underestimates the time taken by the accumulated queue to clear [26]. Using Eq. (9) for each link *a* uniform delay for undersaturated with accumulated queues can be obtained as follows:

(*i*) if $\tau_a \geq t$ than the uniform component of delay on link *a* can be calculated as given by Eqs. (4-6).

(ii) if $\tau_a < t$ than the uniform component of delay on link *a* is calculated by a linear combination of oversaturated and undersaturated conditions as follows:

$$L_a^U = \frac{\mu_a(1-\Lambda_a)C}{2t}\left[\tau_a + \frac{x_a(1-\Lambda_a)(t-\tau_a)}{(1-\Lambda_a x_a)}\right] \tag{10}$$

$$D_a^U = L_a^U \tag{11}$$

$$d_a^U = \frac{D_a^U}{q_a} \tag{12}$$

Following aforementioned statements on link cost function, the upper level objective function can be presented as follows:

Upper level problem:

$$\min TTC(\zeta, \mathbf{q}^*, \boldsymbol{\psi}) = \sum_{a\in A}\left[q_a^*(\zeta, \boldsymbol{\psi})\cdot\left(c_a^0 + d_a^U + d_a^{ro}\right)\right] \tag{13}$$

subject to

$$\boldsymbol{\psi}(\mathbf{C}, \boldsymbol{\varphi}) \in \boldsymbol{\Omega}_0; \quad \begin{cases} C_{\min} \le C \le C_{\max} \\ \varphi_{\min} \le \varphi \le C \\ \sum_{i=1}^{z}(\varphi + I)_i = C \end{cases} \tag{14}$$

$$q_a^*(\zeta, \boldsymbol{\psi}) \le \mu_a(\boldsymbol{\psi}, s_a) \tag{15}$$

where ζ is O-D demand multiplier, $C_{\min}$ and $C_{\max}$ are possible bounds for cycle time C, φ is stage green time, $\varphi_{\min}$ is the minimum stage green time, I is intergreen time, $\boldsymbol{\psi}$ is vector of signal timings, $\boldsymbol{\Omega}_0$ is feasible region for signal timings, s_a is the saturation flow on link a, and z is the number of stages. Additionally, equilibrium link flow on link a, $q_a^*(\zeta, \boldsymbol{\psi})$ can be obtained by solving SUE problem at the lower level as proposed by Bell and Iida [28] as follows.

Lower level problem:

$$\min_{\mathbf{q}(\boldsymbol{\psi})} F(\mathbf{q}(\boldsymbol{\psi}), \boldsymbol{\psi}) = -\zeta\mathbf{p}^{\mathrm{T}}\mathbf{y}(\mathbf{q}(\boldsymbol{\psi}), \boldsymbol{\psi}) + \mathbf{q}^{\mathrm{T}}\mathbf{c}(\mathbf{q}(\boldsymbol{\psi}), \boldsymbol{\psi}) - \sum_{a\in A}\int_0^{q_a(\boldsymbol{\psi})} c_a(\boldsymbol{\psi}, w)dw \tag{16}$$

subject to

$$\zeta\mathbf{p} = \boldsymbol{\Lambda}\mathbf{h}, \qquad \mathbf{q}(\boldsymbol{\psi}) = \boldsymbol{\delta}\mathbf{h}, \qquad \mathbf{h} \ge \mathbf{0} \tag{17}$$

where **p** is the vector of travel demand, **c** and **y** represent vectors of link and path travel times for the given vector of link flows $\mathbf{q}(\boldsymbol{\psi})$, respectively, **h** is the vector of path flows, $\boldsymbol{\delta}$ represents the link/path incidence matrix where $\delta_{ar}=1$ if link *a* is on path *r*, and $\delta_{ar}=0$ otherwise $\left[\delta_{ar};\forall a\in A;\forall r\in R\right]$, and $\boldsymbol{\Lambda}$ is the O-D/path incidence matrix $\left[\Lambda_r;\forall r\in R\right]$.

3. MODEL DEVELOPMENT

As known, *TTC* in a road network can be minimized by optimizing signal timings (i.e. cycle times and stage green times) of isolated intersections. At this point, route choice behavior of road users is considered as a reaction to the changing travel costs based on signal timing adjustments. Therefore, *TTC* minimization problem is formulated as a bi-level programming problem, in which the upper and lower levels represent *TTC* minimization and SUE assignment problems, respectively, as given in Eqs. (13-17). In this study, upper level objective function is modified by transforming the capacity constraint, which is given in Eq. (15), into a penalty function as given in Eq. (18). Note that the second term on the right side of Eq. (18) ensures that the capacity is not violated on any links in the road network.

$$\min TTC(\zeta,\mathbf{q}^*,\boldsymbol{\psi})=\sum_{a\in A}q_a^*(\zeta,\boldsymbol{\psi})\cdot\left(c_a^0+d_a^U+d_a^{ro}\right)+\sum_{a\in A}\sigma\max\left(q_a^*(\zeta,\boldsymbol{\psi})-\mu_a(\boldsymbol{\psi},s_a),0\right) \tag{18}$$

subject to

$$\boldsymbol{\psi}(\mathbf{C},\boldsymbol{\varphi})\in\boldsymbol{\Omega}_0;\quad \begin{cases} C_{\min}\le C\le C_{\max}\\ \varphi_{\min}\le\varphi\le C\\ \sum_{i=1}^{z}(\varphi+I)_i=C \end{cases} \tag{19}$$

where σ is constant weight parameter which is used to include capacity constraint into the objective function given in Eq. (13) as a penalty component. The weight parameter provides a balance between left and right sides of Eq. (18) by reflecting negative effects of links with capacity violation. Considering both non-convexity and the vast search space of the above given optimization problem, the solution is carried out by TRACOM model which is developed based on DE solution framework. The flowchart of TRACOM model is given in Fig. 1.

The stepwise solution procedure of the TRACOM model is given as follows:

Step 1: At this step, the objective function given in Eq. (18), possible bounds for cycle and stage green times, network-related parameters (i.e. free-flow travel times, saturation flows), O-D travel demand, and O-D demand multiplier are initialized. Subsequently, three DE parameters are introduced. These parameters can be explained as follows:

- Population size (*Np*) represents the number of solution namely signal timing vectors in the population pool,

- Mutation factor (F) is used to create a mutant vector,
- Crossover rate (CR) is used to create a trial vector [29].

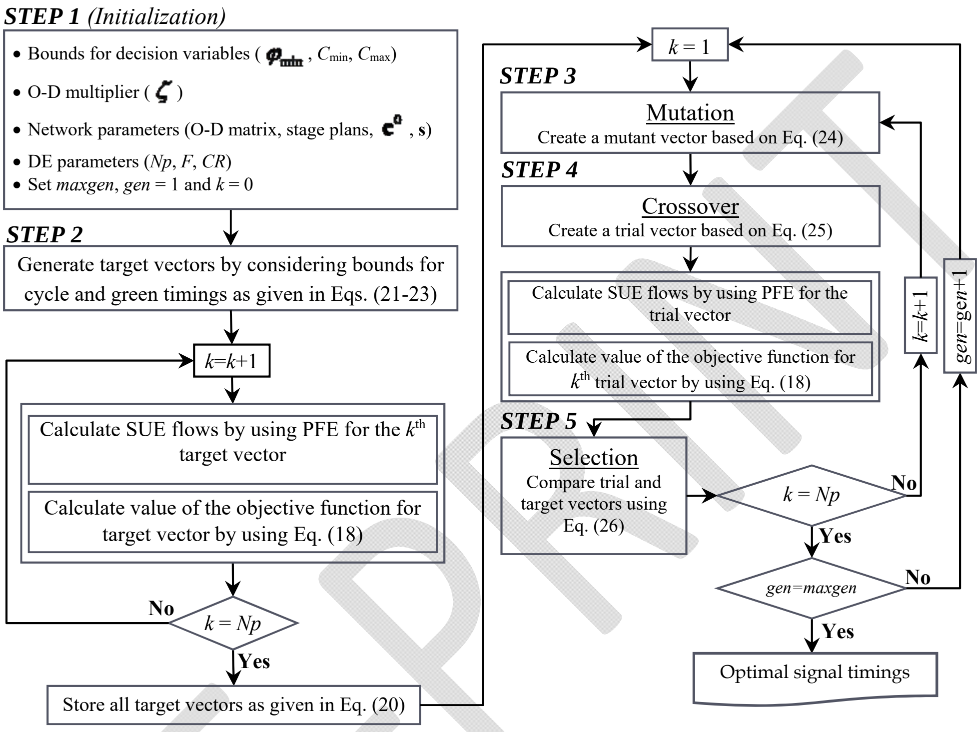

Figure 1 - Flowchart of TRACOM model

Step 2: At this step, initial solution vectors are produced subject to the possible bounds of decision variables which are cycle and green times for each intersection in the road network. Then, *TTC* values are calculated using Eq. (18) and stored as shown in Eq. (20).

$$\left[\begin{array}{cccc|cccccc} \multicolumn{4}{c}{\overbrace{\hphantom{C_1^1 \; C_2^1 \; \cdots \; C_N^1}}^{\text{Cycle times}}} & \multicolumn{6}{c}{\overbrace{\hphantom{\varphi_{1,1}^1 \; \varphi_{1,2}^1 \; \cdots \; \varphi_{1,z_1}^1 \; \varphi_{2,1}^1 \; \cdots \; \varphi_{N,z_n}^1}}^{\text{Green times}}} \\ C_1^1 & C_2^1 & \cdots & C_N^1 & \varphi_{1,1}^1 & \varphi_{1,2}^1 & \cdots & \varphi_{1,z_1}^1 & \varphi_{2,1}^1 & \cdots & \varphi_{N,z_n}^1 \\ C_1^2 & C_2^2 & \cdots & C_N^2 & \varphi_{1,1}^2 & \varphi_{1,2}^2 & \cdots & \varphi_{1,z_1}^2 & \varphi_{2,1}^2 & \cdots & \varphi_{N,z_n}^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ C_1^{Np} & C_2^{Np} & \cdots & C_N^{Np} & \varphi_{1,1}^{Np} & \varphi_{1,2}^{Np} & \cdots & \varphi_{1,z_1}^{Np} & \varphi_{2,1}^{Np} & \cdots & \varphi_{N,z_n}^{Np} \end{array}\right] \left[\begin{array}{c} \overbrace{\hphantom{TTC(\zeta,\mathbf{q}^*,\psi)}}^{\text{Objective functions}} \\ TTC\left(\zeta,\mathbf{q}^*,\psi\right)^1 \\ TTC\left(\zeta,\mathbf{q}^*,\psi\right)^2 \\ \vdots \\ TTC\left(\zeta,\mathbf{q}^*,\psi\right)^{Np} \end{array}\right] \tag{20}$$

where $\varphi_{i,j}$ is j^{th} stage green time of intersection i (i=1,2,…, N and j=1,2,…, z_n), z_n number of stages at n^{th} intersection, and N is the number of intersections. As can be seen in Eq. (20) that the population pool includes initial signal timing vectors and their related *TTC* values as many as *Np*. To generate an initial signal timing vector with TRACOM algorithm, following procedure is executed.

(i). Cycle time values are generated for all intersections in the road network. Following representation is given for the i^{th} intersection:

$$C_i = \text{int}\left[rnd\left(0;1\right) \times \left(C_{\max} - C_{\min} \right) + C_{\min} \right] \tag{21}$$

(ii). Green time values for the stages of all intersections are generated. Eq. (22) illustrates this process for the j^{th} stage of the i^{th} intersection.

$$\varphi_{i,j} = \text{int}\left[rnd\left(0;1\right) \times \left(C_i - \varphi_{\min} \right) + \varphi_{\min} \right] \tag{22}$$

(iii). In order to provide consistency between a cycle time and its components (green and intergreen times), generated green times of each signalized intersection are revised. Eq. (23) illustrates this process for the j^{th} stage of the i^{th} intersection.

$$\varphi_{i,j} = \varphi_{\min} + \frac{\varphi_{i,j}}{\sum_{j=1}^{z_i} \varphi_{i,j}} \left[C_i - z_i \times \left(I + \varphi_{\min} \right) \right] \tag{23}$$

As can be seen in Eq. (18) that the equilibrium link flows, $\mathbf{q}^*$, are required for calculating *TTC* values of signal timing vectors. In TRACOM, the SUE assignment problem is solved using PFE traffic assignment tool. Assuming β is dispersion parameter, Dell'Orco et al. [17] performed sensitivity analysis on SUE assignment by using PFE and found that for values of β up to 1, the objective function values remain stable, and decrease rapidly for bigger values than 1. This result is reasonable: in fact, the higher the value of β, the more deterministic the traffic assignment. Therefore, the value of β is selected 1. The pseudo code of the PFE is illustrated in Fig. 2.

Once the SUE assignments are carried out and equilibrium link flows are obtained for each signal timing vector, their corresponding *TTC* values are calculated. Note that as of the end of Step 2, each solution vector in the population is called as a target vector.

Step 3. At this step, a randomly chosen solution vector is mutated by adding a weighted difference of two randomly selected solution vectors. Note that all three vectors must be different from both each other and the target vector, $\mathbf{\Gamma}_i$. A mutant vector, $\boldsymbol{\rho}_i$ is created as follows:

$$\boldsymbol{\rho}_{i,gen} = \mathbf{\Gamma}_{r0,gen} + F \cdot \left(\mathbf{\Gamma}_{r1,gen} - \mathbf{\Gamma}_{r2,gen} \right) \tag{24}$$

where $r0$, $r1$ and $r2$ are indices of randomly chosen solution vectors.

$q_a \leftarrow 0, \; \forall a \in A$

$c_a \leftarrow c_a(q_a), \; \forall a \in A$

$m \leftarrow 0$

repeat

$m \leftarrow m+1$

Update link travel costs $c_a \leftarrow \frac{1}{m} c_a(q_a) + (1 - \frac{1}{m}) c_a$

For each path r

Calculate new path costs $y_r \leftarrow \sum_{a \in A} \delta_{ar} c_a(q_a)$

Next r

For each path r

Calculate new path flows $h_r \leftarrow p_k \frac{\exp(-\beta y_r)}{\sum_{r \in \mathbf{R}_k} \exp(-\beta y_r)}$

Next r

For each link a

Calculate new link flows $q_a \leftarrow \sum_{r \in \mathbf{R}_k} \delta_{ar} h_r$

Next a

until no new path and link flows converged

Figure 2 - Pseudo code of the PFE

Step 4. At this step, crossover is applied by choosing each member of the trial vector, $\mathbf{E}_i$ from the target or the mutant vectors with the probabilities of *CR* or 1-*CR*, respectively, as given in Eq. (25).

$$\mathbf{E}_{i,gen} = \varepsilon_{j,i,gen} = \begin{cases} \rho_{j,i,gen} & \text{if} \left(rnd_j(0,1) \le CR \text{ or } j = j_{rnd} \right) \\ \gamma_{j,i,gen} & \text{otherwise} \end{cases} \tag{25}$$

where $\varepsilon_{j,i,gen}$, $\rho_{j,i,gen}$, and $\gamma_{j,i,gen}$ are j^{th} members of i^{th} trial, mutant and target vectors, respectively. The condition of $j = j_{rnd}$ provides that target and trial vectors are definitely different from each other. After determining the members of the trial vector, their related SUE link flows and the *TTC* value are calculated.

Step 5. At the last step, the target vector, $\mathbf{\Gamma}_{i,gen+1}$ for the next generation is selected by comparing the *TTC* values of trial and target vectors as given in Eq. (26).

$$\mathbf{\Gamma}_{i,gen+1} = \begin{cases} \mathbf{E}_{i,gen} & \text{if } TTC\left(\mathbf{q}^*\left(\mathbf{E}_{i,gen}\right), \mathbf{E}_{i,gen}\right) \le u\left(\mathbf{q}^*\left(\mathbf{\Gamma}_{i,gen}\right), \mathbf{\Gamma}_{i,gen}\right) \\ \mathbf{\Gamma}_{i,gen} & \text{otherwise} \end{cases} \tag{26}$$

The DE process is repeated until the maximum number of generations, *maxgen*, is reached. In order to provide a more explanatory illustration, pseudo code of TRACOM model is given in Fig. 3.

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for $\zeta \leftarrow 1$ to $\zeta_{\max}$ step α **do** (α is defined as step size for O-D multiplier, ζ)
  for $k \leftarrow 1$ to Np **do**
    for $i \leftarrow 1$ to N **do**
      Generate a cycle time C_i for i^{th} signalized intersection randomly from $\in \{ C_{\min}, \ldots, C_{\max} \}$
      for $j \leftarrow 1$ to z_i **do**
      Generate a green split $\varphi_{i,j}$ for j^{th} stage of i^{th} signalized intersection randomly from $\in \{\varphi_{\min}, \ldots, C_i\}$
      Revise stage green times of i^{th} signalized intersection based on Eq. (23)
    Run PFE to calculate SUE flows $\mathbf{q}^*$ for the k^{th} initial signal timing vector
    Calculate total travel cost for k^{th} target (initial signal timing) vector and current O-D matrix multiplier ζ
  for *gen* ← 1 to *maxgen* **do**
    for $k \leftarrow 1$ to Np **do**
      Perform mutation to the k^{th} solution vector to create a mutant vector based on Eq. (24)
      for $v \leftarrow 1$ to Nd **do** (*Nd* is defined as number of decision variables)
        Perform crossover to obtain the v^{th} decision variable of k^{th} trial vector based on Eq. (25)
      Revise intersection cycle times considering their upper and lower bounds $\{ C_{\min}, \ldots, C_{\max} \}$
      Revise stage green times based on Eq. (23)
      Run PFE to calculate SUE flows $\mathbf{q}^*$ for the trial vector
      Calculate total travel cost for trial vector and current O-D matrix multiplier ζ
      Replace k^{th} target vector in the population with the k^{th} trial vector in case it produces lower total travel cost.
  print optimal/near optimal signal timings and total travel cost for current O-D matrix multiplier ζ

Figure 3 - Pseudo code of TRACOM

4. NUMERICAL APPLICATION

Medium-sized signalized road network firstly presented by Allsop and Charlesworth [30] is chosen for numerical application of TRACOM model. Additionally, by using this benchmark road network we aimed to give readers a chance to compare some results drawn by previous studies in the literature since it has widely been used in most of traffic related studies. Allsop and Charlesworth's network has 23 links and 20 signal setting variables at six junctions by taking each one as isolated into account. The representation of the road network and its stage plans are given in Figs. 4 and 5 while the network data and O-D travel demand matrix are presented in Tables 2 and 3, respectively.

Table 2 - Network data

Intersection number	Link number	Free-flow travel time (c_a^0)	Saturation flow (s_a)	Intersection number	Link number	Free-flow travel time (c_a^0)	Saturation flow (s_a)
1	1	1	2000	4	5	20	1800
	2	1	1600		6	20	1850
	16	10	2900		10	10	2200
	19	10	1500		11	1	2000
					12	1	1800
					13	1	2200
2	3	10	3200	5	8	15	1850
	15	15	2600		9	15	1700
	23	15	3200		17	10	1700
					21	15	3200
3	4	15	3200	6	7	10	1800
	14	20	3200		18	15	1700
	20	1	2800		22	1	3600

Table 3 - O-D travel demand matrix (veh/h)

O-D	A	B	D	E	F
A	--	250	700	30	200
C	40	20	200	130	900
D	400	250	--	0	100
E	300	130	0	--	20
G	550	450	170	60	20

Constraints for each signal timing variable used in TRACOM are set as given below:

$$\boldsymbol{\psi}\left(\mathbf{C},\boldsymbol{\varphi}\right)\in\boldsymbol{\Omega}_0;\ \begin{cases}36\le C\le 120\\ 7\le\varphi\le C\end{cases}\tag{27}$$

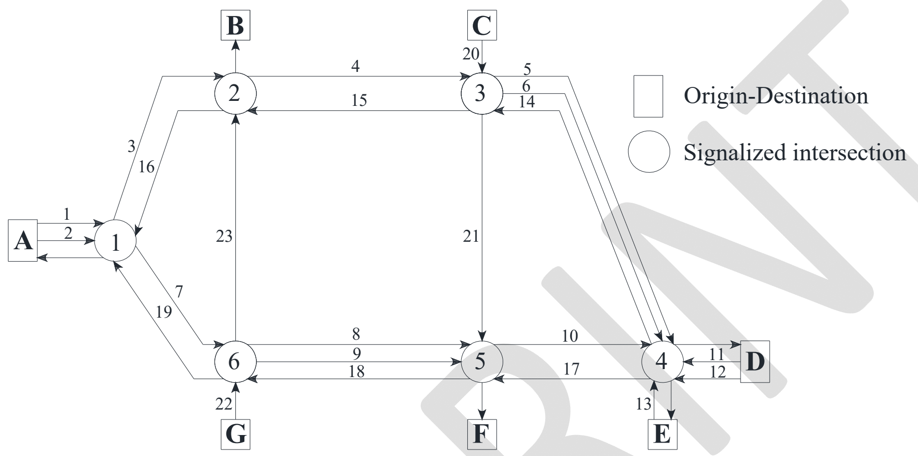

Figure.4 - Allsop & Charlesworth's network

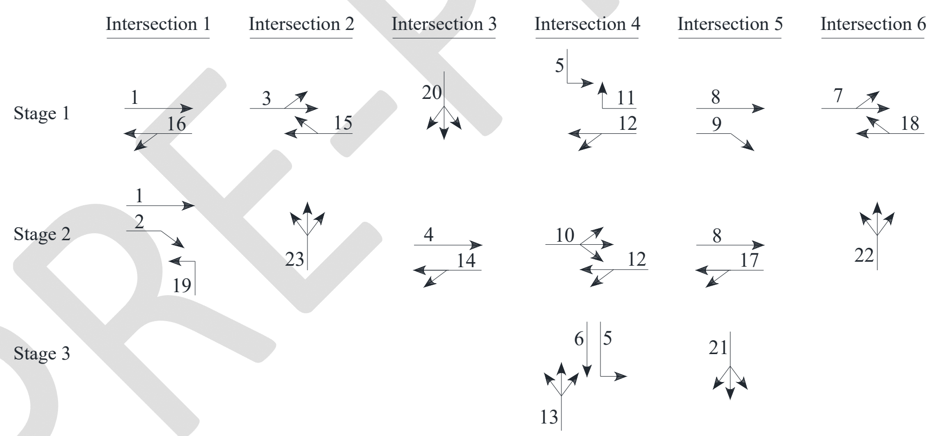

Figure 5 - Stage plans

Considering that the values of DE parameters play an important role on the performance of TRACOM model, a sensitivity analysis has been performed based on the recommended ranges for DE parameters by Storn and Price [31]. These ranges are assumed to be [0.5, 1.0] and [0.8, 1.0] for mutation factor and crossover rate, respectively. For this purpose, 30 different cases are created and each case is solved 10 times with different random seeds for base O-D travel demand matrix. Note that the intergreen time, *I*, between stages was set to 5 seconds, population size, *Np*, was set to 30, and maximum number of generations, *maxgen*, was set to 1000 during the sensitivity analysis. The best *TTC* values after 10 solutions for each case are given in Table 4.

Table 4 - TTC values resulting from the sensitivity analysis (veh-h)

CR \ F	0.50	0.60	0.70	0.80	0.90	1.00
0.80	148.02	147.94	147.49	147.28	148.53	149.66
0.85	148.02	147.94	147.56	147.54	148.91	149.66
0.90	148.55	148.91	148.14	149.49	149.95	149.66
0.95	148.97	148.69	148.69	149.85	149.49	149.78
1.00	148.97	148.97	148.79	149.04	150.23	150.11

As can be seen in Table 4 that the minimum TTC value was obtained as 147.28 veh-h for the case with both *F* and *CR* are 0.80. The computational time for complete run of TRACOM resulted in 2.61 hours. It means that each generation takes about 9.4 seconds of CPU. TRACOM has been executed in MATLAB programming and performed on PC with Intel Core i7 2.10 GHz, RAM 8 GB. Based on the results obtained with the sensitivity analysis, the following user-specified DE parameters were used during the analyses for Allsop and Charlesworth's network: *CR*=0.8, *F*=0.8, *Np*=30, and *maxgen*=1000. The upper bound for the O-D multiplier, ζ_{max} and step size, α are set to 1.30 and 0.02, respectively. *TTC*, ζ and their changes are given in Table 5 after applying the TRACOM model.

Table 5 - Evolution of the change of ζ and TTC

i	O-D demand multiplier ζ_i	Total travel cost TTC_i	Change (%) $\Delta\zeta = \frac{\zeta_i - \zeta_{i-1}}{\zeta_{i-1}}$	Change (%) $\Delta TTC = \frac{TTC_i - TTC_{i-1}}{TTC_{i-1}}$
1	1.00	147.28		
2	1.02	153.80	2.00	4.43
3	1.04	163.00	1.96	5.98
4	1.06	170.76	1.92	4.76
5	1.08	179.10	1.89	4.88
6	1.10	186.53	1.85	4.15
7	1.12	197.50	1.82	5.88
8	1.14	208.80	1.79	5.72
9	1.16	218.39	1.75	4.59
10	1.18	243.15	1.72	11.34

As can be seen in Table 5 that network *TTC* value is increased up to 65% while the O-D demand matrix is increased up to 18%. On the other hand, when ζ equals 1.20, degree of saturation of at least one link exceeds 100% that means the travel demand can be increased only up to 18% by optimizing intersection signal timings. It can also be seen in Table 5 that, changes of *TTC* vary between 4%-6% at each step *i* during the O-D multiplier increases from 1.00 to 1.16. On the other hand, change of *TTC* is about 11% when ζ equals 1.18 that can also be seen as a sudden spike in Fig. 6. For this reason, the critical value of O-D multiplier can be selected as 1.16.

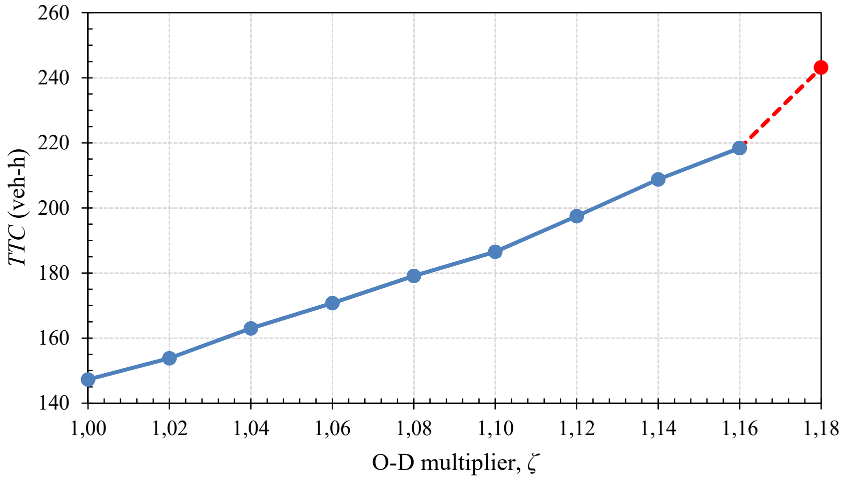

Figure 6 - Relationship between TTC and O-D multiplier ζ

In order to illustrate the convergence behavior of the TRACOM model, Fig. 7 is presented for ζ =1.16 since this value is selected as a critical point for capacity enhancement on the road network to cope with increasing travel demand. As can be seen in Fig. 7, the TRACOM algorithm begins to search optimal signal timings in the feasible search space and finds the initial value of *TTC* as about 435 veh-h. After 10th generation, the TRACOM starts to ignore worse solution vectors and seriously improves the value of *TTC* about 36%. After continuing to improve the objective function, the TRACOM finds the final value of *TTC* as about 218 veh-h. That is, the total improvement rate on the objective function value is about 100% after 1000th generation.

For further information, SUE link flows and corresponding degree of saturations found for ζ =1.16 are given in Table 6. As shown, there is no links exceeding their capacities in the road network since degree of saturations of links are less than 100%. It should be pointed out that the degree of saturations of links numbered 10, 13, 20, 21 and 22 are higher than 90% because SUE flows of those links approach to their capacities. Additionally, optimal signal timings by applying TRACOM algorithm are presented in Table 7. By providing cycle time constraints, the lowest cycle time is found as 55 seconds for the second intersection while the highest cycle time is 111 sec for the fifth intersection.

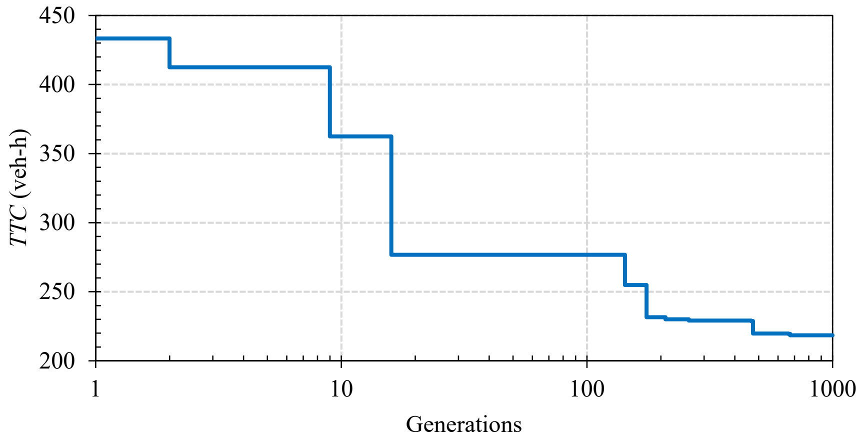

Figure 7 - Convergence graph of TRACOM for ζ =1.16

Table 6 - SUE link flows and corresponding degree of saturations for ζ =1.16

Link number	SUE link flows (veh/h)	Degree of saturation (%)	Link number	SUE link flows (veh/h)	Degree of saturation (%)
1	833	47	13	522	91
2	536	61	14	914	82
3	833	65	15	915	88
4	687	61	16	768	78
5	740	72	17	478	87
6	203	42	18	407	52
7	536	65	19	728	88
8	554	68	20	1496	98
9	128	64	21	1240	92
10	554	93	22	1450	93
11	578	92	23	984	73
12	292	28			

Table 7 - Optimum signal timings for ζ =1.16

Cycle time C (s)	Intersection number i	Duration of stages (s)		
		$\varphi_{i,1}$	$\varphi_{i,2}$	$\varphi_{i,3}$
94	1	32	52	---
55	2	22	23	---
97	3	53	34	---
96	4	30	26	25
111	5	13	36	47
92	6	42	40	---

5. DISCUSSION AND CONCLUSION

The urban population in developing countries is mostly projected to continue to increase. This is likely to mean more drivers result in more traffic day by day. On one hand road users expect reliable journeys, on the other hand local authorities focus on maintaining the traffic operations successfully by implementing traffic control strategies, clearing up the incidents quickly, keeping lanes open. When these tools remain incapable of overcoming traffic congestions, authorities take capacity enhancement countermeasures such as increasing link capacities, lane additions or applying grade-separated junctions. However, such actions bring high investment costs. Therefore, it is crucial to determine the right time for such physical improvements to avoid both premature investments considering limited resources, and late investments to maintain traffic flow properly.

In this study, effectiveness of network wide signal timing optimization on total travel cost in case of increasing travel demand is investigated. For this purpose, a bi-level programming model TRACOM is developed. At the upper level, network TTC, which is a function of link traffic volumes, free flow travel times, uniform and random plus over saturation components of delay, is minimized. At the lower level of the proposed model, drivers' route choice behaviors are taken into consideration in SUE context. The TRACOM model, which is based on DE optimization method, is applied to Allsop and Charlesworths' test network to evaluate its effectiveness. In the case of Allsop and Charlesworth's road network, the traffic agency can manage the network by optimizing signal timings until the O-D travel demand increases 18%. However, the TTC shows a sudden spike while the value of O-D multiplier is between 1.16 and 1.18 although it shows an approximate linear increase while the travel demand is increased up to 16%. Thus, any capacity enhancement countermeasure before or after the 16% increase in travel demand would be a premature or late investment, respectively.

Capacity enhancement represents improvement of an existing road network by investing in new transport construction which plays an important role on the performance of the road network. Timely and efficient investments in roads provide economic and social benefits to the emerging economies. On the other hand, premature investments waste limited resources and it may hinder the realization of more important services. In this context, local authorities who are responsible for urban road network management can benefit from TRACOM model to make their investment decisions timely based on projected travel demand data provided in transportation master plans.

The TRACOM model considers networks including signalized intersections. Since the use of different types of intersections (i.e. stop controlled or roundabout) will clearly affect the results of TRACOM model, this issue will be taken into account in future studies. Moreover, different multipliers for each O-D pair can be used rather than a common O-D matrix multiplier. The effects of signal coordination will also be taken into consideration.

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