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# Generalization of the space $l(p)$ derived by absolute Euler summability and matrix operators

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**Abstract**

The sequence space $l(p)$ having an important role in summability theory was defined and studied by Maddox (Q. J. Math. 18:345–355, 1967). In the present paper, we generalize the space $l(p)$ to the space $|E^r_\phi|(p)$ derived by the absolute summability of Euler mean. Also, we show that it is a paranormed space and linearly isomorphic to $l(p)$. Further, we determine $\alpha$-, $\beta$-, and $\gamma$-duals of this space and construct its Schauder basis. Also, we characterize certain matrix operators on the space.

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**Keywords:** Absolute summability; Euler means; Matrix transformations; Sequences spaces

## 1 Introduction

Let $X$, $Y$ be any subsets of $\omega$, the set of all sequences of complex numbers, and $A = (a_{nv})$ be an infinite matrix of complex numbers. By $A(x) = (A_n(x))$, we indicate the $A$-transform of a sequence $x = (x_v)$ if the series

$$A_n(x) = \sum_{v=0}^{\infty} a_{nv}x_v$$

are convergent for $n \geq 0$. If $Ax \in Y$, whenever $x \in X$, then $A$, denoted by $A : X \to Y$, is called a matrix transformation from $X$ into $Y$, and we mean the class of all infinite matrices $A$ such that $A : X \to Y$ by $(X, Y)$. For $c_s$, $b_s$, and $l_p$ ($p \geq 1$), we write the space of all convergent, bounded, $p$-absolutely convergent series, respectively. Further, the matrix domain of an infinite matrix $A$ in a sequence space $X$ is defined by

$$X_A = \{x = (x_n) \in \omega : A(x) \in X\}. \tag{1}$$

The $\alpha$-, $\beta$-, and $\gamma$-duals of the space $X$ are defined as follows:

$$X^\alpha = \{\epsilon \in \omega : (\epsilon_n x_n) \in l_1 \text{ for all } x \in X\},$$
$$X^\beta = \{\epsilon \in \omega : (\epsilon_n x_n) \in c_s \text{ for all } x \in X\},$$
$$X^\gamma = \{\epsilon \in \omega : (\epsilon_n x_n) \in b_s \text{ for all } x \in X\}.$$

A subspace $X$ is called an $FK$ space if it is a Frechet space, that is, a complete locally convex linear metric space, with continuous coordinates $P_n : X \to C$ $(n = 1, 2, \ldots)$, where $P_n(x) = x_n$ for all $x \in X$; an $FK$ space whose metric is given by a norm is said to be a $BK$ space. An $FK$ space $X$ including the set of all finite sequences is said to have $AK$ if

$$\lim_{m\to\infty} x^{[m]} = \lim_{m\to\infty} \sum_{v=0}^{m} x_v e^{(v)} = x$$

for every sequence $x \in X$, where $e^{(v)}$ is a sequence whose only non-zero term is one in $v$th place for $v \geq 0$. For example, it is well known that the Maddox space

$$l(p) = \left\{ x = (x_n) : \sum_{n=1}^{\infty} |x_n|^{p_n} < \infty \right\}$$

is an $FK$ space with $AK$ with respect to its natural paranorm

$$g(x) = \left( \sum_{n=0}^{\infty} |x_n|^{p_n} \right)^{1/M},$$

where $M = \max\{1, \sup_n p_n\}$; also it is even a $BK$ space if $p_n \geq 1$ for all $n$ with respect to the norm

$$\|x\| = \inf\left\{ \delta > 0 : \sum_{n=0}^{\infty} |x_n/\delta|^{p_n} \leq 1 \right\}$$

([19–21, 29]).

Throughout this paper, we assume that $0 < \inf p_n \leq H < \infty$ and $p_n^*$ is a conjugate of $p_n$, *i.e.*, $1/p_n + 1/p_n^* = 1$, $p_n > 1$, and $1/p_n^* = 0$ for $p_n = 1$.

Let $\sum a_v$ be a given infinite series with $s_n$ as its $n$th partial sum, $\phi = (\phi_n)$ be a sequence of positive real numbers and $p = (p_n)$ be a bounded sequence of positive real numbers. The series $\sum a_v$ is said to be summable $|A, \phi_n|(p)$ if (see [10])

$$\sum_{n=1}^{\infty} (\phi_n)^{p_n-1} \left| A_n(s) - A_{n-1}(s) \right|^{p_n} < \infty.$$

It should be noted that the summability $|A, \phi_n|(p)$ includes some well-known summability methods for special cases of $A$, $\phi$ and $p = (p_n)$. For example, if we take $A = E^r$ and $p_n = k$ for all $n$, then it is reduced to the summability method $|E, r|_k$ (see [12]) where Euler matrix $E^r$ is defined by

$$e_{nk}^r = \begin{cases} \binom{n}{k}(1-r)^{n-k} r^k, & 0 \leq k \leq n, \\ 0, & k > n, \end{cases}$$

for $0 < r < 1$ and

$$e_{nk}^1 = \begin{cases} 0, & 0 \leq k < n, \\ 1, & k = n. \end{cases}$$

Also we refer the readers to the papers [7, 9, 30, 31, 35] for detailed terminology.

A large literature body, concerned with producing sequence spaces by means of matrix domain of a special limitation method and studying their algebraic, topological structure and matrix transformations, has recently grown. In this context, the sequence spaces $\bar{l}(p)$, $r_p^t$, $l(u,v,p)$, and $l(N^t,p)$ were studied by Choudhary and Mishra [8], Altay and Başar [2, 3], Yeşilkayagil and Başar [37] by defining as the domains of the band, Riesz, the factorable, and Nörlund matrices in the $l(p)$ (see also [1, 4–6, 16–18, 23–28]).

Also, some series spaces have been derived and examined by various absolute summability methods from a different point of view (see [13, 14, 32, 34]). In this paper, we generalize the space $l(p)$ to the space $|E_\phi^r|(p)$ derived by the absolute summability of Euler means and show that it is a paranormed space linearly isomorphic to $l(p)$. Further, we determine $\alpha$-, $\beta$-, and $\gamma$-duals of this space and construct its Schauder basis. Finally, we characterize certain matrix transformations on the space.

First, we remind some well-known lemmas which play important roles in our research.

## 2 Needed lemmas

**Lemma 2.1** ([11]) *Let $p=(p_v)$ and $q=(q_v)$ be any two bounded sequences of strictly positive numbers.*

(i) *If $p_v>1$ for all $v$, then $A\in(l(p),l_1)$ if and only if there exists an integer $M>1$ such that*

$$\sup\left\{\sum_{v=0}^{\infty}\left|\sum_{n\in K}a_{nv}M^{-1}\right|^{p_v^*}:K\subset N \text{ finite}\right\}<\infty. \tag{2}$$

(ii) *If $p_v\le 1$ and $q_v\ge 1$ for all $v\in N$, then $A\in(l(p),l(q))$ if and only if there exists some $M$ such that*

$$\sup_v\sum_{n=0}^{\infty}\left|a_{nv}M^{-1/p_v}\right|^{q_n}<\infty.$$

(iii) *If $p_v\le 1$, then $A\in(l(p),c)$ if and only if*

$$\text{(a)}\quad \lim_n a_{nv} \text{ exists for each } v, \qquad \text{(b)}\quad \sup_{n,v}|a_{nv}|^{p_v}<\infty,$$

*and $A\in(l(p),l_\infty)$ iff* (b) *holds.*

(iv) *If $p_v>1$ for all $v$, then $A\in(l(p),c)$ if and only if* (a) (a) *holds, and* (b) *there is a number $M>1$ such that*

$$\sup_n\sum_{v=0}^{\infty}\left|a_{nv}M^{-1}\right|^{p_v^*}<\infty,$$

*and $A\in(l(p),l_\infty)$ iff* (b) *holds.*

It may be noticed that condition (2) exposes a rather difficult condition in applications. The following lemma produces a condition to be equivalent to (2) and so the following lemma, which is more practical in many cases, will be used in the proofs of theorems.

**Lemma 2.2** ([33]) *Let $A = (a_{nv})$ be an infinite matrix with complex numbers and $(p_v)$ be a bounded sequence of positive numbers. If $U_p[A] < \infty$ or $L_p[A] < \infty$, then*

$$(2C)^{-2}U_p[A] \le L_p[A] \le U_p[A],$$

*where $C = \max\{1, 2^{H-1}\}$, $H = \sup_v p_v$,*

$$U_p[A] = \sum_{v=0}^{\infty}\left(\sum_{n=0}^{\infty}|a_{nv}|\right)^{p_v}$$

*and*

$$L_p[A] = \sup\left\{\sum_{v=0}^{\infty}\left|\sum_{n\in K}a_{nv}\right|^{p_v} : K \subset N \text{ finite}\right\}.$$

**Lemma 2.3** ([22]) *Let $X$ be an FK space with AK, $T$ be a triangle, $S$ be its inverse, and $Y$ be an arbitrary subset of $\omega$. Then we have $A \in (X_T, Y)$ if and only if $\widehat{A} \in (X, Y)$ and $V^{(n)} \in (X, c)$ for all $n$, where*

$$\hat{a}_{nv} = \sum_{j=v}^{\infty} a_{nj}s_{jv}; \quad n, v = 0, 1, \ldots,$$

*and*

$$v_{mv}^{(n)} = \begin{cases} \sum_{j=v}^{m} a_{nj}s_{jv}, & 0 \le v \le m, \\ 0, & v > m. \end{cases}$$

## 3 Main theorems

In this section, we introduce the paranormed series space $|E_\phi^r|(p)$ as the set of all series summable by the absolute summability method of Euler matrix and show that this space is linearly isomorphic to the space $l(p)$. Also, we compute the Schauder base, $\alpha$-, $\beta$-, and $\gamma$-duals of the space and characterize certain matrix transformations defined on that space.

First of all, we note that, by the definition of the summability $|A, \phi_n|(p)$, we can write the space $|E_\phi^r|(p)$ as

$$\left|E_\phi^r\right|(p) = \left\{a \in \omega : \sum_{n=0}^{\infty} \phi_n^{p_n-1}\left|\triangle A_n^r(s)\right|^{p_n} < \infty\right\},$$

where

$$\triangle A_n^r(s) = A_n^r(s) - A_{n-1}^r(s)$$

and

$$A_n^r(s) = \sum_{k=0}^{n}\binom{n}{k}(1-r)^{n-k}r^k s_k, \quad n \ge 0, \qquad A_{-1}^r(s) = 0.$$

Also, a few calculations give

$$\begin{aligned}\triangle A_n^r(s) &= \sum_{m=0}^{n}\sum_{k=m}^{n}\binom{n}{k}(1-r)^{n-k}r^k a_m - \sum_{m=0}^{n-1}\sum_{k=m}^{n-1}\binom{n-1}{k}(1-r)^{n-1-k}r^k a_m \\ &= \sum_{m=1}^{n}\sum_{k=m}^{n}(1-r)^{n-1-k}\left[\binom{n-1}{k-1} - r\binom{n}{k}\right]r^k a_m \\ &= \sum_{m=1}^{n}\sigma_{nm}a_m,\end{aligned}$$

where

$$\sigma_{nm} = \begin{cases}\sum_{k=m}^{n}(1-r)^{n-1-k}r^k[\binom{n-1}{k-1} - r\binom{n}{k}], & 1 \le m \le n, \\ 0, & m > n.\end{cases}$$

Further, it follows by putting $r = q(1+q)^{-1}$

$$\begin{aligned}\sigma_{nm} &= (1+q)^{1-n}\sum_{k=m}^{n}q^k\left[\binom{n-1}{k-1} - q(1+q)^{-1}\binom{n}{k}\right] \\ &= (1+q)^{-n}\sum_{k=m}^{n}\left[q^k\binom{n-1}{k-1} - q^{k+1}\binom{n-1}{k}\right] \\ &= q^m(1+q)^{-n}\binom{n-1}{m-1} = \binom{n-1}{m-1}(1-r)^{n-m}r^m.\end{aligned}$$

Now, by considering $T_n^r(\phi,p)(a) = \phi_n^{1/p_n^*} \triangle A_n^r(s)$, we immediately get that $T_0^r(\phi,p)(a) = a_0\phi_0^{1/p_0^*}$ and

$$\begin{aligned}T_n^r(\phi,p)(a) &= \phi_n^{1/p_n^*}\sum_{k=1}^{n}\binom{n-1}{k-1}(1-r)^{n-k}r^k a_k \\ &= \sum_{k=1}^{n}t_{nk}^r(\phi,p)a_k,\end{aligned} \tag{3}$$

where

$$t_{nk}^r(\phi,p) = \begin{cases}\phi_0^{1/p_0^*}, & k = n = 0, \\ \phi_n^{1/p_n^*}\binom{n-1}{k-1}(1-r)^{n-k}r^k, & 1 \le k \le n, \\ 0, & k > n.\end{cases} \tag{4}$$

Therefore, we can state the space $|E_\phi^r|(p)$ as follows:

$$\left|E_\phi^r\right|(p) = \left\{a = (a_k) : \sum_{n=1}^{\infty}\left|\phi_n^{1/p_n^*}\sum_{k=1}^{n}\binom{n-1}{k-1}(1-r)^{n-k}r^k a_k\right|^{p_n} < \infty\right\},$$

or

$$\left|E_\phi^r\right|(p) = \left[l(p)\right]_{T^r(\phi,p)}$$

according to notation (1).

Further, since every triangle matrix has a unique inverse which is a triangle (see [36]), the matrix $T^r(\phi,p)$ has a unique inverse $S^r(\phi,p) = (s_{nk}^r(\phi,p))$ given by

$$s_{nk}^r(\phi,p) = \begin{cases} \phi_0^{-1/p_0^*}, & k = n = 0, \\ \phi_k^{-1/p_k^*}\binom{n-1}{k-1}(r-1)^{n-k}r^{-n}, & 1 \le k \le n, \\ 0, & k > n. \end{cases} \quad (5)$$

Before main theorems, note that if $r = 1$ and $\phi_n = 1$ for all $n \ge 0$, the space $|E_\phi^r|(p)$ is reduced to the space $l(p)$.

**Theorem 3.1** *Let* $0 < r < 1$ *and* $p = (p_n)$ *be a bounded sequence of non-negative numbers. Then*:

(a) *The set* $|E_\phi^r|(p)$ *becomes a linear space with the coordinate-wise addition and scalar multiplication, and also it is an FK-space with respect to the paranorm*

$$\|x\|_{|E_\phi^r|(p)} = \left(\sum_{n=0}^{\infty}\left|T_n^r(\phi,p)(x)\right|^{p_n}\right)^{1/M},$$

*where* $M = \max\{1, \sup p_n\}$.

(b) *The space* $|E_\phi^r|(p)$ *is linearly isomorphic to the space* $l(p)$, *i.e.*, $|E_\phi^r|(p) \cong l(p)$.

(c) *Define a sequence* $(b_n^{(\nu)})$ *by* $S^r((e^{(\nu)})) = (\sum_{\nu=0}^n s_{n\nu}^r(\phi,p)e^{(\nu)})$. *Then the sequence* $(b_n^{(\nu)})$ *is the Schauder base of the space* $|E_\phi^r|(p)$.

(d) *The space* $|E_\phi^r|(p)$ *is separable*.

*Proof* (a) The first part is a routine verification, so it is omitted. Since $T^r(\phi,p)$ is a triangle matrix and $l(p)$ is an $FK$-space, it follows from Theorem 4.3.2 in [36] that $|E_\phi^r|(p) = [l(p)]_{T^r(\phi,p)}$ is an $FK$-space.

(b) We should show that there exists a linear bijection between the spaces $|E_\phi^r|(p)$ and $l(p)$. Now, consider $T^r(\phi,p) : |E_\phi^r|(p) \to l(p)$ given by (3). Since the matrix corresponding this transformation is a triangle, it is obvious that $T^r(\phi,p)$ is a linear bijection. Furthermore, since $T^r(\phi,p)(x) \in l(p)$ for $x \in |E_\phi^r|(p)$, we get

$$\|x\|_{|E_\phi^r|(p)} = \left(\sum_{n=0}^{\infty}\left|T_n^r(\phi,p)(x)\right|^{p_n}\right)^{1/M} = \left\|T^r(\phi,p)(x)\right\|_{l(p)}.$$

So, $T^r(\phi,p)$ preserves the paranorm, which completes this part of the proof.

(c) Since the sequence $(e^{(\nu)})$ is the Schauder base of the space $l(p)$ and $|E_\phi^r|(p) = [l(p)]_{T^r(\phi,p)}$, it can be written from Theorem 2.3 in [15] that $b^{(\nu)} = (S^r(\phi,p)(e^{(\nu)}))$ is a Schauder base of the space $|E_\phi^r|(p)$.

(d) Since the space $|E_\phi^r|(p)$ is a linear metric space with a Schauder base, it is separable. □

**Theorem 3.2** *Let* $0 < r < 1$. *Define*

$$D_1^r = \left\{ a \in \omega : \exists M > 1, \sum_{\nu=0}^{\infty} \left( \sum_{n=\nu}^{\infty} \left| M^{-1} b_n^{(\nu)} a_n \right| \right)^{p_\nu^*} < \infty \right\},$$

$$D_2^r = \left\{ a \in \omega : \exists M > 1, \sup_\nu M^{1/p_\nu} \sum_{n=\nu}^{\infty} \left| b_n^{(\nu)} a_n \right| < \infty \right\},$$

$$D_3^r = \left\{ a \in \omega : \sum_{n=\nu}^{\infty} b_n^{(\nu)} a_n \text{ converges for each } \nu \right\},$$

$$D_4^r = \left\{ a \in \omega : \exists M > 1, \sup_n \sum_{\nu=1}^{n} \left| \sum_{k=\nu}^{n} b_k^{(\nu)} a_k M^{-1} \right|^{p_\nu^*} < \infty \right\},$$

$$D_5^r = \left\{ a \in \omega : \sup_{n,\nu} \left| \sum_{k=\nu}^{n} b_k^{(\nu)} a_k \right|^{p_\nu} < \infty \right\}.$$

(i) *If* $p_\nu > 1$ *for all* $\nu$, *then*

$$\{|E_\phi^r|(p)\}^\alpha = D_1^r, \qquad \{|E_\phi^r|(p)\}^\beta = D_4^r \cap D_3^r, \qquad \{|E_\phi^r|(p)\}^\gamma = D_4^r.$$

(ii) *If* $p_\nu \leq 1$ *for all* $\nu$, *then*

$$\{|E_\phi^r|(p)\}^\alpha = D_2^r, \qquad \{|E_\phi^r|(p)\}^\beta = D_5^r \cap D_3^r, \qquad \{|E_\phi^r|(p)\}^\gamma = D_5^r.$$

*Proof* To avoid the repetition of a similar statement, we only calculate $\beta$-duals of $|E_\phi^r|(p)$.

(i) Let us recall that $a \in \{|E_\phi^r|(p)\}^\beta$ if and only if $ax \in cs$ whenever $x \in |E_\phi^r|(p)$. Now, by using (5), it can be obtained that

$$\begin{aligned} \sum_{k=0}^{n} a_k x_k &= T_0^r(\phi,p)(x)\phi_0^{-1/p_0^*} a_0 + \sum_{k=1}^{n} a_k \sum_{\nu=1}^{k} \phi_\nu^{-1/p_\nu^*} \binom{k-1}{\nu-1} (r-1)^{k-\nu} r^{-k} T_\nu^r(\phi,p)(x) \\ &= T_0^r(\phi,p)(x)\phi_0^{-1/p_0^*} a_0 + \sum_{\nu=1}^{n} \phi_\nu^{-1/p_\nu^*} T_\nu^r(\phi,p)(x) \sum_{k=\nu}^{n} a_k \binom{k-1}{\nu-1} (r-1)^{k-\nu} r^{-k} \\ &= \sum_{\nu=0}^{n} d_{n\nu} T_\nu^r(\phi,p)(x), \end{aligned}$$

where $D = (d_{n\nu})$ is defined by

$$d_{n\nu} = \begin{cases} \phi_0^{-1/p_0^*} a_0, & n = \nu = 0, \\ \sum_{k=\nu}^{n} b_k^{(\nu)} a_k, & 1 \leq \nu \leq n, \\ 0, & \nu > n. \end{cases}$$

Since $T^r(\phi,p)(x) \in l(p)$ whenever $x \in |E_\phi^r|(p)$, $a \in \{|E_\phi^r|(p)\}^\beta$ if and only if $D \in (l(p), c)$. So, it follows from Lemma 2.1 that $a \in D_4^r \cap D_3^r$ if $p_\nu > 1$ for all $\nu$, and also $a \in D_5^r \cap D_3^r$ if $p_\nu \leq 1$ for all $\nu$.

The remaining part of the theorem can be similarly proved by Lemma 2.1. □

**Theorem 3.3** *Let $A = (a_{n\nu})$ be an infinite matrix of complex numbers, $(\phi_n)$ and $(\psi_n)$ be sequences of positive numbers, $p = (p_n)$ and $q = (q_n)$ be arbitrary bounded sequences of positive numbers with $p_n \leq 1$ and $q_n \geq 1$ for all $n$. Further, let the matrix $\hat{A}$ be defined by*

$$\hat{a}_{n\nu} = \sum_{j=\nu}^{\infty} a_{nj} b_j^{(\nu)}$$

*and $F = T^r(\psi, q)\hat{A}$. Then $A \in (|E_\phi^r|(p), |E_\psi^r|(q))$ if and only if there exists an integer $M > 1$ such that, for $n = 0, 1, \ldots,$*

$$\sum_{k=\nu}^{\infty} b_k^{(\nu)} a_{nk} \quad \text{converges for each } \nu, \tag{6}$$

$$\sup_{m,\nu} \left| \sum_{k=\nu}^{m} b_k^{(\nu)} a_{nk} \right|^{p_\nu} < \infty, \tag{7}$$

*and*

$$\sup_{\nu} \sum_{n=0}^{\infty} \left| M^{-1/p_\nu} f_{n\nu} \right|^{q_n} < \infty. \tag{8}$$

*Proof* Suppose that $p_\nu \leq 1$, $q_\nu \geq 1$ for all $\nu$. Note that $|E_\phi^r|(p) = [l(p)]_{T^r(\phi,p)}$ and $|E_\psi^r|(q) = [l(q)]_{T^r(\psi,q)}$. By Lemma 2.3, $A \in (|E_\phi^r|(p), |E_\psi^r|(q))$ if and only if $\hat{A} \in (l(p), |E_\psi^r|(q))$ and $V^{(n)} \in (l(p), c)$, where the matrix $V^{(n)}$ is defined by

$$v_{m\nu}^{(n)} = \begin{cases} \sum_{j=\nu}^{m} b_j^{(\nu)} a_{nj}, & 0 \leq \nu \leq m, \\ 0, & \nu > m. \end{cases}$$

One can see that since $\hat{A}(x) \in |E_\psi^r|(q) = [l(q)]_{T^r(\psi,q)}$ whenever $x \in l(p)$, $\hat{A} \in (l(p), |E_\psi^r|(q))$ iff $F = T^r(\psi, q)\hat{A} \in (l(p), l(q))$. Now, applying Lemma 2.1(ii) and (iii) to the matrices $F$ and $V^{(n)}$, it follows that $V^{(n)} \in (l(p), c)$ iff, for $n = 0, 1, \ldots,$ conditions (6) and (7) hold, and $F \in (l(p), l(q))$ iff there exists an integer $M$ such that

$$\sup_{\nu} \sum_{n=0}^{\infty} \left| M^{-1/p_\nu} f_{n\nu} \right|^{q_n} < \infty,$$

which completes the proof. □

**Theorem 3.4** *Assume that $A = (a_{n\nu})$ is an infinite matrix of complex numbers and $(\phi_n)$, $(\psi_n)$ are sequences of positive numbers. If $p = (p_n)$ is an arbitrary bounded sequence of positive numbers such that $p_n > 1$ for all $n$, and $H = T^r(\psi, 1)\hat{A}$, then $A \in (|E_\phi^r|(p), |E_\psi^r|(1))$ if and only if there exists an integer $M > 1$ such that, for $n = 0, 1, \ldots,$*

$$\sum_{k=\nu}^{\infty} b_k^{(\nu)} a_{nk} \quad \text{converges for each } \nu \tag{9}$$

$$\sup_{n} \sum_{\nu=0}^{\infty} \left| \sum_{k=\nu}^{n} b_k^{(\nu)} a_{nk} M^{-1} \right|^{p_\nu^*} < \infty \tag{10}$$

*and*

$$\sum_{\nu=0}^{\infty}\left(\sum_{n=0}^{\infty}\left|M^{-1}h_{n\nu}\right|\right)^{p_\nu^*}<\infty. \tag{11}$$

*Proof* Let $p_n > 1$ for all $n$. It is clear that $|E_\phi^r|(p) = [l(p)]_{T^r(\phi,p)}$ and $|E_\psi^r|(1) = l_{T^r(\psi,1)}$. So, by Lemma 2.3, we have $A \in (|E_\phi^r|(p), |E_\psi^r|(1))$ if and only if $\hat{A} \in (l(p), |E_\psi^r|(1))$ and $V^{(n)} \in (l(p), c)$, where $\hat{A}$ and $V^{(n)}$ are given in Theorem 3.3. If we take $H = T^r(\psi, 1)\hat{A}$, then it is easily seen that $\hat{A} \in (l(p), |E_\psi^r|(1))$ iff $H \in (l(p), l_1)$ because, if $\hat{A}(x) \in |E_\psi^r|(1)$ for all $x \in l_1(p)$, $H(x) = T^r(\psi, 1)(\hat{A}(x)) \in l_1$. So, applying Lemma 2.1(iv) to the matrix $V^{(n)}$, it is obtained that $V^{(n)} \in (l(p), c)$ iff conditions (9) and (10) are satisfied. Again, if we apply Lemma 2.1(i) and Lemma 2.2 to the matrix $H$, then we have $H \in (l(p), l_1)$ iff the last condition holds. □

## 4 Conclusion

The sequence spaces defined as domains of Riesz, factorable, Nörlund and $S$-matrices in the spaces $l(p)$ and the space of series summable by the absolute Euler have been recently studied by several authors. In this paper, we have defined the new absolute Euler space $|E_\phi^r|(p)$ and investigated some topological and algebraic properties such as isomorphism, duals, base, and also characterized certain matrix transformations on that space. So, we have extended some well-known results.

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