

Are Bitcoin Returns Predictable?

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Abstract

Bitcoin is the most radical of the cryptocurrencies which are becoming popular nowadays. The advantage of the cryptocurrencies is that they are decentralized systems so do not need central banks. The purpose of this study is to determine if there is volatility in the returns of Bitcoin and if so, whether it is predictable. The volatility of the Bitcoin returns was investigated using the log-normal stochastic volatility (SV) model and SV model with leverage for daily data covering the period between 19.12.2011 and 29.01.2018. While there is no significant leverage effect in the Bitcoin returns, it can be said that the volatility is permanent and unpredictable. The unpredictability of Bitcoin returns' fluctuations suggests that it is risky to use it as an investment tool or currency. It is increasing day by day that Bitcoin takes place of banknotes or digital money, which are conventional means of payment. The more widespread the system, the safer and the more resistant to speculative it will be.

Keywords: Bitcoin, log-normal stochastic volatility model, stochastic volatility model with leverage, leverage effect

JEL Codes: C11, C63, G10

1. Introduction

Bitcoin is the most popular of the 897 cryptocurrencies as of February 2018. It is the first cryptocurrency proposed by Nakamoto (2008) and was released as open-source software in 2009. Bitcoin works with Blockchain technology, one of the world's leading software platforms for digital assets. The most important feature of cryptocurrencies is that they are decentralized and that central banks are not needed in this mechanism. As the world has never seen such a fictional currency, it's really exciting to imagine how the cryptocurrencies will advance. In this study, our aim is to determine if there is volatility in the Bitcoin returns and if so, whether it is predictable.

In the related literature, Bech & Garratt (2017) suggest the central bank cryptographic currency concept. Nagpal (2018), explains the emergence, functioning, and risks of digital money.

Baur & Dimpfl (2017) examine the volatility of four different Bitcoin markets by comparing US dollar, the euro and the Japanese yen via fractional integration and Granger causality tests. Eross, McGroarty, Urquhart, & Wolfe (2017) investigate the relationship between returns, volume, bid-ask spread (BAS) and volatility of Bitcoin by using Granger causality. Chengyuan (2017) analyzes the volatility transmission of Bitcoin price between Chinese and US markets through the Granger causality test and the BEKK model. Urquhart

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(2018) ascertains the relationship between realized volatility and volume of Bitcoin and investor attention by way of Vector Autoregression, Granger causality and Impulse Response analysis. Ciaian, Rajcaniova, & Kancs (2018) examine interdependencies between Bitcoin and 16 altcoin markets in the short- and long-run by using Autoregressive Distributed Lag (ARDL) model.

Fiser (2015) analyzes Bitcoin bubbles using ARIMA, GARCH and Log-Periodic Power Law model (LPPL) models. Kasper (2017) compares Bitcoin volatility to volatility of currencies of least developed countries and other cryptocurrencies by GARCH model. Stavroyiannis (2017) investigates volatility dynamics of the six major digital currencies, Bitcoin, Ethereum, Ripple, Litecoin, Dash, and NEM, using GARCH models. Corbet, Larkin, Lucey, Meegan, & Yarovaya (2017) examine the reaction of 100 digital assets to US policy announcements by use of GARCH model. Cermak (2017) researches whether Bitcoin is a useful currency by using GARCH model. Liu, Shao, & Wei (2017) compare the empirical performance of a standard normal distribution, the Student's t distribution and the normal reciprocal inverse Gaussian (NRIG) for the daily Bitcoin exchange rate returns via GARCH model. Bouri, Azzi, & Dyrberg (2018) investigate the return-volatility relationship for Bitcoin using the asymmetric GARCH model both before and after the price crash of 2013 and throughout the study period. Shi (2018) examines the effect of Bitcoin futures on the volatility and liquidity of the Bitcoin spot market through the asymmetric EGARCH model with a generalized error distribution (GED).

Cheah & Fry (2015) investigate whether there are the bubbles in Bitcoin markets by using a stochastic bubble model. Catania & Grassi (2017) use 289 cryptocurrencies and focus on four of them, Bitcoin, Ethereum, Ripple, and Litecoin, and extend Score-Driven GHSKT model considering the properties of long memory, leverage effect and time-varying higher order moments. Balcilar, Bouri, Gupta, & Roubaud (2017) examine the causality relationship between Bitcoin returns, volatility and trading volume by considering nonlinearity and structural breaks via a non-parametric causality-in-quantiles test. Lahmiri & Bekiros (2018) reveal the chaos, randomness, and multi-fractality in Bitcoin prices and returns which are separated into the low regime and high regime by using the largest Lyapunov exponent, Shannon entropy, and the multi-fractal detrended fluctuation analysis.

Johnson (2017) estimates SV models with heavy tails, leverage, and covariates by using particle Markov Chain Monte Carlo (MCMC) method for Bitcoin exchange rate data. Johnson (2017) analyzes the volatility of the Bitcoin returns using the SV models. The rest of the study has been organized as follows. Section 2 presents the methodology related to log-normal SV model and SV model with leverage. Section 3 introduces the dataset used in the analysis and reports the empirical results. Finally, Section 4 concludes.

2. Methodology

The studies in order to model the volatility are based on Clark (1973). The increases in the price process $\Delta x_t = x_t - x_{t-1}$ are stationary in the mean and uncorrelated, where x_t indicates the price at time t. This situation can best be explained by a random walk as follows (Clark, 1973: 135);

$$x_t = x_{t-1} + \varepsilon_t, E(\varepsilon_t) = 0, E(\varepsilon_t \varepsilon_s) = 0, \text{ for } t \neq s \quad (1)$$

Clark (1973) laid the foundations for the SV model by modelling the σ^2 process, which is the variance of ε_t . There are several models in which variance and covariance change. The basis of models in which volatility is defined as an observable and deterministic variable is the autoregressive conditional variance (ARCH) model proposed by Engle (1982). The first order autoregressive model can be written as follows;

$$Y_t = \gamma Y_{t-1} + \varepsilon_t \quad (2)$$

where Y_t is a random variable drawn from the conditional density function $f(Y_t|Y_{t-1})$, ε_t is the white noise process which has the variance σ^2 . The conditional mean and unconditional mean of Y_t are γY_{t-1} and zero, respectively. The conditional variance of Y_t is σ^2 while the unconditional variance is $\sigma^2 / (1 - \gamma^2)$. The bilinear model which allows the conditional variance to depend on the past realization of the series is as follows (Engle, 1982: 987-988);

$$Y_t = \varepsilon_t Y_{t-1} \quad (3)$$

where the conditional variance is $\sigma^2 Y_{t-1}^2$. However, the unconditional variance is zero or infinite. Despite the fact that this problem is avoided with light measures, this is an undesirable situation. An ARCH model is defined as follows (Engle, 1982: 988);

$$Y_t = \varepsilon_t h_t^{1/2} \quad (4)$$

$$h_t = \alpha_0 + \alpha_1 Y_{t-1}^2 \quad (5)$$

where $\text{var}(\varepsilon_t) = 1$. In this model, the variance in time t is allowed to be a linear function of the squares of past observations. The generalized form of this model can be stated as in (6) (Engle, 1982: 988);

$$h_t = \alpha_0 + \alpha_1 Y_{t-1}^2 + \dots + \alpha_p Y_{t-p}^2 \quad (6)$$

Taylor (1982), proposed a SV model that allows the volatility to be a function of the unobserved or latent component in parameter-based or state-space models. The volatility, which can be observed and modeled as a deterministic variable in ARCH-GARCH models, has an unobservable and stochastic structure in SV models. The volatility in SV models is determined by an unpredictable shock. While the conditional mean has a stochastic process in ARCH-GARCH models, both the conditional mean and the conditional variance follow the stochastic process in the SV models (Göktaş & Hepsağ, 2016: 4).

Classical SV model is defined as follows (Taylor, 2008: 74);

$$y_t = \varepsilon_t \exp(h_t/2), \varepsilon_t \sim N(0,1) \quad (7)$$

$$h_t = \gamma + \phi h_{t-1} + \eta_t, \eta_t \sim N(0, \sigma_\eta^2) \quad (8)$$

where is assumed that ε_t and η_t are mutually independent and identically distributed (iid) random variables. h_t refers to the unobservable volatility. (7) and (8) are the mean and volatility equation, respectively. Because of the Gaussianity of η_t , this model is called as log-normal SV model. ϕ indicates the continuity (permanence) of the volatility. If ϕ is close to 1, it can be concluded the existence of volatility clustering in the data. σ_η also indicates the variability of

volatility. If σ_η^2 is close to 0, it can be expressed that the volatility is predictable (Göktaş & Hepsağ, 2016: 11).

The SV model with dynamic leverage effect, including the asymmetric relations due to the direct negative correlation between the changes in volatility, and returns, is as follows (Asai & McAleer, 2005: 320);

$$y_t = \varepsilon_t \exp(h_t/2), \varepsilon_t \sim N(0,1) \quad (9)$$

$$h_{t+1} = \mu + \varphi h_t + \eta_t, \eta_t \sim N(0, \sigma_\eta^2) \quad (10)$$

$$E(\varepsilon_t, \eta_t) = \rho \sigma_\eta \quad (11)$$

While it is assumed that ε_t and η_t are mutually independent in SV models, ε_t and η_t are allowed to be the contemporaneous relationship in SV models with dynamic leverage effect. In this case, the relationship becomes asymmetric (Ghysels, Harvey, & Renault, 1996: 139). ρ is the relationship between the variation in volatility, and return series, and can be stated as $\text{corr}(\varepsilon_t, \eta_t) = \rho$. If ρ is negative, the negative changes in ε_t cause the higher volatility in contemporaneous and following periods. On the other hand, positive changes in ε_t are associated with decreases in volatility. That is, when ρ is negative, negative shocks increase the fluctuation more than positive shocks. This asymmetry is called as leverage effect (Jacquier, Polson, & Rossi, 2004: 193).

As the time dimension increases or the dimension of h_t goes above 1, the sample size grows (Shephard, 1996: 28). Because the sample size is often large and no traditional integral technique can be used to estimate the model, the Bayesian method MCMC procedure is used to estimate SV model and SV model with dynamic leverage effect (Danielsson, 1994: 376). In the MCMC method, parameters are estimated using Gibbs or Metropolis sampling.

3. Dataset and Empirical Results

In this study, volatility behavior of Bitcoin returns for closing prices is investigated using SV and SV with dynamic leverage effect. The Bitcoin price index is taken in USD-denominated for the period between 19.12.2011 and 29.01.2018 on Bitstamp, which is obtained from www.bitcoincharts.com. Bitstamp is the most rooted European Bitcoin exchange and dates back 2011, also mainly focus on trading Bitcoin. Table 1 reports the descriptive statistics for the return of the Bitcoin index for 2234 observations.

Table 1: Descriptive statistics

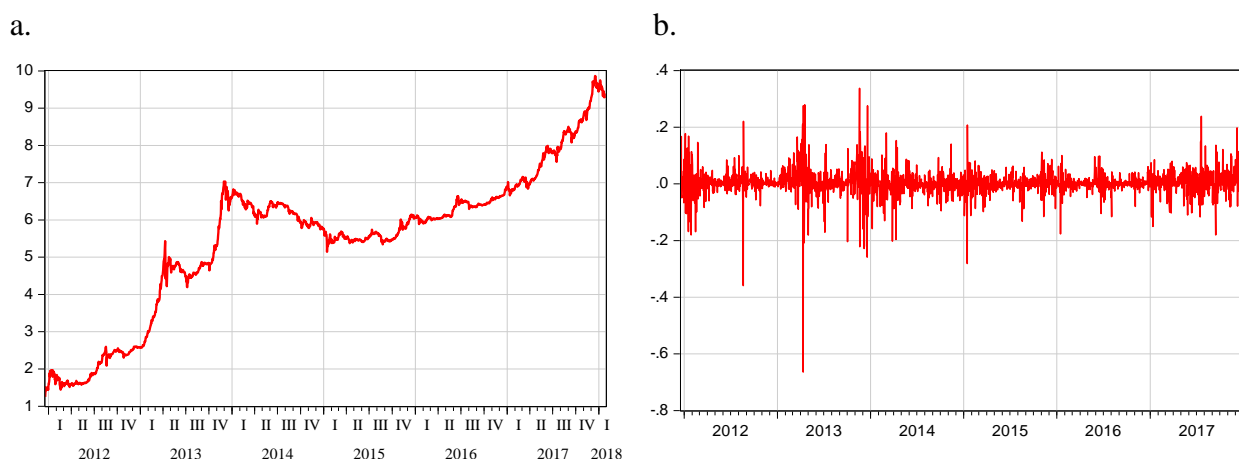
Mean	Median	Minimum	Maximum	Standard Deviation	Skewness	Kurtosis	Jarque-Bera	Probability
0.003613	0.002553	-0.663948	0.337486	0.048971	-1.365440	26.41153	51713.13	0.0000

Considering the descriptive statistics of the Bitcoin returns, it is observed that the mean value of the return series is smaller than the standard deviation. This situation is consistent with the fact that the financial time series generally follow the random walk process. It is seen that

the series of returns are negative skewed and has higher kurtosis than normal distribution. The Jarque-Bera test statistic also shows that the distribution of return series is not normal. By virtue of these features, it can be stated that the Bitcoin index carries the typical financial time series feature.

The natural logarithm of the data and the return series for the bitcoin index, are presented in Figure 1. It is clear that there is an increasing trend in the bitcoin returns and that there are volatility clusters in the return series.

Figure 1: a) Natural logarithm of Bitcoin returns. b) Bitcoin returns



The preliminary distributions for the parameters to be estimated using the Gibbs sampler in the estimation of SV model and SV model with dynamic leverage effect were obtained from the codes used in the studies performed by Yasuhiro Omori. In the estimates made using the WinBUGS 1.4.3 package program, the initial values for the estimation method were determined by the package program and 100,000 samples were made.

Table 2 reports the results of the estimated log-normal SV model for the Bitcoin returns.

Table 2: The results of the estimated log-normal SV model

	Coefficient	Standard Deviation	MC Error	Confidence Interval
μ	-7.263*	0.1755	0.005369	(-7.604, -6.908)
ϕ	0.9243*	0.01364	4.193E-4	(0.8964, 0.9488)
σ_{η}	0.5865*	0.0469	0.001607	(0.4975, 0.6796)

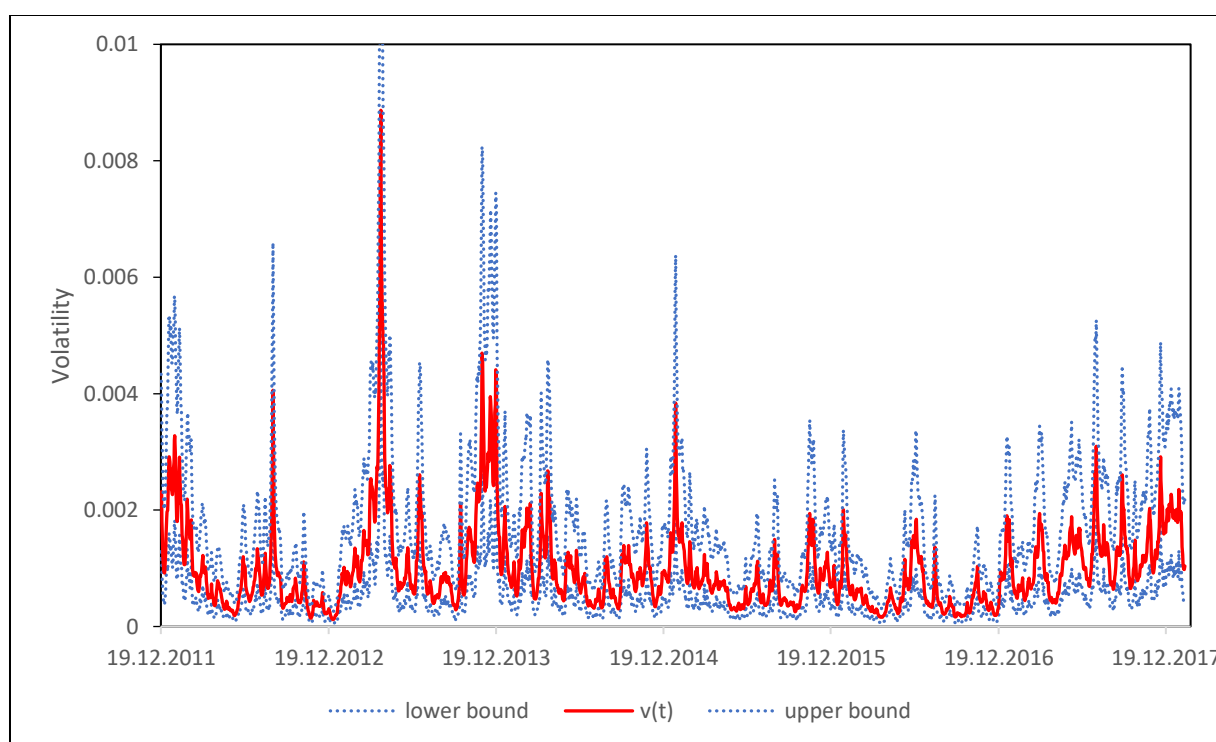
Note: * denotes the rejection of the null hypothesis at the 5% significance level.

According to the estimation results of SV model shown in Table 2, the ϕ coefficient indicating the permanence of Bitcoin volatility is statistically significant at 5% significance level and is obtained as 0.9243. It is understood that the volatility is permanent and volatility clusters have arisen in Bitcoin returns take part in the Bitstamp market. The σ_{η} coefficient indicating the variability of the volatility is also statistically significant at the 5% significance

level and is obtained as 0.5865 ($\sigma_{\eta}^2 = 0.3440$). Accordingly, there is a high level of variability in Bitstamp market volatility. Given the fact that the σ_{η}^2 coefficient is not close to 0, it can be made the interpretation that the volatility is not predictable. To sum up, it can be said that the volatility is permanent, the variability of the volatility is high and the volatility is not predictable.

Figure 2 shows the graph of estimated volatility values obtained from the model results belongs to Bitcoin returns. It is observed that the return volatility on 11.04.2013 is the highest level. It may be the cause of this volatility that the intense demand for Bitcoin, which has reached its highest closing price on 10.04.2013, was locked the system and not allowed new purchases for three days (www.bitstamp.net). It is also seen that the return volatility is at the lowest level on 28.12.2012.

Figure 2: The volatility of Bitcoin returns



The estimation results for the SV model with dynamic leverage effect belongs to the Bitcoin returns are given in Table 3.

Table 3: The results of the estimated SV model with dynamic leverage effect

	Coefficient	Standard Deviation	MC Error	Confidence Interval
μ	-7.229*	0.1684	0.00532	(-7.568, -6.923)
ϕ	0.9233*	0.01293	6.46E-04	(0.8975, 0.9472)
ρ	-0.02426	0.04674	0.001848	(-0.1186, 0.06251)
σ_{ε}	0.02703*	0.002259	7.21E-05	(0.02273, 0.03139)

σ_{η}	0.5883*	0.04543	0.002126	(0.5037, 0.6831)
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Note: * denotes the rejection of the null hypothesis at the 5% significance level.

According to the estimation results of SV model with dynamic leverage effect shown in Table 3, the ϕ coefficient indicating the permanence of Bitcoin volatility is statistically significant at 5% significance level and is obtained as 0.9233. It is understood that the volatility is permanent and volatility clusters have arisen in Bitcoin returns take part in the Bitstamp market. The σ_{η} coefficient indicating the variability of the volatility is also statistically significant at the 5% significance level and is obtained as 0.5883 ($\sigma_{\eta}^2 = 0.3461$). Accordingly, there is a high level of variability in Bitstamp market volatility. Given the fact that the σ_{η}^2 coefficient is not close to 0, it can be made the interpretation that the volatility is not predictable. To sum up, it can be said that the volatility is permanent, the variability of the volatility is high and the volatility is not predictable. These findings are consistent with those of the log-normal SV model. The ρ coefficient indicating the relationship between Bitcoin returns and the changes in the volatility is not statistically significant at the 5% significance level. For this reason, it cannot be said that the leverage effect exists in Bitcoin returns.

4. Conclusion

SV models are used in order to model time series data, especially in financial applications. The motivation for this study is to determine if there is volatility in the Bitcoin returns and if so, whether it is predictable. Although the use of Bitcoin is now not popular enough to have a huge impact on the real economy, it is worth to examine in terms of understanding the trajectory of cryptocurrencies.

In this study, it is found that the volatility is permanent in Bitcoin returns, the variability of the volatility is high, and the volatility is not predictable. Also, there is no leverage effect, which refers to the asymmetric reaction of the volatility process to past positive and negative returns, on Bitcoin returns. The unpredictability of Bitcoin fluctuations suggests that it is risky to use it as an investment tool or currency.

This study can be developed by examining other cryptocurrencies with SV models or searching the volatility of Bitcoin returns with other methods. In addition, the presence of structural breaks or regime changes in the analysis can also be taken into account.

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