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A New Heuristic Routing Algorithm for Fleet Size and Mix Vehicle Routing Problem

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ABSTRACT

Ochi's approach solves the heterogenous vehicle routing problem using the constraint having fixed costs as a multiplier of residuals. However, in this approach, there is not any information about which vehicle will be assigned to the route related to this constraint. In our study, Ochi's approach is interpreted again in terms of vehicle capacity and number of customers assigned to each route. The proposed routing approach is taking the higher capacity vehicle for improving the performance. Then the solution phases of a sample problem are shown by using the given algorithm. In order to highlight the performance of the routing approach, Golden's 12 test problems (Fleet Size and Mix Vehicle Routing Problem with Fixed Cost) are used. It is seen that the proposed method has better average time complexity and equal cost performances than Ochi's routing approach. Therefore, the solutions with higher capacity vehicle of the proposed method that uses vehicle type information are better than those of the methods that use residual cost based on the vehicle type information.

Key Words: *Fleet Size and Mix Vehicle Routing Problem, Constructive Routing Heuristics, Vehicle Routing Problem, Routing Algorithm, Ochi's routing approach*

1. INTRODUCTION

In fleet size and mix vehicle routing problem (HFVRP) each customer is visited by exactly one route. HFVRP consists of designing a number of feasible paths having minimum total cost / total distance. Aim of HFVRP is mainly to determine the best fleet composition as well as the set of paths that minimize the sum of fixed and travel costs in such a way that:

(a) every route starts and ends at the depot and is associated to a vehicle type;
(b) each customer belongs to exactly one route; and
(c) the vehicle's capacity is not exceeded.

The HFVRP is an NP-hard problem and numerous methods have been proposed as it is a natural generalization of the travelling salesman problem (TSP) and as it includes the classical vehicle routing problem (VRP) [1-4].

Some researchers developed algorithms such as the savings algorithm of Clarke and Wright [5], the sweep algorithm of Gillett and Miller [6] and the generalized assigment of Fisher and Jaikumar [7]. Matching based saving algorithms were also proposed by Desrochers and Verhoog [8], Salhi and Rand [9] and Osman and Salhi [10]. Evolutionary algorithms have been attempted by Ochi et al. [1] and Lima et al. [2] on FSMF (Fleet Size and Mix Vehicle Routing Problem with Fixed Cost). However, the vehicle type information is always ignored in these methods [11]. In our study, a new constructive routing algorithm is proposed incorporating the vehicle type information.

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2. OCHI AND PROPOSED CONSTRUCTIVE ROUTING ALGORITHMS

A undirected graph is defined by $G = (V, A)$ where $V = \{0,1,2,...,n\}$ is a set composed of $n+1$ vertices, and $A = \{(i, j): i, j \in V, i \neq j\}$ is the set of arcs. The vertex 0 denotes the depot, where the vehicle fleet is initially located, while the set $V' = V - \{0\}$ is composed of the remaining vertices that represent n customers.

It is assumed that each customer $i \in V'$ has a positive demand q_i and depot's demand is always zero. $C = [c_{ij}]$ is the distance matrix where the parameter c_{ij} represents a positive cost or distance between vertices *i* and *j*. A heterogenous fleet of vehicles must be used to supply the customers. The vehicle fleet is composed by a set $\psi(k) \in \{1,2,...,t\}$ of different vehicle types where t is the number of vehicle types associated with the route, and it is assumed that each vehicle type is available at unlimited numbers. For each vehicle type $i \in \psi$, Q_i is the capacity, f_i is the fixed cost to be paid, and D_i is the amount of demand collected from or loaded to the vehicle. It is assumed that the fixed costs are increasing with the capacity i.e. $Q_1 < Q_2 < ... < Q_t$ and $f_1 < f_2 < ... < f_t$ [1-4].

A route for vehicle type *k* is defined by the pair $(R, \psi(k))$ where $R = (i_1, i_2, ..., i_{|R|})$, with $i_1 = i_{|R|} = 0$ and $\{i_2, i_3, ..., i_{|R|-1}\} \subseteq V$, is a simple circuit in G containing the depot. Here, R will be used to refer both to visiting sequence and to the set of customers (including depot) of the route. A route $(R, \psi(k))$ is feasible if the total demand of the customers visited by the route does not exceed the vehicle capacity Q_k, that is, $\sum_{h=2}^{|R|-1} q_{i_h} \le Q_k$. The cost of a route corresponds to the sum of the costs of the edges forming the route, plus the fixed cost of the associated vehicle, that is, $\sum_{h=1}^{|R|-1} c^k_{i_h i_{h+1}} + f_k$ [3].

The route configuration proposed by Ochi et al is achieved by selecting the minimum from the alternatives obtained by the constraint $(Q_k - D_k) * f_k$. However, our study is based on $(Q_k - D_k)$ constraint for route configuration and then selecting the minimum from the alternatives obtained. The constraints of the related routing strategies are given in Table 1. Ochi's approach solves the HVRP with Petal algorithm using the constraint $(Q_k - D_k) * f_k$. However, in [1], there is not any information about which vehicle will be assigned to the route related to this constraint.

Table 1. Proposed Approach and Ochi Approach routing constraint for HFVRP

	Ochi (1998)	Karagul
Route Construction / Route Selection Strategy	$\min_k (Q_k - D_k) * f_k$	$\min_k (Q_k - D_k)$

In our study, Ochi's approach is interpreted in terms of vehicle capacity and number of customers assigned to each route and with this interpretation Ochi's approach is denoted as **Ochi Minumum Distance Maximum Vertex Algorithm** (**Ochi MinDis-MaxVer Algorithm**). The proposed routing approach is denoted as **Karagul Minumum Distance Maximum Vertex Algorithm** (**Karagul MinDis-MaxVer Algorithm**). In Figure 1, the Ochi MinDis-MaxVer algorithm is demonstrated where the constraint $(Q_k - D_k) * f_k$ is defined as (Residuals*FixedCost). If any two paths has equal values of minimum(Residuals*FixedCost), the one with the maximum vertex is used.

```
start with the Initial Solution Space (TSP_order)
for each TSP_order
  while {end of the TSP_order}
     while {end of the number of vehicle_type}
        Construct  temporary routings for each vehicle_type
     end {of while}
        Find  the minimum(Residuals*FixedCost) that is the temporary Path
        If
                  there is only one min temporary routes
          Assign  the vertex  and vehicle type  to Route
        else
          there are equal  residuals  for temporary routes  more than one
          Find temporary route  with max  vertex
          Assign  the vertex  and vehicle type  to  Route
        end
        Assign  the vertex  and vehicle type  to the Path
        Calculate Routing_Cost, TSP_order, vehicle_type
   end {of while}
     TSP order solution: [Total_Cost  Routings  Type_of_Vehicles
TSP_order]
 end {of for}

Solution Space:[TSP_order solution [Total_Cost Routings Type_of_Vehicles
TSP_order] ]
```

Figure 1. Ochi Minimum Distance Maximum Vertex Algorithm (Ochi MinDis-MaxVer Algorithm)

In Figure 2, Karagul MinDis-MaxVer algorithm is given. For this algorithm, the constraint $\left(Q_k - D_k\right)$ is defined with (Residuals) instead of (Residuals*FixedCost). Similar to Ochi MinDis-MaxVer Algorithm, If any two paths has equal values of minimum(Residuals), the one with the maximum vertex is used.

```
start with the Initial Solution Space (TSP orders)
for each TSP order
  while {end of the TSP order}
     while {end of the number of vehicle type}
        Construct  temporary routings for each type of vehicle
     end {of while}
        Find  the minimum(Residuals) that is the temporary paths
        If
          there is only one min temporary routes
          Assign  the vertex  and vehicle type  to Route
        else
          there are equal  residuals  for temporary routes  more than one
          Find temporary route  with max  vertex
          Assign  the vertex  and vehicle type  to  Route
        end
        Assign  the vertex  and vehicle type  to Path
        Calculate Routing Cost, path part of TSP order, TSP order part,
vehicle type
   end {of while}
     TSP order solution: [Total Cost  Routings  Type of Vehicles  TSP
order]
 end {of for}

Solution Space:[TSP order solution [Total Cost Routings Type of Vehicles
TSP order] ]
```

Figure 2. Karagul Minimum Distance Maximum Vertex Algorithm (Karagul MinDis-MaxVer Algorithm)

3. SAMPLE PROBLEMS AND SOLUTION PHASES

In Figure 3, an HFVRP problem defined by Liu-Huang-Ma [13] is used in order to show the effectiveness of the proposed method. The problem is composed of a depot, two types of vehicles (t_1, t_2), and 6 customers. In Figure 4, the parameters defining the problem are given.

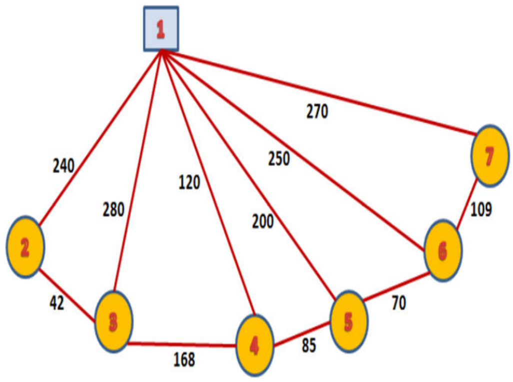

Figure 3. Representation of sample problem 1 and sample problem 2 with vertices and some connections

Sample Problem 1

Distance Matrix (C) ($c_{ij} = c_{ji}$)

	1	2	3	4	5	6	7
1	0	240	280	120	200	250	270
2		0	42	210	295	365	474
3			0	168	253	323	432
4				0	85	155	264
5					0	70	179
6						0	109
7							0

Vehicle Capacity	Vehicle Types
$Q_1 = 60$	t_1
$Q_2 = 90$	t_2
Fixed Costs	**Variable Costs**
$f_1 = 40$	$v_1 = 1$
$f_2 = 50$	$v_2 = 1$
Sum of Demand for Vehicle k	
D_k	

Depot/Vertex	1	2	3	4	5	6	7	
Demand Quantity	0	60	32	30	15	50	50	q_i

Figure 4. Sample problem 1's distance matrix, customer demands, vehicle types fixed costs and vehicle type's capacity.

In Figure 5, the solution routes of Ochi MinDis-MaxVer algorithm for a random TSP order {7, 3, 2, 4, 6, 5} is shown step by step.

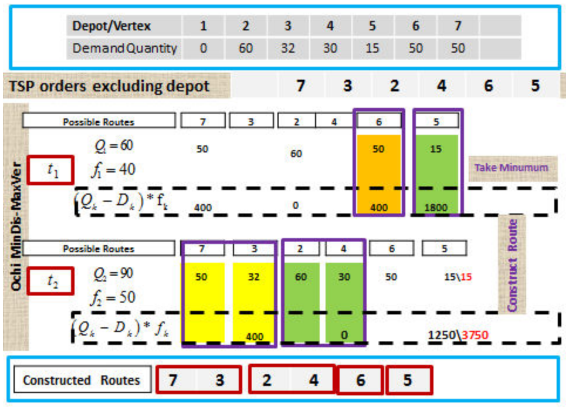

Figure 5. Ochi Minimum Distance Maximum Vertex Algorithm solution phases for routing TSP order {7, 3, 2, 4, 6, 5}

The vehicle routes are constructed with respect to $(Q_k - D_k) * f_k$ constraint and vehicles with higher capacities are considered first. As shown in Figure 5, when customer 7 is considered for t_1, the demand is 50 units. As the vehicle capacity will exceed 32, customer 3 cannot be added. Therefore, for vehicle t_1 the temporary route is {7}, the total load quantity is 50 units and residual is 60-50=10 units, and the residual cost is 10*40=400 unit cost.

Similar to t_1, when the demands of 50 units from customer 2, 32 units from customer 3 are loaded to the vehicle t_2 not to exceed 90 units capacity, 82 units loading is made in total. The temporary path {7, 3} is obtained. The residual for t_2 is 90-82=8 units and the residual cost is 8*50=400 unit cost.

When the residual costs of two vehicles are considered, it is seen that there is 400 units cost for t_1 and 400 units cost for t_2. In this case, there are two equal residual costs for temporary routes and the one with the maximum vertex is chosen. Therefore, the first constructed path R1=({1-7-3-1}, t_2) is taken as it has minimum residual cost. Then, customer {7, 3} is discarded from TSP order.

The unrouted customers {2, 4, 6, 5} are reconstructed for temporary routes. As can be seen from the second phase in Figure 5, firstly for vehicle t_1, 60 units from {2} is loaded. The residual is 0 units and the residual cost is 0 unit cost. Similarly, for vehicle t_2, 60 units from {2}, 30 units from {4} are loaded which in total compose 90 unit load. The residual is 0 units and the residual cost is 0 unit cost. In this case, there are two equal residual costs for temproray routes and therefore the one with the maximum vertex is chosen. Therefore, the second constructed path R2=({1-2-4-1}, t_2) is taken as it has the maximum vertex.

Then, customer {2, 4} are discarded from TSP order. The unrouted customers {6, 5} are reconstructed for temporary routes. As can be seen from the second phase in Figure 5, firstly for vehicle t_1, 50 units from {6} is loaded. The residual is 10 units and the residual cost is 400 units cost. Similarly, for vehicle t_2, 50 and 15 units from {6, 5} are loaded which in total compose 65 unit load. The residual is 25 units and the residual cost is 1250 unit cost. When the residual costs of two vehicles are considered for the second phase, it is seen that there is 400 units cost for t_1 and 1250 unit cost for t_2. Therefore, the third constructed path R3={1-6-1}, t_1) is taken as it has minimum residual cost. Then, customer {6} is discarded from TSP order and the unrouted customers {5} are reconstructed for temporary routes.

When the same process is executed for remaining customer {6}, the route and assigned vehicle is R4=({1-6-1}, t_1). Thus, for Ochi MinDis-MaxVer algorithm the routing process for the TSP orders is completed. The summary table for the routings and costs are given in Figure 6 and graph solution is given in Figure 7. Figure 8 gives us Sample Problem 2. And the solution routes of Karagul MinDis-MaxVer algorithm for different TSP orders are shown step by step in Figure 9.

Ochi MinDis-MaxVer

TSP orders excluding depot		7 3	2 4	6	5
Routes	T	Fixed Cost	Route Distances	Total Distance	Route Cost
R1=[1-7-3-1]	t2	50	(270+432+280)	982	1032
R2=[1-2-4-1]	t2	50	(240+210+120)	570	620
R3=[1-6-1]	t1	40	(250+250)	500	540
R4=[1-5-1]	t1	40	(200+200)	400	440
Total Cost					2632
Best Known Solution (Estimated Value)					2228

Figure 6. Ochi Minimum Distance Maximum Vertex Algorithm routes solutions for TSP order {7, 3, 2, 4, 6, 5}

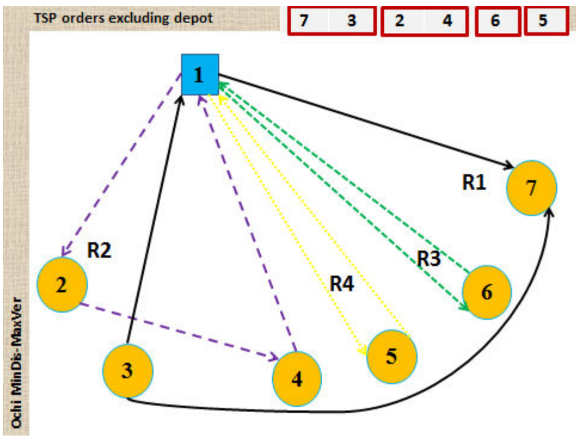

Figure 7. Ochi Minimum Distance Maximum Vertex Algorithm graph solution for routing TSP order {7, 3, 2, 4, 6, 5}

Sample Problem 2

Distance Matrix (C) ($c_{ij} = c_{ji}$)

	1	2	3	4	5	6	7
1	0	240	280	120	200	250	270
2		0	42	210	295	365	474
3			0	168	253	323	432
4				0	85	155	264
5					0	70	179
6						0	109
7							0

Vehicle Capacity	Vehicle Types
$Q_1 = 600$	t_1
$Q_2 = 900$	t_2
Fixed Costs	**Variable Costs**
$f_1 = 400$	$v_1 = 1$
$f_2 = 500$	$v_2 = 1$
Sum of Demand for Vehicle k	
D_k	

Depot/Vertex	1	2	3	4	5	6	7	
Demand Quantity	0	600	320	300	150	500	500	q_i

Figure 8. Sample problem 2's distance matrix, customer demands, vehicle types fixed costs and vehicle types capacity

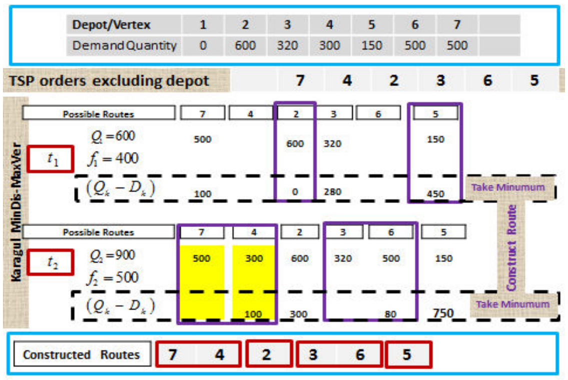

Figure 9. Karagul Minimum Distance Maximum Vertex Algorithm solution phases for routing TSP order {7, 4, 2, 3, 6, 5}

The vehicle routes are constructed with respect to $(Q_k - D_k)$ constraint and vehicles with higher capacities are considered first. As shown in Figure 9, when customer {7} is considered for t_1, the demand is 500 units. As the vehicle capacity will exceed 300, customer {4} cannot be added. Therefore, for vehicle t_1 the temporary route is {7}, the total load quantity is 500 units and residual is 600-500=100 units.

Similar to t_1, when the demands of 500 units from customer {7}, 300 units from customer {4} are loaded to the vehicle t_2 not to exceed 900 units capacity, 800 units loading is made in total. The temporary path {7, 4} is obtained. The residual for t_2 is 900-800=100 units.

When the residuals of two vehicles are considered, it is seen that there is 100 units for t_1 and 100 units for t_2. There are two equal residuals, this means to take path with maximum vertex. Therefore, the first constructed path R1=({1-7-4-1}, t_2) is taken as it has minimum residual with maximum vertex. Then, customer {7, 4} is discarded from TSP order.

The unrouted customers {2, 3, 6, 5} are reconstructed for temporary routes. As can be seen from the second phase in Figure 9, firstly for vehicle t_1, 600 units from {2} is loaded which in total compose 600 unit load. The residual is 0 units. Similarly, for vehicle t_2, 600 units from {2}, 300 units from {4} are loaded which in total compose 900 units load. The residual is 0 units. When the residual of two vehicles are considered for the second phase, it is seen that there is 0 units for t_1 and 0 units for t_2. Therefore, the second constructed path R2= ({1-2-4-1}, t_2) is taken as it has minimum residual and maximum vertex. Then, customer {2,4} are discarded from TSP order. The unrouted customers {6, 5} are reconstructed for temporary routes. As can be seen from the second phase in Figure 9, firstly for vehicle t_1, 500 units from {6} is loaded. The residual is 100 units. Similarly, for vehicle t_2, 650 units from {6, 5} are loaded. The residual is 250 units. Therefore, the third constructed path R3= ({1-6-1}, t_1) is taken as it has minimum residual. Then, customer {6} is discarded from TSP order and the unrouted customer {5} is reconstructed for temporary routes.

When the same process is executed for remaining customer {5}, the route and assigned vehicle are R4=({1-5-1}, t_1). Thus, for Karagul MinDis-MaxVer algorithm the routing process for the TSP orders is completed. The summary table for the routings and costs are given in Figure 10. Karagul MinDis-MaxVer algorithm graph solution is given in Figure 11.

Karagul MinDis-MaxVer	TSP orders excluding depot		7	4	2	3	6	5
	Routes	T	Fixed Cost	Route Distances		Total Distance		Route Cost
	R1=[1-7-4-1]	t2	500	(270+264+120)		654		1154
	R2=[1-2-1]	t1	400	(240+240)		480		880
	R3=[1-3-6-1]	t2	500	(280+323+250)		853		1353
	R4=[1-5-1]	t1	400	(200+200)		400		800
	Total Cost							4187
	Best Known Solution							3908

Figure 10. Karagul Minimum Distance Maximum Vertex Algorithm routes solutions for TSP order {7, 4, 2, 3, 6, 5}

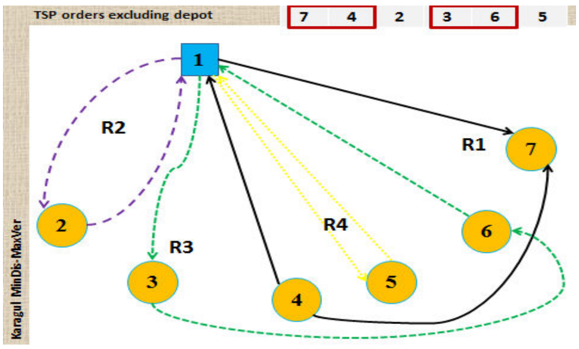

Figure 11. Karagul Minimum Distance Minimum Vertex Algorithm graph solution for TSP order {2, 3, 4, 5, 6, 7}

TSP orders excluding depot		R Name	Route	T	Fixed Costs	Route Distance	Route Cost
7 4 2 3 6 5	Ochi MinDis-MaxVer	R1	1-7-3-1	t2	50	982	1032
		R2	1-2-4-1	t2	50	570	620
		R3	1-6-1	t1	40	500	540
		R4	1-5-1	t1	40	400	440
					Total	2452	2632
7 3 2 4 6 5	Karagul MinDis-MaxVer	R1	1-7-4-1	t2	500	654	1154
		R2	1-2-1	t1	400	480	880
		R3	1-3-6-1	t2	500	853	1353
		R4	1-5-1	t1	400	400	800
					Total	2387	4187

Figure 12. Ochi and Karagul Routings Algorithms solutions results for TSP orders that are {7, 4, 2, 3, 6, 5} and {7, 3, 2, 4, 6, 5}

As can be seen from Figure 12, different routes are obtained by using the given algorithms. The graph of routes for both Ochi and Karagul routing algorithms are given in Figure 12.
From the sample routing problems, In the next section, to see the performance of the proposed method and to analyses the difference of the proposed method from Ochi's approach, some known test problems from the literature, especially Golden's test instances, are used.

4. COMPUTATIONAL RESULT

The proposed method is tested by using 12 sample problems obtained by Golden et al. [12] and extensively used in the literature for FSMF.

The calculations are constructed from two phases: the first step is obtaining the initial solution space, and the second step is the route configuration and the selection of the appropriate constraint. The initial solution space is generated based on the method presented by Liu et al. [13] where the initial solution space is composed of 3 parts: first part from the Savings Algorithm, second part from the Sweep Algorithms and the rest of the individuals are generated randomly. In our study, on the other hand, the randomly generated individuals are not used. The solutions of the Savings and Sweep algorithms are obtained by using "**Matlog**: Matlab Logistic Engineering Toolbox" [14].

The problems are tested on a computer with Pentium Core Duo i7 processor and 4 GB RAM. The results obtained on the basis of the initial solutions from Sweep and Savings algorithms are listed in Table 2 where **P.No** is the problem number as given by Golden et al. [12], **BKS** is the best known solution in the literature, **Solution** is the Karagul and Ochi solutions obtained for the given problems with this study, **Deviation** is the percent deviation from the best known solution, Time is the solution time in seconds and **S.S**. is the dimension of the initial solution space. The initial solutions are obtained excluding the depot in the form of **TSP order**. Then the routes are configured with respect to the related methods. From the alternative route solutions, the type of vehicle that provides the minimum condition is selected as the optimal route. For the solution times in Table 2, the period for obtaining the initial solutions are not considered. Therefore, the solution times are solely giving the execution times (seconds) of the algorithms.

Table 2. Ochi MinDis-MaxVer Algorithm and Karagul MinDis-MaxVer Algorithm computational results for FSMVRP with fixed cost (FSMF) on 12 test problems

		Ochi MinDis-MaxVer Algorithm			Karagul MinDis-MaxVer Algorithm			
P.No	BKS	Solution	Deviation	Time	Solution	Deviation	Time	S. S.
3	961,03	1.088,70	-13,28	0,1085	999,20	**-3,97**	0,0890	4
4	6.437,30	7.324,70	-13,78	0,0898	7324,7	-13,78	0,0840	6
5	1.007,10	1.153,00	-14,49	0,0711	1097,4	**-8,97**	0,0629	4
6	6.516,50	7.031,40	-7,90	0,0582	7.031,40	-7,90	0,0565	6
13	2.406,40	2.670,70	**-10,98**	0,2457	2.680,20	-11,38	0,1692	8
14	9.119,00	9.214,40	-1,05	0,0681	9.214,40	-1,05	0,0721	6
15	2.586,40	2.800,10	**-8,26**	0,0869	2.861,20	-10,63	0,0780	6
16	2.720,40	3.063,80	-12,62	0,0730	2.899,00	**-6,56**	0,0668	4
17	1.734,50	2.088,90	-20,43	0,1572	1.954,10	**-12,66**	0,1133	8
18	2.369,70	2.992,40	-26,28	0,3156	2.986,50	-26,03	0,1776	10
19	8.661,80	9.599,20	**-10,82**	0,0889	9.824,80	-13,43	0,0992	6
20	4.039,50	4.459,10	**-10,39**	0,1197	4.498,90	-11,37	0,1119	6
Average	4.046,64	4.457,20	-12,52	0,1235	**4.447,65**	**-10,65**	**0,0984**	

When Table 2 is reviewed, for 4 of 12 test problems **Karagul MinDis-MaxVer Algorithm** has better total cost values. Also, for 4 problems it has same total costs with Ochi MinDis-MaxVer Algorithm. For 4 of 12 test problems, Ochi MinDis-MaxVer Algorithm have best total cost values. When the average performances are compared, the proposed method has better characteristics from time complexity and total cost point of view. Based on the given tests, Karagul MinDis-MaxVer Algorithm can be proposed as a new constructive heuristic routing algorithm for HFVRP.

5. CONCLUSION AND DISCUSSION

In this study, a new constructive route configuration different from the method recommended by Ochi et al and an approach certainly competitive with their method is proposed. The problems in the literature are solved using a seeding with Sweep and Savings algorithms proposed by Liu-Huang-Ma [13]. When the proposed method is logically compared for different situations, it gives better results than the approach of Ochi et al [1]. Thus, the new method can be suggested both for route configuration and route selection in heterogeneous VRPs. The solution given in this study can be enriched using different initial solution generation methods and new hybrid solution methods can be obtained by combining with heuristic search methods.

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CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

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