Non-minimal RF^2-type corrections to holographic superconductor

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Abstract

We study (2+1)-dimensional holographic superconductors in the presence of non-minimally coupled electromagnetic field to gravity by considering an arbitrary linear combination of RF^2-type invariants with three parameters. Our analytical analysis shows that the non-minimal couplings affect the condensate and the critical temperature.

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1 Introduction

Anti-de Sitter/conformal field theory (AdS/CFT) duality emerged from string theory has given rise to novel deductions on the research of superconductivity in condensed matter physics. According to AdS/CFT dictionary a suitably chosen gravitational theory in four dimensions (bulk) can describe basic properties of a superconductor in three dimensions (boundary). Holographic superconductors have been worked extensively in literature, see some selected papers [1]-[6] and references therein. Although in this regard generally numerical solutions were investigated, endeavors of finding analytic solutions occurred after the paper [3] in which the authors found that higher curvature corrections make condensation harder by using an analytic approximation method.

In Ref.[4], based on a numerical approach, the author found that the critical temperature depends on the rotation by applying the method developed by Hartnoll et al [2] to a rotating holographic superconductor. In Ref.[5] the authors considered the Maxwell field strength corrections by following the technique in [4] and concluded that the higher correction to the Maxwell field makes the condensation harder to form. In Ref.[6], based on an analytic method, the author investigated several properties of holographic superconductors by incorporating separately the Born-Infeld term and the Weyl-invariant correction term to the standard bulk lagrangian. As distinct from those, in this Letter we aim to analyze the effects of non-minimal RF^2-type contributions to the standard holographic superconductor lagrangian. Similar terms have appeared in a calculation in QED of the photon effective action from 1-loop vacuum polarization on a curved background [7], and in the Kaluza-Klein reduction of R^2 lagrangian from five dimensions to four dimensions [8]. Furthermore, as the relevances of such terms to the dark matter, the dark energy, the primordial magnetic fields in the universe have been investigated in the references [9]-[12], the gravitational wave concerns have been discussed by searching pp-wave solutions in [13]. Therefore, it is worthwhile to work possible effects of RF^2-terms on holographic superconductor. In fact, some specific combinations of that type interactions have appeared in previous holographic studies. For example, in Ref.[14] the authors have investigated the effects on the charge transport properties of the holographic CFT resulting from the extra four-derivative interaction formulated in terms of the Weyl tensor which is constructed as a particular linear combination of our c_1, c_2, c_3 terms in the lagrangian density (1). Thus, one novelty of this work is to keep three (perturbatively small) unspecified coupling constants $c_{1,2,3}$ independent.

We work in the probe limit in which the electromagnetic field and scalar field do not backreact on the geometry, and use the analytic approximation method developed by Gregory et al in [3]. Their method explains the qualitative features of superconductors and expects quantitatively accurate numerical results. Consequently, we obtain an analytic expression for the condensate and the critical temperature in the existence of the non-minimal couplings. Correspondingly, we observe that they are are affected critically by the non-minimal coupling parameters.

2 The Model

The Riemannian bulk spacetime is denoted by $\{M, g\}$ where M is four-dimensional differentiable and orientable manifold endowed with a non-degenerate metric g. We will be using orthonormal 1-form e^a such that $g = \eta_{ab} e^a \otimes e^b$ where $\eta_{ab} = \mathrm{diag}(-1, 1, 1, 1)$. Orientation is fixed through the Hodge map $*$ such that $*1 = e^0 \wedge e^1 \wedge e^2 \wedge e^3$ where $\wedge$ figures the exterior product. The Riemann curvature 2-form is defined by $R^a{}_b = d\omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b$ where $\omega_{ab} = -\omega_{ba}$ is the Levi-Civita connection 1-form $\omega^a{}_b \wedge e^b = -de^a$. We will use the following shorthand notations throughout the Letter; $e^a \wedge e^b \wedge \cdots = e^{ab\cdots}$, $\iota_a \iota_b \cdots = \iota_{ab\cdots}$, $\iota_a F = F_a$, $\iota_{ba} F = F_{ab}$, $\iota_a R^a{}_b = R_b$, $\iota_{ba} R^{ab} = R$ where ι_a denotes the interior product such that $\iota_b e^b = \delta^a_b$. Here δ^a_b is the Kronecker symbol.

We consider the following lagrangian density 4-form

$$\begin{aligned}
\mathcal{L} \;=\; & \frac{1}{\kappa^2}\left[R_{ab} \wedge *e^{ab} + \Lambda * 1 + (de^a + \omega^a{}_b \wedge e^b) \wedge \lambda_a\right] \\
& -\frac{1}{2} F \wedge *F - D\psi^\dagger \wedge D\psi - m^2 \psi^\dagger \psi * 1 \\
& +\frac{c_1}{2} F^{ab} R_{ab} \wedge *F + \frac{c_2}{2} F^a \wedge R_a \wedge *F + \frac{c_3}{2} R F \wedge *F \qquad (1)
\end{aligned}$$

where κ is the gravitational coupling coefficient, $F = dA$ is the Maxwell 2-form with the electromagnetic potential 1-form A, Λ is the cosmological constant, ψ is the complex scalar field (hair), the dagger symbol signifies complex conjugation, m is the mass of hair, c_i, $i = 1, 2, 3$, are non-minimal coupling coefficients, λ_a is lagrange multiplier constraining connection to be Levi-Civita and $D\psi = d\psi + iA\psi$. The first two lines of (1), i.e. the choice of $c_1 = c_2 = c_3 = 0$, is the standard holographic superconductor lagrangian introduced by Gubser in [1]. The third line is a linear combination of three non-minimal RF^2-type terms. The case for vanishing hair and cosmological constant has been worked much for various reasons such as the dark matter, the dark energy and the primordial magnetic fields in the universe, the gravitational waves [7]-[12]. Another special choice in which the RF^2-lagrangian is conformally invariant is $c_3 = \gamma/3$, $c_1 = c_2 = \gamma$. Here it is also important to note that these non-minimal terms are taken to be perturbative, otherwise the model suffers from a number of problems, e.g. the presence of ghosts. Therefore we consider the perturbatively small coefficients c_i.

Now, since we will be working in the probe limit, as usual we concentrate only on the electromagnetic field equation and the hair field equation obtained by independent variations with respect to A and $\psi^\dagger$, respectively,

$$\begin{aligned} d\Big\{ -*F + \frac{c_2}{2}[R_a \wedge \imath^a * F - R * F + *(F^a \wedge R_a)] & \\ +c_1 * F^{ab} R_{ab} + c_3 R * F\Big\} - 2|\psi|^2 * A &= 0, \end{aligned} \tag{2}$$

$$D * D\psi - m^2 \psi * 1 = 0\,. \tag{3}$$

We can assume that ψ is everywhere real and the mass is $m^2 = -2/L^2$ which satisfies the Breitenlohner-Freedman bound, $m^2L^2 \geq -9/4$. In order to find a solution to those equations we start with the metric of a planar Schwarzschild-AdS black hole

$$g = -f(r)dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{L^2}(dx^2 + dy^2) \tag{4}$$

where

$$f(r) = \frac{r^2}{L^2}(1 - \frac{r_H^3}{r^3})\,. \tag{5}$$

Here we write the cosmological constant in terms of the AdS radius L as $\Lambda = 6/L^2$ and the mass of the black hole M in terms of the position of the horizon r_H as $M = r_H^3/L^2$. Now we think of the case $\psi = \psi(r)$ and $A = \phi(r)dt$. Then the passage to a new independent variable through $z = r_H/r$ brings the outer region $r_H \leq r \leq \infty$ to the interval $0 \leq z \leq 1$. Correspondingly, the equations (2) and (3) turn out to be

$$\left[1 + \frac{2c_1}{\beta L^2}(1 - z^3)\right]\phi_{zz} - \frac{6c_1}{\beta L^2} z^2 \phi_z - \frac{2L^2}{\beta}\frac{\psi^2}{z^2(1 - z^3)}\phi = 0\,, \tag{6}$$

$$\psi_{zz} - \frac{2 + z^3}{z(1 - z^3)}\psi_z + \left[\frac{L^4\phi^2}{r_H^2(1 - z^3)^2} + \frac{2}{z^2(1 - z^3)}\right]\psi = 0\,, \tag{7}$$

where $\beta = 1 - 6c_2/L^2 + 12c_3/L^2$ and a subindex z denotes d/dz. Thus, we continue tracking β apart from c_1 in order to see the novel RF^2-contributions.

Now we enumerate the steps that we will pursue. Firstly, we make a usual solution ansatz in the AdS region, $z \to 0$,

$$\phi(z) = \mu - qz\,, \quad \psi(z) = \psi_1 z + \psi_2 z^2\,. \tag{8}$$

ψ_1 and ψ_2 are interpreted as condensates, and μ and q as the chemical potential and the charge density, respectively, in the dual theory. According to [1], for $-9/4 < m^2L^2 < -5/4$ one can choose either $\psi_1 = 0$ or $\psi_2 = 0$. So we take safely $\psi_1 = 0$. Secondly, we find an approximate solution at the horizon, $z = 1$, by using the Taylor expansion technique and then apply the regularity condition at the boundary

$$\phi(1) = 0\,, \quad \psi_z(1) = \frac{2}{3}\psi(1)\,, \tag{9}$$

Thirdly, we match two solutions at an intermediate point $0 < z_m < 1$. Finally we deduce ψ_2 and the critical temperature, and comment on them.

Since the first step is already there, let us start with the second step by writing down the Taylor expansion of $\psi(z)$ and $\phi(z)$ around $z = 1$

$$\phi(z) = \phi(1) - \phi_z(1)(1-z) + \frac{1}{2}\phi_{zz}(1)(1-z)^2 + \cdots\,, \tag{10}$$

$$\psi(z) = \psi(1) - \psi_z(1)(1-z) + \frac{1}{2}\psi_{zz}(1)(1-z)^2 + \cdots\,. \tag{11}$$

We calculate $\phi_{zz}(1)$ from (6) and $\psi_{zz}(1)$ from (7)

$$\phi_{zz}(1) = \frac{6c_1}{\beta L^2}\phi_z(1) - \frac{2}{3\beta}L^2\phi_z(1)\psi(1)^2\,, \tag{12}$$

$$\psi_{zz}(1) = -\frac{2}{3}\psi_z(1) - \frac{L^4}{18r_H^2}\phi_z(1)^2\psi(1)\,. \tag{13}$$

By substituting these into above and also incorporating (9) we obtain a set of two serial solutions at the horizon

$$\phi(z) = -\phi_z(1)(1-z) + \left[\frac{3c_1}{\beta L^2} - \frac{1}{3\beta}L^2\psi(1)^2\right]\phi_z(1)(1-z^2) + \cdots\,, \tag{14}$$

$$\psi(z) = \frac{1}{3}\psi(1) + \frac{2}{3}\psi(1)z - \frac{2}{9}\left[1 + \frac{L^4}{8r_H^2}\phi_z(1)^2\right]\psi(1)(1-z^2) + \cdots\,. \tag{15}$$

Thus we are in the third step in which we equate $\phi(z)$ and $\psi(z)$ in (8) to (14) and to (15), respectively, at the intermediate point z_m. Since allowing z_m to be arbitrary does not alter quantitative behaviors of the analytic method [3], we fix it as $z_m = 1/2$. Smooth matching yields four conditions

$$\begin{aligned}
\mu - \frac{q}{2} &= \left(\frac{1}{2} - \frac{3c_1}{4\beta L^2}\right) b + \frac{L^2}{12\beta} ba^2 \,, && (16)\\
-q &= \left(\frac{3c_1}{\beta L^2} - 1\right) b - \frac{L^2}{3\beta} ba^2 \,, && (17)\\
\frac{\psi_2}{4} &= \frac{11}{8} a - \frac{L^4}{144 r_H^2} ab^2 \,, && (18)\\
\psi_2 &= \frac{8}{9} a + \frac{L^4}{36 r_H^2} ab^2 \,, && (19)
\end{aligned}$$

where we have renamed $\psi(1) \equiv a$ and $-\phi_z(1) \equiv b$ $(a, b > 0)$ for plain. These equations yield

$$a^2 = \frac{3q\beta}{bL^2}\left(1 - \alpha^2 \frac{b}{q}\right) , \quad \psi_2 = \frac{5}{3} a \,, \quad b = \frac{2\sqrt{7} r_H}{L^2} \,, \tag{20}$$

where $\alpha^2 = 1 - 3c_1/\beta L^2$. From now on we need to track α and β for our novel results coming from RF^2-terms. By fixing the charge density $\rho = q r_H$ and using the Hawking temperature $T = 3r_H/(4\pi L^2)$, we calculate the expectation value of the dimension 2 operator $\langle \mathcal{O}_2 \rangle = \sqrt{2}\psi_2 r_H^2/L^3$ as

$$\langle \mathcal{O}_2 \rangle = \frac{80\pi^2}{9} \sqrt{\frac{2\beta}{3}} T T_c \sqrt{1 + \frac{T}{T_c}} \sqrt{1 - \frac{T}{T_c}} \tag{21}$$

$$\tag{22}$$

where we defined the critical temperature

$$T_c = \frac{3\sqrt{\rho}}{4\pi L \alpha \sqrt{2\sqrt{7}}} \,. \tag{23}$$

For the case $\alpha \to 0$ or $c_1/\beta \to L^2/3$ the critical temperature T_c goes to infinity. This case corresponds to having the higher order corrections of the same order as the leading order result, which can not be admissible for arising ghosts in such case. Hence, we perturbatively expand the term $1/\beta$ in α^2 in terms of the small coefficients c_i up to leading order.

$$\alpha^2 = 1 - 3c_1/L^2 + \mathcal{O}(c_i^2) \tag{24}$$

Then, the critical temperature T_c is affected from the non-minimal correction in the leading order as follows,

$$T_c = \frac{3\sqrt{\rho}}{4\pi L\sqrt{2\sqrt{7}}}(1 + \frac{3c_1}{2L^2}) + \mathcal{O}(c_i^2)\,. \tag{25}$$

Thus, T_c increases as c_1 increases from zero to $c_1 = L^2/3$.

3 Concluding remarks

As expected, the condensation occurs below the critical point T_c and the mean field theory result $\langle\mathcal{O}_2\rangle \propto (1 - T/T_c)^{1/2}$ is valid. Since the condensate is proportional to $\sqrt{\beta}$, the consistency condition $\beta > 0$ causes a restriction between the non-minimal coupling constants c_2 and c_3. The condensation is low for $\beta < 1$ and the reverse is hold for $\beta > 1$. Besides, if the special case $c_2 = 2c_3$ is encountered, β goes to unity which means that the condensate is not influenced.

We notice that for the special case $c_1 = \beta L^2/3$ in (23) the critical temperature T_c goes to infinity. This case corresponds to having the higher order corrections of the same order as the leading order result, which should not be admissible for arising ghosts in such cases. In order to avoid such problems we expand the temperature in terms of c_i. From (25) we see that the critical temperature depends on c_1 in the leading order. We see that as c_1 increases, T_c increases. This case allows very high temperature superconductor. But, there is a constrain on the coupling parameter which is $c_1 \leq L^2/3$ in the leading order.

We notice also that for the conformally invariant case $c_1 = c_2 = \gamma$ and $c_3 = \gamma/3$ the Weyl parameter must respect an upper bound $\gamma \leq L^2/5$ for a non-zero condensate. This result is in favor of [6] and [14] in which certain aspects of the Weyl corrections to holographic superconductor in a five and a four dimensional bulk space-times have been discussed, respectively.

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References

[1] S. S. Gubser, *Phys. Rev. D* **78** 065034 (2008) [arXiv:0801.2977]

[2] S. A. Hartnoll, C.P. Herzog and G. T. Horowitz, *JHEP* **12** 015 (2008) [arXiv:0810.1563]

[3] R. Gregory, S. Kanno and J. Soda, *JHEP* **10** 010 (2009) [arXiv:0907.3203]

[4] J. Sonner, *Phys. Rev. D* **80** 084031 (2009) [arXiv:0903.0627]

[5] Q. Pan, J. Jing and B. Wang, *Phys. Rev. D* **84** 126020 (2011) [arXiv:1111.0714]

[6] D. Roychowdhury, *Phys. Rev. D* **86** 106009 (2012) [arXiv:1211.0904]

[7] I. T. Drummond and S. J. Hathrell, *Phys. Rev. D* **22** 343 (1980)

[8] T. Dereli and G. Üçoluk, *Class. Q. Grav.* **7** 1109 (1990)

[9] G. Lambiase, S. Mohanty and G. Scarpetta, *JCAP* **07** 019 (2008)

[10] K. Bamba, S. Nojiri and S. D. Odintsov, *JCAP* **10** 045 (2008) [arXiv:0807.2575]

[11] K. E. Kunze *Phys. Rev. D* **81** 043526 (2010) [arXiv:0911.1101]

[12] T. Dereli and Ö. Sert, *Mod. Phys. Lett. A* **26** 1487 (2011) [arXiv:1105.4579]

[13] T. Dereli and Ö. Sert, *Phys. Rev. D* **83** 065005 (2011) [arXiv:1101.1177]

[14] R. C. Myers, S. Sachdev and A. Singh, *Phys. Rev. D* **83** 066017 (2011) [arXiv:1010.0443]