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Abstract. In this study, determining the β dual of the space of bv_k^{θ} we characterize the matrix class $\left(bv_k^{\theta}, bv\right)$, where θ is a sequence of positive numbers and $bv_k^{\theta} = \left\{x \in w : \left(\theta_v^{1/k^*} \Delta x_v\right) \in \ell_k\right\}$ for $1/k + 1/k^* = 1$ $(1 < k < \infty)$.

Keywords: Sequence spaces, matrix transformations, bv_k^{θ} space.
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INTRODUCTION

Let ω be the set of all complex sequences, ℓ_k and c be the sets of k-absolutely convergent series and convergent sequences, respectively. By bv we denote the space of all sequences of bounded variation, i.e.,

$$bv = \left\{x \in w : \Sigma_{v=0}^{\infty} |x_v - x_{v-1}| < \infty, \quad x_{-1} = 0\right\}.$$

Let U and V be subspaces of w and (θ_n) be positive sequence, and $A = (a_{nv})$ be an arbitrary infinite matrix of complex numbers. By $A(x) = (A_n(x))$, we denote the A-transform of the sequence $x = (x_v)$, i.e.,

$$A_n(x) = \sum_{v=0}^{\infty} a_{nv} x_v$$

provided that the series is convergent for $n \geq 0$. Then, we say that A defines a matrix transformation from U into V, and denote it by $A \in (U, V)$ if the sequence $Ax = (A_n(x)) \in V$ for all sequence $x \in U$, and the set

$$U^{\beta} = \{\varepsilon \in \omega : \Sigma \varepsilon_v x_v \text{ converges for all } x \in U\}$$

is called the β dual of U.

An infinite matrix $A = (a_{nv})$ is called a triangle if $a_{nn} \neq 0$ and $a_{nv} = 0$ for all $v > n$ for all n, v [10]. Throughout k^* denotes the conjugate of $k > 1$, i.e., $1/k + 1/k^* = 1$.

We define the notations Γ_c and Γ_s , $v = 1, 2, ...$, as follows:

$$\Gamma_c = \left\{\varepsilon = (\varepsilon_v) : \lim_m \sum_{v=r}^{m} \varepsilon_v \quad \text{exists for } r = 1, 2, ...\right\},$$

$$\Gamma_s = \left\{\varepsilon = (\varepsilon_v) : \sup_m \sum_{r=1}^{m} \left|\theta_r^{-1/k^*} \sum_{v=r}^{m} \varepsilon_v\right|^{k^*} < \infty\right\}.$$

Sequence spaces and matrix operators are very important topics in the summability, which have been studied by several authors in many research papers (for example, see $[1-7]$) . In this study, by determining the β dual of bv_k^{θ} we characterize some matrix operators involving the space bv_k^{θ}.

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The following lemmas are needed in proving our theorems.

Lemma 1 *Let $1 < k < \infty$. Then, $A \in (\ell_k, \ell)$ if and only if*

$$\sum_{v=0}^{\infty}\left(\sum_{n=0}^{\infty}|a_{nv}|\right)^{k^*} < \infty \ \ [8].$$

Lemma 2 *Let $1 < k < \infty$. Then $A \in (\ell_k, c) \Leftrightarrow$*

$$a-) \ \lim_n a_{nv} \ \text{ exists for each } \ v, \qquad b-) \sup_n \sum_{v=0}^{\infty}|a_{nv}|^{k^*} < \infty \, [9] \, .$$

RELATED MATRIX OPERATORS

In [2] , the space bv_k^θ has been defined by

$$bv_k^\theta = \left\{x = (x_k) \in w : \sum_{n=0}^{\infty}\theta_n^{k-1}\,|\triangle x_n|^k < \infty, \ \ x_{-1} = 0\right\}$$

which is a complete normed space where (θ_n) is a sequence of nonnegative terms, $1 \le k < \infty$ and $\triangle x_n = x_n - x_{n-1}$ for all n. Also, it is reduced to bv^k for $\theta_n = 1$ for all n and $bv_1^\theta = bv$, which have been studied by Malkowsky et all [6] and Jarrah and Malkowsky [5]. Moreover, recently, Başar et all [1] have defined the sequence space $bv\,(u, p)$ and proved that this space is linearly isomorphic to the space $\ell(p)$ of Maddox as generalized to paranormed space.

Now we begin with β dual of bv_k^θ, which also can be deduced from [1].

Lemma 3 *Let $1 < k < \infty$ and (θ_n) be a sequence of nonnegative numbers. Then, $\left(bv_k^\theta\right)^\beta = \Gamma_c \cap \Gamma_s$.*

Proof Let $1 < k < \infty$. Now, $\varepsilon = (\varepsilon_n) \in \left(bv_k^\theta\right)^\beta$ iff $\Sigma\varepsilon_n x_n$ is convergent for all $x \in bv_k^\theta$. Also, it can be written that

$$\sum_{n=0}^{m}\varepsilon_n x_n = \sum_{v=0}^{m}\left(\sum_{n=v}^{m}\varepsilon_n\right)\theta_v^{-1/k^*}y_v = \sum_{j=0}^{m}a_{mv}y_v$$

where

$$a_{mv} = \begin{cases} \theta_v^{-1/k^*}\sum\limits_{n=v}^{m}\varepsilon_n, & 0 \le v \le m \\ 0, & v > m. \end{cases}$$

So it follows that $\varepsilon \in \left(bv_k^\theta\right)^\beta$ iff $A \in (\ell_k, c)$, which completes the proof together with Lemma 2.

The following theorem is the main result of our study which characterize the matrix class $\left(bv_k^\theta, bv\right)$, where θ is a sequence of positive numbers.

Theorem 1 *Let $A = (a_{nv})$ be an infinite matrix of complex numbers for all $n, v \ge 0$ and $1 < k < \infty$. Then, $A \in \left(bv_k^\theta, bv\right)$ if and only if*

$$\lim_{n\to\infty}\sum_{j=v}^{\infty}a_{nj} \ \text{ exists for each } \ v, \tag{1}$$

$$\sup_m \sum_{v=0}^{m}\left|\theta_v^{-1/k^*}\sum_{j=v}^{m}a_{nj}\right|^{k^*} < \infty \ \text{ for each } \ n \tag{2}$$

and

$$\sum_{v=0}^{\infty}\left(\sum_{n=0}^{\infty}\left|\theta_v^{1/k^*}\sum_{j=v}^{\infty}\left(a_{nj} - a_{n-1,j}\right)\right|\right)^{k^*} < \infty. \tag{3}$$

Proof Now, $A \in \left(bv_k^\theta, bv\right)$ iff $(a_{nv})_{v=0}^{\infty} \in \left(bv_k^\theta\right)^\beta$ and $A(x) \in bv$ for every $x \in bv_k^\theta$. Also, by Lemma 3, $(a_{nv})_{v=0}^{\infty} \in \left(bv_k^\theta\right)^\beta$ iff the conditions (1) and (2) hold. On the other hand, consider the operators $T : bv_k^\theta \to \ell_k$ and $B : bv \to \ell$ defined by $T(x) = \left(\theta_n^{1/k^*}\Delta x_n\right)$ and $B(x) = (\Delta x_n)$ and also denote inverse of T by G. Then, it is easily seen that the matrix G is given by

$$g_{nv} = \begin{cases} \theta_v^{-1/k^*}, 0 \le v \le n, \\ 0, \quad\quad v > n, \end{cases}$$

and if we say that $\widetilde{D} = Bo\widetilde{A}$, where $\widetilde{A} = AoG$, then, $A : bv_k^\theta \to bv$ if and only if $\widetilde{D} : \ell_k \to \ell$. So, it can be deduced that

$$\widetilde{a}_{nv} = \theta_v^{-1/k^*} \sum_{j=v}^{\infty} a_{nj},$$

which implies

$$\widetilde{d}_{nv} = \theta_v^{1/k^*} \sum_{j=v}^{\infty} \left(a_{nj} - a_{n-1,j}\right).$$

Therefore, by applying Lemma 1 with the matrix $\widetilde{D}$, it can be achieved that $\widetilde{D} : \ell_k \to \ell$ iff the condition (3) holds. This completes the proof.

If we take $\theta_v = 1$ for all $v \ge 0$, we get the well known result in [6].

Corollary 1 *Let $A = (a_{nv})$ be an infinite matrix of complex numbers for all $n, v \ge 0$ and $1 < k < \infty$. Then, $A \in (bv_k, bv)$ if and only if* (1) *holds,*

$$\sup_m \sum_{v=0}^{m} \left| \sum_{j=v}^{m} a_{nj} \right|^{k^*} < \infty \quad \text{for each} \quad n,$$

$$\sum_{v=0}^{\infty} \left(\sum_{n=0}^{\infty} \left| \sum_{j=v}^{\infty} \left(a_{nj} - a_{n-1,j}\right) \right| \right)^{k^*} < \infty.$$

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