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New exact solutions for the conformable space-time fractional KdV, CDG, (2+1)-dimensional CBS and (2+1)-dimensional AKNS equations

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ABSTRACT

In the present paper, G'/G^2 expansion method is applied to the space-time fractional third order Korteweg-De Vries (KdV) equation, space-time fractional Caudrey-Dodd-Gibbon (CDG) equation, space-time fractional (2+1)-dimensional Calogero-Bogoyavlenskii-Schiff (CBS) equation and space-time fractional (2+1)-dimensional Ablowitz-Kaup-Newell-Segur (AKNS) equation. Here, the fractional derivatives are described in conformable sense. The obtained traveling wave solutions are expressed by the hyperbolic, trigonometric, exponential and rational functions. The graphs for some of these solutions have been presented by choosing suitable values of parameters to visualize the mechanism of the given nonlinear fractional evolution equations.

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Space-time fractional third order KdV equation; space-time fractional CDG equation; space-time fractional (2+1)-dimensional CBS equation; space-time fractional (2+1)-dimensional AKNS equation; conformable derivative; G'/G^2 expansion method

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1. Introduction

The nonlinear evolution equations are widely used as models to describe complex physical phenomena in various field of science, particularly in fluid mechanics, solid state physics, plasma waves and chemical physics (see, for example, [1–4]). In this paper, we apply G'/G^2 expansion method (see, for example, [5]) to four space-time fractional nonlinear evolution equations: space-time fractional third-order KdV equation, space-time fractional CDG equation, space-time fractional (2+1)-dimensional CBS equation and space-time fractional (2+1)-dimensional AKNS equation. Here, fractional derivatives are defined in conformable sense. In the literature, the solutions of these equations have been investigated by many authors using various methods (see, for example, [6–29]).

KdV equation was first introduced by Boussinesq in 1877 and rediscovered by Diederik Korteweg and Gustav de Vries in 1895. It describes surface waves of long wavelength and small amplitude on shallow water and internal waves in a shallow density-stratified fluid. Natural transform and Homotopy perturbation methods, Homotopy Perturbation Transform Method, Riccati Equation Approach, extended hyperbolic function method, projective Riccati equation method and the Exp-function method have been applied to the third

order KdV equation in [6–10]. Jacobi elliptic function expansion method has been applied to conformable space-fractional KdV equation in [11].

Physical understanding of the CDG equation has been investigated in [30] and its solutions have been studied in [12–15]. The sin-cosine method, the rational Exp-Function, sinh method, G'/G -expansion method, Hirota's bilinear method and exp-function method have been used to obtain solutions of the fifth order CDG equation in [12–14]. G'/G -expansion method has been applied to conformable time fractional CDG equation in [15].

The CBS equation was first constructed by Bogoyavlenskii and Schiff in different ways [31]. The modified simple equation method, the exp-function methods, Sine-Gordon expansion method, the simplest equation method, G'/G -expansion method, a modified version of the Fan sub-equation method, improved G'/G -expansion and extended tanh-function method, symmetry method, Cole-Hopf transformation and the Hirota bilinear method have been implemented to compute solutions of the nonlinear (2+1)-dimensional CBS equation in [16–23], respectively.

The AKNS equation is one of the most important physical models (see, for example, [32]). In 1997, Lou and Hu have obtained the (2+1)-dimensional AKNS

equation from the inner parameter dependent symmetry constraints of the KP equation [33]. Solutions of the AKNS equation have been investigated by many researchers. Hirota’s bilinear method, TANF($\xi/2$)-expansion method, the ansatz method, the improved tanh method, the simplified form of the bilinear method to obtain some new exact solutions for high nonlinear form of (2+1)-dimensional AKNS equation have been presented in [24–28]. Bilinear Backlund transformation has been presented to obtain periodic wave solutions of (2+1)-dimensional AKNS equation in [29].

2. Description of the conformable fractional derivative and its properties

For a function $f : (0, \infty) \rightarrow R$, the conformable fractional derivative of f of order $0 < \alpha < 1$ is defined as (see, for example, [34])

$$T_t^\alpha f(t) = \lim_{\varepsilon \rightarrow 0} \frac{f(t + \varepsilon t^{1-\alpha}) - f(t)}{\varepsilon}. \tag{1}$$

Some important properties of the the conformable fractional derivative are as follows:

$$T_t^\alpha (af + bg)(t) = aT_t^\alpha f(t) + bT_t^\alpha g(t), \quad \forall a, b \in R, \tag{2}$$

$$T_t^\alpha (t^\mu) = \mu t^{\mu-\alpha}, \tag{3}$$

$$T_t^\alpha (f(g(t))) = t^{1-\alpha} g'(t) f'(g(t)). \tag{4}$$

3. Analytic solutions to the conformable space-time fractional KdV equation

Conformable space-time fractional KdV equation is given as follows (see, for example, [8])

$$T_t^\alpha u + auT_x^\beta u + bT_x^\beta T_x^\beta T_x^\beta u = 0, \quad 0 < \alpha \leq 1, 0 < \beta \leq 1, \tag{5}$$

where $a \neq 0$ and b are constants. Let us consider the following transformation

$$u(x, t) = U(\xi), \quad \xi = k \frac{t^\alpha}{\alpha} + m \frac{x^\beta}{\beta}, \tag{6}$$

where k, m are constants. Substituting (6) into Equation (5) we obtain the following ordinary differential equation (ODE)

$$kU' + amUU' + bm^3U''' = 0. \tag{7}$$

Integrating of Equation (7) with zero constant of integration, we have

$$kU + am \frac{U^2}{2} + bm^3U'' = 0. \tag{8}$$

Let us suppose that the solution of Equation (8) can be expressed in the following form:

$$U(\xi) = a_0 + \sum_{i=1}^N a_i \left(\frac{G(\xi)'}{G(\xi)^2} \right)^i + \sum_{i=1}^N b_i \left(\frac{G(\xi)'}{G(\xi)^2} \right)^{-i}, \tag{9}$$

where $G(\xi)$ satisfies the following ODE

$$\left(\frac{G'}{G^2} \right)' = \mu + \lambda \left(\frac{G'}{G^2} \right)^2, \quad \lambda \neq 0, \mu \neq 0, \tag{10}$$

where $a_0, a_i, b_i (i = 1, 2, \dots, N), \lambda$ and μ are constants to be determined. Equation (10) has different solutions as follows (see, for example, [5]):

When $\mu\lambda > 0$,

$$\frac{G'}{G^2} = \sqrt{\frac{\mu}{\lambda}} \frac{C \cos(\sqrt{\mu\lambda}\xi) + D \sin(\sqrt{\mu\lambda}\xi)}{D \cos(\sqrt{\mu\lambda}\xi) - C \sin(\sqrt{\mu\lambda}\xi)}. \tag{11}$$

When $\mu\lambda < 0$,

$$\frac{G'}{G^2} = -\sqrt{\frac{|\mu\lambda|}{\lambda}} \frac{C \sinh(2\sqrt{|\mu\lambda|}\xi) + C \cosh(2\sqrt{|\mu\lambda|}\xi) + D}{C \sinh(2\sqrt{|\mu\lambda|}\xi) + C \cosh(2\sqrt{|\mu\lambda|}\xi) - D}. \tag{12}$$

When $\mu = 0, \lambda \neq 0$,

$$\frac{G'}{G^2} = -\frac{C}{\lambda(C\xi + D)}. \tag{13}$$

Here C and D are nonzero constants. Substituting Equation (9) into Equation (8) and then by balancing the highest order derivative term and nonlinear term in result equation, the value of N can be determined as 2. The solution can be expressed as follows

$$U(\xi) = a_0 + a_1 \left(\frac{G'}{G^2} \right) + a_2 \left(\frac{G'}{G^2} \right)^2 + b_1 \left(\frac{G'}{G^2} \right)^{-1} + b_2 \left(\frac{G'}{G^2} \right)^{-2}. \tag{14}$$

Substituting Equation (14) into Equation (8), collecting all the coefficients with the same power of G'/G^2 , we can obtain a set of algebraic equations for the unknowns $a_0, a_1, b_1, a_2, b_2, \lambda, \mu, k, m$:

$$\begin{aligned} aa_2^2m + 12ba_2\lambda^2m^3 &= 0, \\ 4a_1b\lambda^2m^3 + 2aa_1a_2m &= 0, \\ aa_1^2m + 16a_2b\lambda\mu m^3 + 2aa_0a_2m + 2a_2k &= 0, \\ 2a_1k + 2aa_0a_1m + 2aa_2b_1m + 4a_1b\lambda m^3\mu &= 0, \\ 2a_0k + aa_0^2m + 4bb_2\lambda^2m^3 + 4a_2bm^3\mu^2 \\ &+ 2aa_1b_1m + 2aa_2b_2m = 0, \\ 2b_1k + 2aa_0b_1m + 2aa_1b_2m + 4bb_1\lambda m^3\mu &= 0, \\ ab_1^2m + 16bb_2\lambda\mu m^3 + 2aa_0b_2m + 2b_2k &= 0, \\ 4bb_1m^3\mu^2 + 2ab_1b_2m &= 0, \\ ab_2^2m + 12bb_2m^3\mu^2 &= 0. \end{aligned}$$

Solving the algebraic equations in the Mathematica 10.0, we obtain the following set of solutions:

Case 1: $a_0 = ((-24b\lambda m^2\mu)/a), a_1 = 0, a_2 = ((-12b\lambda^2 m^2)/a), b_1 = 0, b_2 = ((-12b\mu^2 m^2)/a), k = 16b\lambda m^3\mu :$

When $\mu\lambda > 0$,

$$u(x, t) = \frac{-24b\lambda m^2 \mu}{a} + \frac{-12b\lambda^2 m^2}{a} \times \left(\frac{\sqrt{\mu} C \cos(\sqrt{\mu\lambda}\xi) + D \sin(\sqrt{\mu\lambda}\xi)}{\lambda D \cos(\sqrt{\mu\lambda}\xi) - C \sin(\sqrt{\mu\lambda}\xi)} \right)^2 + \frac{-12b\mu^2 m^2}{a} \left(\frac{C \cos(\sqrt{\mu\lambda}\xi)}{\sqrt{\frac{\mu}{\lambda} + D \sin(\sqrt{\mu\lambda}\xi)}} \right)^{-2} \cdot \left(\frac{D \cos(\sqrt{\mu\lambda}\xi)}{-C \sin(\sqrt{\mu\lambda}\xi)} \right) \quad (15)$$

When $\mu\lambda < 0$,

$$u(x, t) = \frac{-24b\lambda m^2 \mu}{a} + \frac{-12b\lambda^2 m^2}{a} \times \left(\frac{C \sinh(2\sqrt{|\mu\lambda|}\xi)}{\sqrt{\frac{|\mu\lambda|}{\lambda} + C \cosh(2\sqrt{|\mu\lambda|}\xi) + D}} \right)^2 + \frac{-12b\mu^2 m^2}{a} \left(\frac{C \sinh(2\sqrt{|\mu\lambda|}\xi)}{\sqrt{\frac{|\mu\lambda|}{\lambda} + C \cosh(2\sqrt{|\mu\lambda|}\xi) + D}} \right)^{-2} \cdot \left(\frac{C \cosh(2\sqrt{|\mu\lambda|}\xi) + D}{+C \cosh(2\sqrt{|\mu\lambda|}\xi) - D} \right) \quad (16)$$

When $\mu = 0, \lambda \neq 0$,

$$u(x, t) = \frac{-24b\lambda m^2 \mu}{a} + \frac{-12b\lambda^2 m^2}{a} \left(\frac{C}{\lambda(C\xi + D)} \right)^2 + \frac{-12b\mu^2 m^2}{a} \left(\frac{C}{\lambda(C\xi + D)} \right)^{-2} \quad (17)$$

Here $\xi = 16b\lambda m^3 \mu(t^\alpha/\alpha) + m(x^\beta/\beta)$.

Case 2: $a_0 = ((8b\lambda m^2 \mu)/a), a_1 = 0, a_2 = ((-12b\lambda^2 m^2)/a), b_1 = 0, b_2 = ((-12b\mu^2 m^2)/a), k = -16b\lambda m^3 \mu$:

When $\mu\lambda > 0$,

$$u(x, t) = \frac{8b\lambda m^2 \mu}{a} + \frac{-12b\lambda^2 m^2}{a} \times \left(\frac{\sqrt{\mu} C \cos(\sqrt{\mu\lambda}\xi) + D \sin(\sqrt{\mu\lambda}\xi)}{\lambda D \cos(\sqrt{\mu\lambda}\xi) - C \sin(\sqrt{\mu\lambda}\xi)} \right)^2 + \frac{-12b\mu^2 m^2}{a} \left(\frac{C \cos(\sqrt{\mu\lambda}\xi)}{\sqrt{\frac{\mu}{\lambda} + D \sin(\sqrt{\mu\lambda}\xi)}} \right)^{-2} \cdot \left(\frac{D \cos(\sqrt{\mu\lambda}\xi)}{-C \sin(\sqrt{\mu\lambda}\xi)} \right) \quad (18)$$

When $\mu\lambda < 0$,

$$u(x, t) = \frac{8b\lambda m^2 \mu}{a} + \frac{-12b\lambda^2 m^2}{a} \times \left(\frac{\sqrt{|\mu\lambda|} C \sinh(2\sqrt{|\mu\lambda|}\xi) + C \cosh(2\sqrt{|\mu\lambda|}\xi) + D}{\lambda C \sinh(2\sqrt{|\mu\lambda|}\xi) + C \cosh(2\sqrt{|\mu\lambda|}\xi) - D} \right)^2 + \frac{-12b\mu^2 m^2}{a} \cdot \left(\frac{\sqrt{|\mu\lambda|} C \sinh(2\sqrt{|\mu\lambda|}\xi) + C \cosh(2\sqrt{|\mu\lambda|}\xi) + D}{\lambda C \sinh(2\sqrt{|\mu\lambda|}\xi) + C \cosh(2\sqrt{|\mu\lambda|}\xi) - D} \right)^{-2} \cdot \left(\frac{C \cosh(2\sqrt{|\mu\lambda|}\xi) + D}{-D} \right) \quad (19)$$

When $\mu = 0, \lambda \neq 0$,

$$u(x, t) = \frac{8b\lambda m^2 \mu}{a} + \frac{-12b\lambda^2 m^2}{a} \left(\frac{C}{\lambda(C\xi + D)} \right)^2 + \frac{-12b\mu^2 m^2}{a} \left(\frac{C}{\lambda(C\xi + D)} \right)^{-2} \quad (20)$$

Here $\xi = -16b\lambda m^3 \mu(t^\alpha/\alpha) + m(x^\beta/\beta)$.

Figure 1 shows 3D plot of the traveling wave solution $u(x, t)$ in Equation (15) for $a = 6, b = 2, \alpha = 0.5, \beta = 0.7, \lambda = 0.01, \mu = 12, m = 0.05, D = 0.5, C = 1/3$.

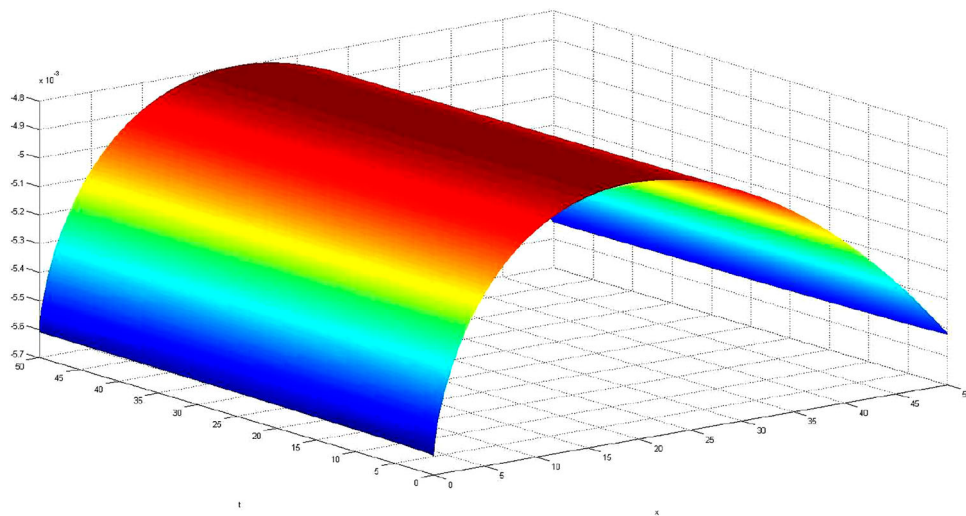


Figure 1. 3D plot of the obtained traveling wave solution $u(x, t)$ of Equation (15) for $a = 6, b = 2, \alpha = 0.5, \beta = 0.7, \lambda = 0.01, \mu = 12, m = 0.05, D = 0.5, C = 1/3, 0 \leq x, t \leq 50$.

4. Analytic solutions to the conformable space-time fractional CDG equation

Conformable space-time fractional CDG equation is given as follows (see, for example, [15])

$$T_t^\alpha u + 30uT_x^\beta T_x^\beta T_x^\beta u + 30T_x^\beta uT_x^\beta T_x^\beta u + 180u^2T_x^\beta u + T_x^\beta T_x^\beta T_x^\beta T_x^\beta T_x^\beta u = 0$$

$$0 < \alpha \leq 1, \quad 0 < \beta \leq 1. \tag{21}$$

Using the transformations (6), Equation (21) reduces to the following ordinary differential equation

$$kU' + 30m^3UU''' + 30m^3U'U'' + 180mU^2U' + m^5U^{(5)} = 0. \tag{22}$$

Integrating of Equation (22) with zero constant of integration, we have

$$kU + 30m^3UU'' + 60mU^3 + m^5U^{(4)} = 0. \tag{23}$$

Let us suppose that the solution of Equation (23) can be expressed in the form of Equation (9). Substituting Equation (9) into Equation (23) and then by balancing the highest order derivative term and nonlinear term in result equation, the value of N can be determined as 2. Therefore, Equation (9) reduces to

$$U(\xi) = a_0 + a_1 \left(\frac{G'}{G^2}\right) + a_2 \left(\frac{G'}{G^2}\right)^2 + b_1 \left(\frac{G'}{G^2}\right)^{-1} + b_2 \left(\frac{G'}{G^2}\right)^{-2}. \tag{24}$$

Substituting Equation (24) into Equation (23), collecting all the terms with the same power of G'/G^2 , we can obtain a set of algebraic equations for the unknowns $a_0, a_1, b_1, a_2, b_2, \lambda, \mu, k, m, n$:

$$60a_2^3m + 180a_2^2\lambda^2m^3 + 120a_2\lambda^4m^5 = 0,$$

$$180a_1a_2^2m + 240a_1a_2\lambda^2m^3 + 24a_1\lambda^4m^5 = 0,$$

$$180a_1^2a_2m + 60a_1^2\lambda^2m^3 + 240\mu a_2^2\lambda m^3 + 180a_0a_2^2m + 240\mu a_2\lambda^3m^5 + 180a_0a_2\lambda^2m^3 = 0,$$

$$60a_1^3m + 300\mu a_1a_2\lambda m^3 + 360a_0a_1a_2m + 40\mu a_1\lambda^3m^5 + 60a_0a_1\lambda^2m^3 + 180b_1a_2^2m + 180b_1a_2\lambda^2m^3 = 0,$$

$$180a_0^2a_2m + 180a_0a_2^2m + 240a_0a_2\lambda m^3\mu + 60a_2^2\lambda m^3\mu + 360b_1a_1a_2m + 60b_1a_1\lambda^2m^3 + 60a_2^2m^3\mu^2 + 180b_2a_2^2m + 136a_2\lambda^2m^5\mu^2 + 240b_2a_2\lambda^2m^3 + ka_2 = 0,$$

$$180a_0^2a_1m + 60a_0a_1\lambda m^3\mu + 360a_2b_1a_0m + 180b_1a_1^2m$$

$$+ 16a_1\lambda^2m^5\mu^2 + 120b_2a_1\lambda^2m^3 + 60a_2a_1m^3\mu^2 + 360a_2b_2a_1 * m + ka_1 + 300a_2b_1\lambda m^3\mu = 0,$$

$$60a_0^3m + 360a_0a_1b_1m + 60b_2a_0\lambda^2m^3 + 60a_2a_0m^3\mu^2 + 360a_2b_2a_0m + ka_0 + 180b_2a_1^2m + 120a_1b_1\lambda m^3\mu + 180a_2b_1^2m + 16b_2\lambda^3m^5\mu + 16a_2\lambda m^5\mu^3 + 480a_2b_2\lambda m^3\mu = 0,$$

$$180a_0^2b_1m + 60a_0b_1\lambda m^3\mu + 360a_1b_2a_0m + 180a_1b_1^2m + 16b_1\lambda^2m^5\mu^2 + 60b_2b_1\lambda^2m^3 + 120a_2b_1m^3\mu^2 + 360a_2b_2b_1m + kb_1 + 300a_1b_2\lambda m^3\mu = 0,$$

$$180a_0^2b_2m + 180a_0b_2^2m + 240a_0b_2\lambda m^3\mu + 60b_1^2\lambda m^3\mu + 360a_1b_1b_2m + 60a_1b_1m^3\mu^2 + 60b_2^2\lambda^2m^3 + 180a_2b_2^2m + 136b_2\lambda^2m^5\mu^2 + 240a_2b_2m^3\mu^2 + kb_2 = 0,$$

$$60b_1^3m + 300\lambda b_1b_2m^3\mu + 360a_0b_1b_2m + 40\lambda b_1m^5\mu^3 + 60a_0b_1m^3\mu^2 + 180a_1b_2^2m + 180a_1b_2m^3\mu^2 = 0,$$

$$180b_2^2b_2m + 60b_2^2m^3\mu^2 + 240\lambda b_2^2m^3\mu + 180a_0b_2^2m + 240\lambda b_2m^5\mu^3 + 180a_0b_2m^3\mu^2 = 0,$$

$$180b_1b_2^2m + 240b_1b_2m^3\mu^2 + 24b_1m^5\mu^4 = 0,$$

$$60b_2^3m + 180b_2^2m^3\mu^2 + 120b_2m^5\mu^4 = 0.$$

Solving the algebraic equations in the Mathematica 10.0, we obtain the following set of solutions:

Case 1: $a_0 = 2\sqrt{\frac{7}{15}}\lambda m^2\mu, a_1 = 0, a_2 = -\lambda^2m^2, b_1 = 0, b_2 = -m^2\mu^2, k = -32(11\lambda^2m^5\mu^2 + \sqrt{105}\lambda^2m^5\mu^2)$:
When $\mu\lambda > 0$,

$$u(x, t) = 2\sqrt{\frac{7}{15}}\lambda m^2\mu - \lambda^2m^2 \times \left(\sqrt{\frac{\mu}{\lambda}} \frac{C \cos(\sqrt{\mu\lambda}\xi) + D \sin(\sqrt{\mu\lambda}\xi)}{D \cos(\sqrt{\mu\lambda}\xi) - C \sin(\sqrt{\mu\lambda}\xi)}\right)^2 - m^2\mu^2 \times \left(\sqrt{\frac{\mu}{\lambda}} \frac{C \cos(\sqrt{\mu\lambda}\xi) + D \sin(\sqrt{\mu\lambda}\xi)}{D \cos(\sqrt{\mu\lambda}\xi) - C \sin(\sqrt{\mu\lambda}\xi)}\right)^{-2}. \tag{25}$$

When $\mu\lambda < 0$,

$$u(x, t) = 2\sqrt{\frac{7}{15}}\lambda m^2\mu - \lambda^2m^2 \times \left(\sqrt{\frac{|\mu\lambda|}{\lambda}} \frac{C \sinh(2\sqrt{|\mu\lambda}|\xi) + C \cosh(2\sqrt{|\mu\lambda}|\xi) + D}{C \sinh(2\sqrt{|\mu\lambda}|\xi) + C \cosh(2\sqrt{|\mu\lambda}|\xi) - D}}\right)^2 - m^2\mu^2 \left(\sqrt{\frac{|\mu\lambda|}{\lambda}} \frac{C \sinh(2\sqrt{|\mu\lambda}|\xi) + C \cosh(2\sqrt{|\mu\lambda}|\xi) + D}{C \sinh(2\sqrt{|\mu\lambda}|\xi) + C \cosh(2\sqrt{|\mu\lambda}|\xi) - D}}\right)^{-2}. \tag{26}$$

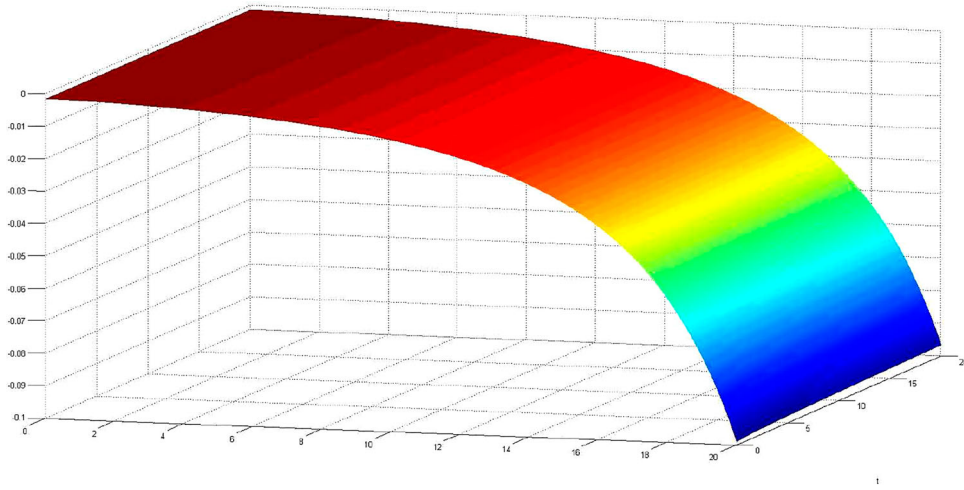


Figure 2. 3D plot of the obtained traveling wave solution $u(x, t)$ of Equation (25) for $\alpha = 0.5, \beta = 0.75, \lambda = 0.05, \mu = 0.2, m = 0.5, D = 1, C = 1, 0 \leq x, t \leq 50$ $0 \leq x, t \leq 20$.

When $\mu = 0, \lambda \neq 0$,

$$u(x, t) = 2\sqrt{\frac{7}{15}}\lambda m^2 \mu - \lambda^2 m^2 \left(\frac{C}{\lambda(C\xi + D)}\right)^2 - m^2 \mu^2 \left(\frac{C}{\lambda(C\xi + D)}\right)^{-2}. \quad (27)$$

Here $\xi = -32(11\lambda^2 m^5 \mu^2 + \sqrt{105}\lambda^2 m^5 \mu^2)(t^\alpha/\alpha) + m(x^\beta/\beta)$.

Case 2: $a_0 = -2\lambda m^2 \mu, a_1 = 0, a_2 = -\lambda^2 m^2, b_1 = 0, b_2 = -m^2 \mu^2, k = -256\lambda^2 m^5 \mu^2$:

When $\mu\lambda > 0$,

$$u(x, t) = -2\lambda m^2 \mu - \lambda^2 m^2 \times \left(\frac{\sqrt{\mu} C \cos(\sqrt{\mu\lambda}\xi) + D \sin(\sqrt{\mu\lambda}\xi)}{\lambda D \cos(\sqrt{\mu\lambda}\xi) - C \sin(\sqrt{\mu\lambda}\xi)}\right)^2 - m^2 \mu^2 \times \left(\frac{\sqrt{\mu} C \cos(\sqrt{\mu\lambda}\xi) + D \sin(\sqrt{\mu\lambda}\xi)}{\lambda D \cos(\sqrt{\mu\lambda}\xi) - C \sin(\sqrt{\mu\lambda}\xi)}\right)^{-2}. \quad (28)$$

When $\mu\lambda < 0$,

$$u(x, t) = -2\lambda m^2 \mu - \lambda^2 m^2 \times \left(\frac{C \sinh(2\sqrt{|\mu\lambda}|\xi)}{\sqrt{|\mu\lambda}| + C \cosh(2\sqrt{|\mu\lambda}|\xi) + D} + \frac{C \sinh(2\sqrt{|\mu\lambda}|\xi)}{+C \cosh(2\sqrt{|\mu\lambda}|\xi) - D}\right)^2 - m^2 \mu^2 \times \left(\frac{C \sinh(2\sqrt{|\mu\lambda}|\xi)}{\sqrt{|\mu\lambda}| + C \cosh(2\sqrt{|\mu\lambda}|\xi) + D} + \frac{C \sinh(2\sqrt{|\mu\lambda}|\xi)}{+C \cosh(2\sqrt{|\mu\lambda}|\xi) - D}\right)^{-2}. \quad (29)$$

When $\mu = 0, \lambda \neq 0$,

$$u(x, t) = -2\lambda m^2 \mu - \lambda^2 m^2 \left(\frac{C}{\lambda(C\xi + D)}\right)^2 - m^2 \mu^2 \left(\frac{C}{\lambda(C\xi + D)}\right)^{-2}. \quad (30)$$

Here $\xi = -256\lambda^2 m^5 \mu^2 (t^\alpha/\alpha) + m(x^\beta/\beta)$.

Figure 2 shows 3D plot of the traveling wave solution $u(x, t)$ in Equation (25) for $\alpha = 0.5, \beta = 0.75, \lambda = 0.05, \mu = 0.2, m = 0.5, D = 1, C = 1$.

5. Analytic solutions to the conformable space-time fractional (2 + 1)-dimensional CBS Equation

Conformable space-time fractional CBS equation is given in the following form: (see, for example, [21])

$$T_x^\beta T_t^\alpha u + T_x^\beta T_x^\beta T_x^\beta T_y^\theta u + 4T_x^\beta u T_x^\beta T_y^\theta u + 2T_x^\beta T_x^\beta u T_y^\theta u = 0, \quad 0 < \alpha \leq 1, \quad 0 < \beta \leq 1, \quad 0 < \theta \leq 1. \quad (31)$$

Using the following transformation

$$u(x, y, t) = U(\xi), \quad \xi = k \frac{t^\alpha}{\alpha} + m \frac{x^\beta}{\beta} + n \frac{y^\theta}{\theta}, \quad (32)$$

Equation (31) can be transformed into the following ordinary differential equation

$$kmU'' + m^3 n U^{(4)} + 4m^2 n U' U'' + 2m^2 n U'' U' = 0. \quad (33)$$

Integrating of Equation (33) with zero constant of integration, we have

$$kmU' + m^3 n U''' + 3m^2 n (U')^2 = 0. \quad (34)$$

Let us suppose that the solution of Equation (34) can be expressed in the form of Equation (9). Substituting Equation (9) into Equation (34) and then by balancing

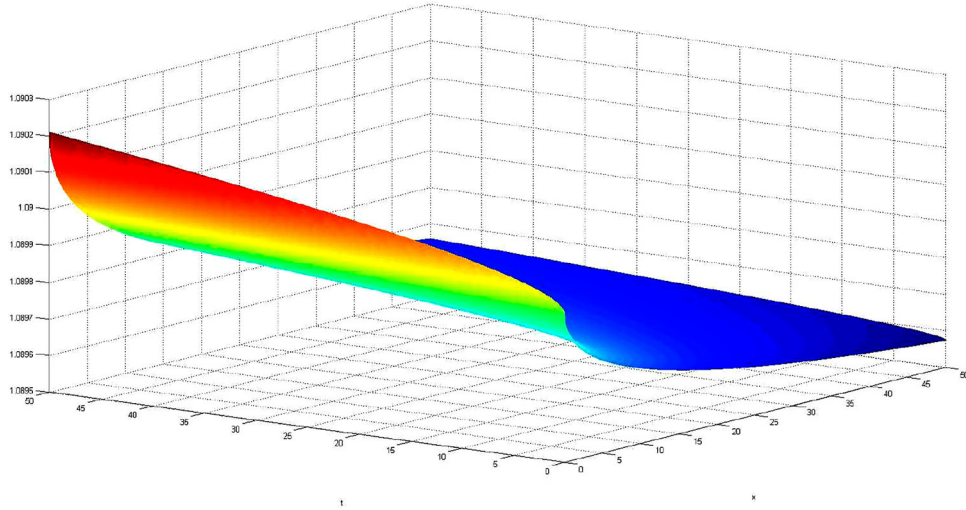


Figure 3. 3D plot of the obtained traveling wave solution $u(1, y, t)$ of Equation (37) for $a_0 = 1, \alpha = 0.5, \beta = 0.5, \theta = 0.75, \lambda = -5, \mu = 1, m = 0.01, n = 0.5, D = 10, C = -10, 0 \leq x, t \leq 50$.

the highest order derivative term and nonlinear term in result equation, the value of N can be determined as 1. Therefore, Equation (9) reduces to

$$U(\xi) = a_0 + a_1 \left(\frac{G'}{G^2} \right) + b_1 \left(\frac{G'}{G^2} \right)^{-1}. \quad (35)$$

Substituting Equation (35) into Equation (34), collecting all the terms with the same power of $\frac{G'}{G^2}$, we can obtain a set of algebraic equations for the unknowns $a_0, a_1, b_1, \lambda, \mu, k, m, n$:

$$\begin{aligned} 3na_1^2\lambda^2m^2 + 6na_1\lambda^3m^3 &= 0, \\ 6\mu na_1^2\lambda m^2 + 8\mu na_1\lambda^2m^3 - 6b_1na_1\lambda^2m^2 + ka_1\lambda m &= 0, \\ 3na_1^2m^2\mu^2 - 12na_1b_1\lambda m^2\mu + 2na_1\lambda m^3\mu^2 + ka_1m\mu \\ + 3nb_1^2\lambda^2m^2 - 2nb_1\lambda^2m^3\mu - kb_1\lambda m &= 0, \\ 6\lambda nb_1^2m^2\mu - 8\lambda nb_1m^3\mu^2 - 6a_1nb_1m^2\mu^2 - kb_1m\mu &= 0, \\ 3nb_1^2m^2\mu^2 - 6nb_1m^3\mu^3 &= 0. \end{aligned}$$

Solving these algebraic equations in the Mathematica 10.0, we obtain the following set of solutions: $a_1 = -2\lambda m, b_1 = 2m\mu, k = 16\lambda m^2\mu n$:

When $\mu\lambda > 0$,

$$\begin{aligned} u(x, y, t) = a_0 - 2\lambda m \sqrt{\frac{\mu}{\lambda}} \frac{C \cos(\sqrt{\mu\lambda}\xi) + D \sin(\sqrt{\mu\lambda}\xi)}{D \cos(\sqrt{\mu\lambda}\xi) - C \sin(\sqrt{\mu\lambda}\xi)} \\ + 2m\mu \left(\frac{C \cos(\sqrt{\mu\lambda}\xi)}{\sqrt{\frac{\mu}{\lambda}} + D \sin(\sqrt{\mu\lambda}\xi)} \right)^{-1} - C \sin(\sqrt{\mu\lambda}\xi)}. \quad (36) \end{aligned}$$

When $\mu\lambda < 0$,

$$\begin{aligned} u(x, y, t) = a_0 - 2\lambda m \sqrt{\frac{|\mu\lambda|}{\lambda}} \frac{C \sinh(2\sqrt{|\mu\lambda}|\xi) + D \cosh(2\sqrt{|\mu\lambda}|\xi)}{C \sinh(2\sqrt{|\mu\lambda}|\xi) + C \cosh(2\sqrt{|\mu\lambda}|\xi) - D} \\ + 2m\mu \left(\frac{C \sinh(2\sqrt{|\mu\lambda}|\xi)}{\sqrt{\frac{|\mu\lambda|}{\lambda}} + C \cosh(2\sqrt{|\mu\lambda}|\xi) + D} \right)^{-1}. \quad (37) \end{aligned}$$

When $\mu = 0, \lambda \neq 0$,

$$\begin{aligned} u(x, y, t) = a_0 + 2\lambda m \left(\frac{C}{\lambda(C\xi + D)} \right) \\ - 2m\mu \left(\frac{C}{\lambda(C\xi + D)} \right)^{-1}. \quad (38) \end{aligned}$$

Here $\xi = 16\lambda m^2\mu n(t^\alpha/\alpha) + m(x^\beta/\beta) + n(y^\theta/\theta)$.

Figure 3 shows 3D plot of the traveling wave solution $u(x, 1, t)$ in Equation (37) for $a_0 = 1, \alpha = 0.5, \beta = 0.5, \theta = 0.75, \lambda = -5, \mu = 1, m = 0.01, n = 0.5, D = 10, C = -10$.

6. Analytic solutions to the conformable space-time fractional (2 + 1)-dimensional AKNS equation

Finally, we consider conformable space-time fractional AKNS equation as follows (see, for example, [25])

$$\begin{aligned} 4T_x^\beta T_t^\alpha u + T_x^\beta T_x^\beta T_y^\beta T_y^\theta u + 8T_x^\beta T_y^\theta u T_x^\beta u \\ + 4T_y^\theta u T_x^\beta T_x^\beta u - a T_x^\beta T_x^\beta u = 0, \\ 0 < \alpha \leq 1, 0 < \beta \leq 1, 0 < \theta \leq 1, \quad (39) \end{aligned}$$

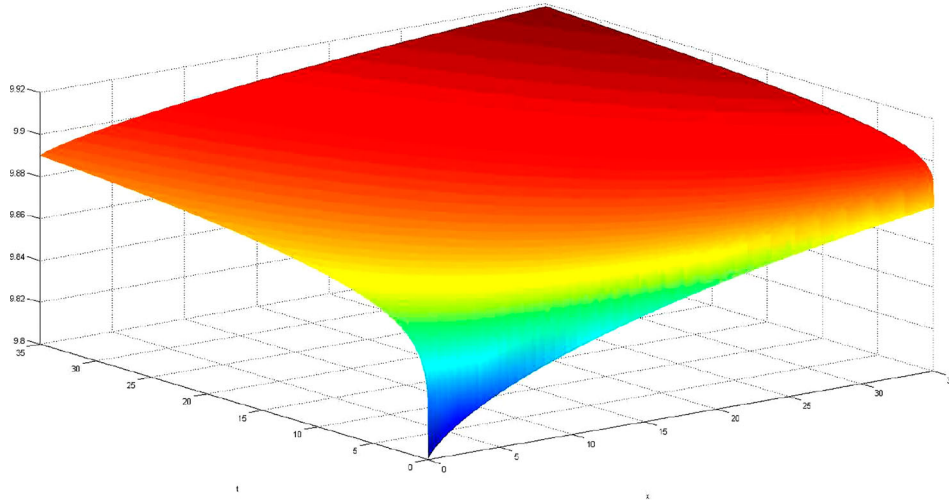


Figure 4. 3D plot of the obtained traveling wave solution $u(x, 1, t)$ of Equation (45) for $a_0 = 10, \alpha = 0.25, \beta = 0.75, \theta = 0.5, \lambda = 2, \mu = 0, m = 0.5, n = 5, D = 5, C = -6, 0 \leq x, t \leq 35$.

where a is constant. Using the transformation (32) for Equation (39) we have the following ordinary differential equation

$$4kmU'' + m^3nU^{(4)} + 8m^2nU''U' + 4m^2nU'U'' - am^2U'' = 0. \quad (40)$$

Integrating of Equation (40) with zero constant of integration, we have

$$(4k - am)U' + m^2nU''' + 6mn(U')^2 = 0. \quad (41)$$

Let us suppose that the solution of Equation (41) can be expressed in the form of Equation (9). Substituting Equation (9) into Equation (41) and then by balancing the highest order derivative term and nonlinear term in result equation, the value of N can be determined as 1. Therefore, Equation (9) reduces to

$$U(\xi) = a_0 + a_1 \left(\frac{G'}{G^2}\right) + b_1 \left(\frac{G'}{G^2}\right)^{-1}. \quad (42)$$

Substituting Equation (42) into Equation (41), collecting all the terms with the same power of $\frac{G'}{G^2}$, we can obtain a set of algebraic equations for the unknowns $a_0, a_1, b_1, \lambda, \mu, k, m, n$:

$$\begin{aligned} 6na_1^2\lambda^2m + 6na_1\lambda^3m^2 &= 0, \\ 12\mu na_1^2\lambda m + 8\mu na_1\lambda^2m^2 - 12b_1na_1\lambda^2m^2 \\ - aa_1\lambda m + 4ka_1\lambda &= 0, \\ 6na_1^2m\mu^2 - 24na_1b_1\lambda m\mu + 2na_1\lambda m^2\mu^2 \\ - aa_1m\mu + 4ka_1\mu + 6nb_1^2\lambda^2m - 2nb_1\lambda^2m^2\mu \\ + ab_1\lambda m - 4kb_1\lambda &= 0, \\ 12\lambda nb_1^2m\mu - 8\lambda nb_1m^2\mu^2 - 12a_1nb_1m\mu^2 \\ + ab_1m\mu - 4kb_1\mu &= 0, \\ 6nb_1^2m\mu^2 - 6nb_1m^2\mu^3 &= 0. \end{aligned}$$

Solving these algebraic equations in the Mathematica 10.0, we obtain the following set of solutions: $a_1 = -\lambda m, b_1 = m\mu, k = ((m(a + 16\lambda m\mu n))/4)$:

When $\mu\lambda > 0$,

$$\begin{aligned} u(x, y, t) &= a_0 - \lambda m \\ &\times \left(\sqrt{\frac{\mu}{\lambda}} \frac{C \cos(\sqrt{\mu\lambda}\xi) + D \sin(\sqrt{\mu\lambda}\xi)}{D \cos(\sqrt{\mu\lambda}\xi) - C \sin(\sqrt{\mu\lambda}\xi)} + m\mu \right) \\ &\times \left(\sqrt{\frac{\mu}{\lambda}} \frac{C \cos(\sqrt{\mu\lambda}\xi) + D \sin(\sqrt{\mu\lambda}\xi)}{D \cos(\sqrt{\mu\lambda}\xi) - C \sin(\sqrt{\mu\lambda}\xi)} \right)^{-1}. \end{aligned} \quad (43)$$

When $\mu\lambda < 0$,

$$\begin{aligned} u(x, y, t) &= a_0 + \lambda m \\ &\times \left(\sqrt{\frac{|\mu\lambda|}{\lambda}} \frac{C \sinh(2\sqrt{|\mu\lambda|}\xi) + C \cosh(2\sqrt{|\mu\lambda|}\xi) + D}{C \sinh(2\sqrt{|\mu\lambda|}\xi) + C \cosh(2\sqrt{|\mu\lambda|}\xi) - D} \right) \\ &- m\mu \left(\sqrt{\frac{|\mu\lambda|}{\lambda}} \frac{C \sinh(2\sqrt{|\mu\lambda|}\xi) + C \cosh(2\sqrt{|\mu\lambda|}\xi) + D}{C \sinh(2\sqrt{|\mu\lambda|}\xi) + C \cosh(2\sqrt{|\mu\lambda|}\xi) - D} \right)^{-1}. \end{aligned} \quad (44)$$

When $\mu = 0, \lambda \neq 0$,

$$\begin{aligned} u(x, y, t) &= a_0 + m \left(\frac{C}{(C\xi + D)} \right) \\ &- m\mu \left(\frac{C}{\lambda(C\xi + D)} \right)^{-1}. \end{aligned} \quad (45)$$

Here $\xi = ((m(a + 16\lambda m\mu n))/4)(t^\alpha/\alpha) + m(x^\beta/\beta) + n(y^\theta/\theta)$.

Figure 4 shows 3D plot of the traveling wave solution $u(x, 1, t)$ in Equation (45) for $a_0 = 10, \alpha = 0.25, \beta = 0.75, \theta = 0.5, \lambda = 2, \mu = 0, m = 0.5, n = 5, D = 5, C = -6$.

7. Conclusion

In this article, G'/G^2 expansion method has been applied to obtain new exact solutions of the conformable space-time fractional third-order KdV, CDG, (2+1)-dimensional CBS and (2+1)-dimensional AKNS equations. The exact solutions include hyperbolic, trigonometric, exponential and rational functions. Note that the obtained solutions are new form of the solutions and are not available in the literature. In this work, we have solved only four conformable nonlinear fractional differential equations. This method is useful in solving wide classes of conformable nonlinear fractional differential equations such as Sharma-Tasso-Olever, Zakharov Kuznetsov, Benjamin Bona Mahony, Boussinesq, Jimbo-Miwa, Burger equations.

Disclosure statement

No potential conflict of interest was reported by the authors.

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