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RESEARCH ARTICLE

Model-based robust suppression of epileptic seizures without sensory measurements

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Abstract

Uncontrolled seizures may lead to irreversible damages in the brain and various limitations in the patient's life. There exist experimental studies to stabilize the patient seizures. However, the experimental setups have many sensory devices to measure the dynamics of the brain cortex. These equipments prevent to produce small portable stabilizers for patients in everyday life. Recently, a comprehensive cortex model is introduced to apply model-based observers and controllers. However, this cortex model can be uncertain and have time-varying parameters. Therefore, in this paper, a robust Takagi–Sugeno (TS) controller and observer are designed to suppress the epileptic seizures without sensory measurements. The unavailable sensory measurements are provided by the designed nonlinear observer. The exponential convergence of the observer and controller is satisfied by the feedback parameter design using linear matrix inequalities. In addition, TS fuzzy observer–controller design has been compared with the conventional PID method in terms of control performance and design problem. The numerical computations show that the epileptic seizures are more effectively suppressed by the TS fuzzy observer-based controller under uncertain membrane potential dynamics.

Keywords Cortex model · Epileptic seizure · Uncertain dynamics · Takagi–Sugeno fuzzy modeling · Observer-based stabilization · PID control

Introduction

The functions of the human brain, one of the most complex systems known, are investigated by analysis of neuronal excitability and synaptic transmissions. Simulation of mesoscopic cortical electrical activity with a mathematical model of the brain cortex system is very important for the treatment of seizures such as epilepsy, Parkinson, cortical spreading depression and etc. (Traub et al. 2005; Kramer et al. 2007; Wang et al. 2015). Such neurological disorders, which can be assessed by electroencephalogram (EEG), is characterized by genetic or developmental anomalies, trauma, central nervous system infections or tumor-induced chaotic electrical brain activity (Iasemidis 2003). In addition, there are several studies that treat these neurological disorders with deep brain stimulating voltage. In these studies, it was observed that epileptic seizures were controlled by clinical parameters adjusted periodically to certain values (Hu et al. 2018). Thanks to EEG-based approaches to drug discovery and optimization, changes in brain activity, drug effects on structural and functional recovery are better understood (Mumtaz et al. 2018). Epilepsy is not only a discomfort in the central nervous system, but also a change in the different disorders of the brain activity into seizures. The chaotic dynamics in seizure-phase consist of high amplitude regular spike wave oscillations, in contrast to low amplitude irregular oscillations in the non-seizure-phase (Taylor et al. 2015). Control signals produced by known feedback control methods (direct electrical stimulation, magnetic stimulation and optogenetics) are applied as the treatment of instant seizures (Ratnadurai-Giridharan et al. 2017). However, the designed control methods are based on the assumption that the exact mathematical model of the cortex is known. Since it is not effective to measure all dynamics of the cortex model (CM), the control methods applied to this model are

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generally designed based on the minimization of the output error.

Nonlinear observers have an important role in monitoring and control for state and parameter estimation. Although the sensor technology is evolved for measurement of the states, the observers are preferred due to the weight/size limitations of the sensors. In addition to this, the observers are more advantageous in terms of cost since they are software-based. In this study, an observer-based controller has been designed to estimate all unmeasurable states of cortex dynamics. There are several studies in the literature that include different mechanisms of control and estimation using the brain cortex model. For instance, in Çetin and Beyhan (2018), an adaptive unscented Kalman filter-based optimal controller is proposed to control the dynamics of uncertain cortex with a single membrane potential measurement in their recent study. In Shan et al. (2015), to reproduce the dynamics and to estimate the unmeasurable parameters of the model, a control framework has been proposed to inhibit epilepticform wave in a neural mass model by external electric field. The values of neurophysiological parameters were estimated using the detailed biophysical model of brain activity in Rowe et al. (2004). In López-Cuevas et al. (2015), a cubature Kalman filter was used to estimate the parameters and status of the model during seizure from observed electrophysiological signals. In Tsakalis et al. (2006), the problem of controlling or suppressing seizures by means of feedback control was investigated. Kramer et al. (2006) showed that three controllers could be used to eliminate the seizure activity. The authors presented new approaches to investigate a feedback control model for epileptic seizures in humans with Lopour and Szeri (2010). In Wang et al. (2016), a Proportional Integral (PI) type closed-loop controller was designed to suppress the epileptic activity in the neural-mass model of Jansen where the controller parameters were optimized to keep the system in stable region. Haghighi and Markazi (207) has led to further investigation of possible seizure prevention approaches.

Nerve cells communicate with the generation and transmission of short electrical pulses. It is possible to obtain the control of the membrane potential and ionic currents, which is important for suppressing oscillations, blocking the action potential transmission and neuromodulation, by an observer-based control method (Fröhlich and Jezernik 2005; Beyhan 2017). Recently, efficient applications of Takagi–Sugeno (TS) fuzzy control methodologies have been developed for complex dynamic systems in various neuroscience applications. TS fuzzy models, expressed by a group of linear sub-models, are considered as a useful tool for approaching such complex nonlinear systems. These models are preferred as modern control tools due to their success in accurate modeling, prediction, estimation, control and fault tolerance in the control of such nonlinear systems (Tseng et al. 2001; Tanaka and Wang 2004; Ho and Chou 2007; Lendek et al. 2009; Wu et al. 2010; Li et al. 2015; Tong et al. 2016; Dahmani et al. 2016; Beyhan et al. 2017; Wei et al. 2017). In TS fuzzy modeling, stability analysis and controller/observer gain design associated with each sub-model is obtained by linear matrix inequality (LMI) tools. In addition, the fuzzy controller asymptotically stabilizes the TS fuzzy model, if there is a common solution to the LMI-based stability conditions (Boyd et al. 1997). A fuzzy Proportional−Integral−Derivative (PID) controller was designed for a class of neural mass models in Liu et al. (2013). For Hindmarsh Rose neuronal model, an affine TS fuzzy modeling-based observer and controller has been proposed in Beyhan (2017). A fuzzy interpolation method was used to approach the nonlinear stochastic Hodgkin–Huxley neuron system (Chen and Li 2010). In Aly and Tapus (2015), an online incremental learning system was developed to understand and produce multimodal actions from a cognitive perspective using TS fuzzy model.

In this study, observer-based stabilization of the epileptic cortex dynamics is investigated while the introduced mathematical model is assumed under unknown uncertainties and noise. In order that, a robust TS fuzzy observer/controller is designed and applied for the observer-based stabilization. Except the membrane potential, all the states are estimated and utilized in state feedback control. Note that the estimated states are trusted for the feedback control since the designed observer feedback gains satisfy the exponential stability and so the convergence of the estimates. In addition, the standard PID stabilization results of the cortex model have been presented comparatively to enhance the contribution of the designed controller for uncertain and noisy cases. In numerical computations, acceptable and applicable results are obtained for a real time treatment. It is expected that a low-cost software based portable device can be produced in the future, and many patient’s life will be healed by the stabilization of the epileptic seizure.

The organization of the paper follows that: in Sect. 2, the human brain cortex model used in this study is presented. TS fuzzy observer-based controller design is introduced in Sect. 3. The computational results are given in Sect. 4 and the conclusions are discussed in Sect. 5.

Chaotic dynamics of brain cortex model

Since animal implementation experiments encourage new studies, some tests should be done on mathematical models in order to understand the effects of experiments on humans. The cortex model, which represents the electrical activity of the human brain cortex, is expressed involving stochastic partial differential equations (SPDEs). According to Lopour and Szeri (2010), this model is a mean-field model, meaning that all of its variables represent spatially averaged properties of populations of neurons. Electroencephalography (EEG) based applications such as epilepsy (Kramer et al. 2007), sleep (Wilson et al. 2006) and anesthesia (Steyn-Ross et al. 2003) can be investigated in consideration of the stochastic and nonlinear behavior of cortex model. The cortex model presented SPDEs in second-order terms as in Liley et al. (1999) is converted into a simpler system (Kramer et al. 2007) as

$$
\begin{aligned}
\dot{h}_e(t) =& ((h_e^{rest} - h_e) + \psi_{ee}(h_e)I_{ee} \\
& + \psi_{ie}(h_e)I_{ie} + u + v)/\tau_e, \\
\dot{h}_i(t) =& ((h_i^{rest} - h_i) + \psi_{ei}(h_i)I_{ei} \\
& + \psi_{ii}(h_i)I_{ii})/\tau_i, \\
\dot{I}_{ee}(t) =& J_{ee}, \\
\dot{J}_{ee}(t) =& -2\gamma_e J_{ee} - \gamma_e^2 I_{ee} + [N_{ee}^{\beta} S_e(h_e) \\
& + \phi_e + p_{ee}] G_e \gamma_e e + \Gamma_1, \\
\dot{I}_{ei}(t) =& J_{ei}, \\
\dot{J}_{ei}(t) =& -2\gamma_e J_{ei} - \gamma_e^2 I_{ei} + [N_{ei}^{\beta} S_e(h_e) \\
& + \phi_i + p_{ei}] G_e \gamma_e e + \Gamma_2, \\
\dot{I}_{ie}(t) =& J_{ie}, \\
\dot{J}_{ie}(t) =& -2\gamma_i J_{ie} - \gamma_i^2 I_{ie} + [N_{ie}^{\beta} S_i(h_i) + p_{ie}] \\
& G_i \gamma_i e + \Gamma_3, \\
\dot{I}_{ii}(t) =& J_{ii}, \\
\dot{J}_{ii}(t) =& -2\gamma_i J_{ii} - \gamma_i^2 I_{ii} + [N_{ii}^{\beta} S_i(h_i) + p_{ii}] \\
& G_i \gamma_i e + \Gamma_4, \\
\dot{\phi}_e(t) =& \chi_e, \\
\dot{\chi}_e(t) =& -2\bar{v}\Lambda_{ee}\chi_e - (\bar{v}\Lambda_{ee})^2 \phi_e \\
& + \bar{v}\Lambda_{ee} N_{ee}^{\alpha} \left(\frac{\partial}{\partial t} + \bar{v}\Lambda_{ee}\right) S_e(h_e), \\
\dot{\phi}_i(t) =& \chi_i, \\
\dot{\chi}_i(t) =& -2\bar{v}\Lambda_{ei}\chi_i - (\bar{v}\Lambda_{ei})^2 \phi_i \\
& + \bar{v}\Lambda_{ei} N_{ei}^{\alpha} \left(\frac{\partial}{\partial t} + \bar{v}\Lambda_{ei}\right) S_e(h_e),
\end{aligned}
\tag{1}
$$

where the indexes e and i indicate excitatory and inhibitory neuron populations, the states h_e (mV) and h_i (mV) imply that the excitatory and inhibitory mean soma potential for a neuronal population, respectively. I_{ee} (mV) is the postsynaptic activation of the excitatory population due to inputs from the excitatory population and I_{ei} (mV) is the postsynaptic activation of the inhibitory population due to inputs from excitatory population. Similarly, I_{ie} (mV) is the postsynaptic activation of the excitatory population due to inputs from the inhibitory population and I_{ii} (mV) is the postsynaptic activation of the inhibitory population due to inputs from inhibitory population. $\phi_e(\mathrm{s}^{-1})$ and $\phi_i(\mathrm{s}^{-1})$ are corticocortical inputs to excitatory and inhibitory populations, respectively. The variables $\Gamma_1, \Gamma_2, \Gamma_3$ and Γ_4 are the stochastic inputs. v is uncertainty term that is considered to cause external disturbances, system failures or noise. In Eq. (1), the term u which was calculated by TS fuzzy model based feedback control and applied by the cortical surface electrode was added. $\psi_{jk}(h_k)(j, k \in e, i) = \frac{h_j^{rev} - h_k}{|h_j^{rev} - h_k^{rest}|}, (j, k \in e, i)$, terms are weighting factors for I_{jk} inputs. The sigmoid functions mapping to the soma potential to the firing rate are expressed as $S_e(h_e) = \frac{S_e^{max}}{1 + exp^{[-g_e(h_e - \theta_e)]}}$ and $S_i(h_i) = \frac{S_i^{max}}{1 + exp^{[-g_i(h_i - \theta_i)]}}$. The definition of the P_{ee} and Γ_e parameters in the dimensionless form of the cortex model are as follows.

$$
P_{ee} = \frac{p_{ee}}{S_e^{max}}, \quad \Gamma_e = \frac{G_e e S_e^{max}}{\gamma_e |h_e^{rev} - h_e^{rest}|} \tag{2}
$$

The parameters of the cortex dynamics are given in Table 1. In Fig. 1, the chaotic behavior of the cortex model without controller design was investigated and illustrated to show how change in pathological parameters (Kramer et al. 2006) (subcortical spike input to excitatory population (p_{ee}) and peak amplitude of excitatory postsynaptic potential (G_e)) with the influence of the stochastic input (Γ_e) in the dynamics. According to Kramer et al. (2006), the "healthy state" occurs when the typical values of pathological parameters are $p_{ee} = 1100$ and $G_e = 0.18$ mV with $\Gamma_e = 1.42 \times 10^{-3}$. However, the "epileptic state" occurs when $p_{ee} = 54{,}800$ and $G_e = 0.1$ mV with $\Gamma_e = 0.8 \times 10^{-3}$. Figure 1a illustrates the bifurcation diagram for unstabilized dynamic of $h_e(t)$ versus the variation of the Γ_e. The numerical solution of $h_e(t)$ at the pathological parameters with $\Gamma_e = 0.8 \times 10^{-3}$ is given in Fig. 1b. While the dynamics of the healthy state is similar to a damping behavior, regular oscillations are observed on the mean soma potential signal for epileptic state.

Table 1 Parameters of the cortex model (Steyn-Ross et al. 2003)

τ_e, τ_i	Membrane time constant	0.04, 0.04 s
h_e^{rest}, h_i^{rest}	Resting potential	−70, −70 mV
h_e^{rev}, h_i^{rev}	Reversal potential	45, −90 mV
p_{ee}, p_{ie}	Subcortical spike input to e population	1100, 1600 s^{-1}
p_{ei}, p_{ii}	Subcortical spike input to i population	1600, 1100 s^{-1}
$\wedge_{ee}, \wedge_{ei}$	Corticotical inverse-length	0.04, 0.065 mm^{-1}
γ_e, γ_i	Neurotransmitter rate constant for e, i postsynaptic potential	300, 65 s^{-1}
G_e, G_i	Peak amplitude of e i postsynaptic potential	0.18, 0.37 mV
$N_{ee}^{\beta}, N_{ei}^{\beta}$	Total number of local synaptic connections of e	3034, 3034
$N_{ie}^{\beta}, N_{ii}^{\beta}$	Total number of local synaptic connections of i	536, 536
$N_{ee}^{\alpha}, N_{ei}^{\alpha}$	Total number of synaptic connections from distant e populations	4000, 2000
$\bar{v}$	Mean axonal conduction speed	7000 mm s^{-1}
S_e^{max}, S_i^{max}	Maximum of sigmoid function	100, 100 s^{-1}
θ_e, θ_i	Inflection-point potential for sigmoid function	−60, −60 mV
g_e, g_i	Sigmoid slope at inflection point	0.28, 0.14 mV^{-1}

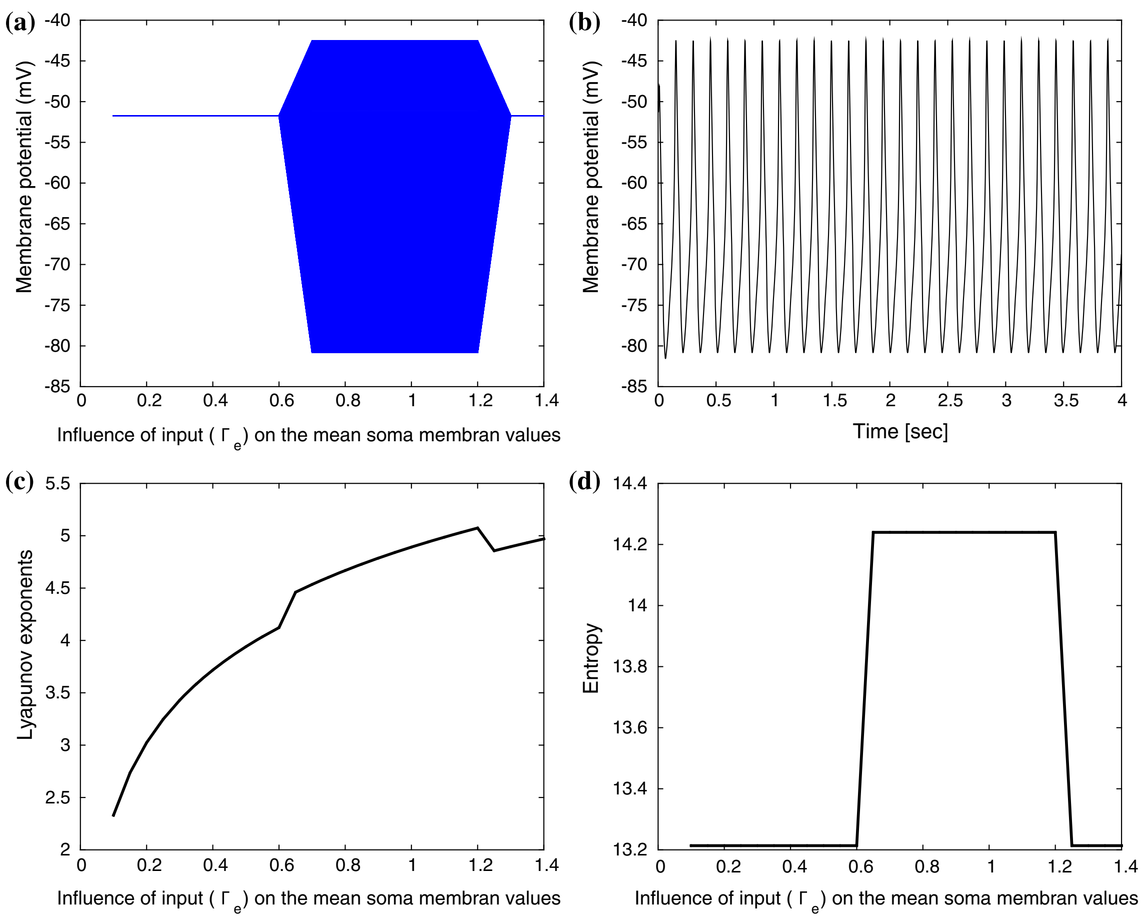


Fig. 1 **a** Bifurcation diagram for unstabilized dynamics of Eq. (1) at the pathological parameter values ($p_{ee} = 54{,}800, G_e = 0.1\times 10^{-3}, \Gamma_e = 0.8 \times 10^{-3}$). **b** Numerical solution of the cortex model at the pathological parameters. **c** Lyapunov exponents of the cortex model. **d** The variation of entropy versus Γ_e

Deterministic or statistical methods based on dynamic system theory are used for analysis of neurophysiological signals and measuring complexity. Among these methods, the results of electrophysiological recordings analyzed by entropy measurement for epilepsy, schizophrenia, abnormal cognitive disorders, coma and sleep are presented in Mateos et al. (2018). The entropy criterion as well as the large Lyapunov exponent (LLE) are important statistics used to analyze chaotic behaviors. Therefore, in this paper, chaotic behaviors observed in cortex dynamics have been examined by LLE and entropy criteria. The critical value range of Γ_e ($[0.64, 1.15] \times 10^{-3}$) shows its effect on LLE and entropy in Fig. 1c, d, respectively.

Observer and controller design for cortex model

The TS fuzzy model-based dynamic system was defined by fuzzy IF–THEN rules that represent local linear input–output relations of a nonlinear system in Takagi and Sugeno (1985). The ith rule of the TS fuzzy model for continuous fuzzy system (Tanaka et al. 1998) with the initial state vector $x(0)$ is as $(i = 1, 2, \ldots, r)$

$$\text{IF} \quad z_1(t) \quad \text{is} \quad M_{i1} \quad \text{and} \quad z_p(t) \quad \text{is} \quad M_{ip}$$
$$\text{THEN} \begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t) \\ \mathbf{y}(t) = \mathbf{C}_i\mathbf{x}(t) \end{cases} \tag{3}$$

where M_{ij} is the fuzzy set, r is the rule number and $z_1(t) \sim z_p(t)$ are the premise variables. $\mathbf{x}(t) \in \Re^{n}$ is the state vector, $\mathbf{u}(t) \in \Re^{m}$ is the input vector and $\mathbf{y}(t) \in \Re^{q}$ is the output vector, respectively. $\mathbf{A}_i \in \Re^{n \times n}$, $\mathbf{B}_i \in \Re^{n \times m}$ and $\mathbf{C}_i \in \Re^{q \times n}$ constant suitable matrices and the linear equation set denoted by $\dot{\mathbf{x}}(t)$ is called the subsystem. According to this definitions, TS fuzzy-model based system is inferred from (3) as

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \sum_{i=1}^{r} h_i(\mathbf{z}(t))\{\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)\} \\ \mathbf{y}(t) &= \sum_{i=1}^{r} h_i(\mathbf{z}(t))\mathbf{C}_i\mathbf{x}(t) \end{aligned} \tag{4}$$

where $h_i(\mathbf{z}(t)) = \frac{w_i(\mathbf{z}(t))}{\sum_{i=1}^{r} w_i(\mathbf{z}(t))} > 0$, $w_i(\mathbf{z}(t)) = \prod_{j=1}^{p} M_{ij}(z_j(t)) \geq 0$ for all t. $M_{ij}(z_j(t))$ is the grade membership of $z_j(t)$ in M_{ij}.

Using Eq. (4), we have that $\sum_{i=1}^{r} h_i(\mathbf{z}(t)) = 1$ and $h_i(\mathbf{z}(t)) \geq 0$ for all t. Then, the fuzzy system rules can be represented as

$$\text{IF} \quad x(t) \quad \text{is} \quad M_{i1} \quad \text{and} \quad x(t-n+1) \quad \text{is} \quad M_{in}$$
$$\text{THEN} \begin{cases} \mathbf{x}(t+1) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t) \\ \mathbf{y}(t) = \mathbf{C}_i\mathbf{x}(t) \end{cases} \tag{5}$$

where $\mathbf{x}(t) = [x(t) \quad x(t-1)\ldots x(t-n+1)]^T$. The stability conditions of continuous fuzzy system (4) is investigated in Tanaka et al. (1998).

Sector nonlinearity-based TS fuzzy modeling

The brain cortex model in (1) can be referred to produce sector nonlinearities, which is used for the design of model-based TS fuzzy systems as follows

$$\begin{aligned} &\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, u)\mathbf{x} + \mathbf{g}(\mathbf{x}, u)u + \mathbf{a}(\mathbf{x}, u) + \mathbf{v}(x), \\ &y = \mathbf{h}(\mathbf{x}, u)\mathbf{x} + \mathbf{d}(\mathbf{x}, u), \end{aligned} \tag{6}$$

where $\mathbf{f}(.), \mathbf{g}(.)$ and $\mathbf{h}(.)$ are nonlinear functions where $(\mathbf{x}, u)$ are defined in compact sets. The input (control voltage), state and output (membrane potential) variables are defined in compact sets with $u \in \mathrm{U}$, $\mathbf{x} \in \mathrm{X}$ and $y \in \mathrm{Y}$ and $\mathbf{a}(.)$ and $\mathbf{d}(.)$ are bounded affine vector terms. $\mathbf{v}(.)$ is uncertainty function that is considered to cause external disturbances, system failures or noise. The output (y) is measured where the all states are stabilized to the equilibrium points with estimated variables. The nonlinear state-space representation of the cortex model (CM) model is as follows.

$$\begin{aligned} &\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) + \mathbf{v} \\ &\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \end{aligned} \tag{7}$$

where

$$\mathbf{A}=\begin{bmatrix} \frac{-1}{\tau_e} & 0 & \frac{z_1}{\tau_e} & 0 & 0 & 0 & \frac{z_2}{\tau_e} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{\tau_i} & 0 & 0 & \frac{z_3}{\tau_i} & 0 & 0 & 0 & \frac{z_4}{\tau_i} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p_1 & 0 & -\gamma_e^2 & -2\gamma_e & 0 & 0 & 0 & 0 & 0 & 0 & p_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p_2 & 0 & 0 & 0 & -\gamma_e^2 & -2\gamma_e & 0 & 0 & 0 & 0 & 0 & 0 & p_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & p_3 & 0 & 0 & 0 & 0 & -\gamma_i^2 & -2\gamma_i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & p_4 & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma_i^2 & -2\gamma_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ p_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_8 & p_9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ p_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{10} & p_{11} \end{bmatrix}$$

$$\mathbf{B}=\begin{bmatrix} \frac{1}{\tau_e} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$\mathbf{C}=[1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \tag{8}$$

and the parameters in Eq. (8)

$$\begin{aligned} &p_1 = N_{ee}^{\beta} G_e \gamma_e z_5 e, \quad p_2 = N_{ei}^{\beta} G_e \gamma_e z_5 e, \quad p_3 = N_{ie}^{\beta} G_i \gamma_i z_6 e, \\ &p_4 = N_{ii}^{\beta} G_i \gamma_i z_6 e, p_5 = \bar{v} \wedge_{ee} N_{ee}^{\alpha} z_7 + (\bar{v}\wedge_{ee})^2 N_{ee}^{\alpha} z_5, \\ &p_6 = \bar{v} \wedge_{ei} N_{ei}^{\alpha} z_7 + (\bar{v}\wedge_{ei})^2 N_{ei}^{\alpha} z_5, \quad p_7 = G_e \gamma_e e, \\ &p_8 = -(\bar{v}\wedge_{ee})^2, \quad p_9 = -2\bar{v}\wedge_{ee}, \quad p_{10} = -(\bar{v}\wedge_{ei})^2, \\ &p_{11} = -2\bar{v}\wedge_{ei} \end{aligned}$$

In the nonlinear state-space form of brain cortex model, the minimum and maximum values of the design functions $(z_1(x), \ldots, z_7(x))$ can be calculated according to the minimum and maximum values of the CM model dynamics where $h_e \in [-70, 45]$ mV, $h_i \in [-90, -70]$ mV.

As an example, let's explain how $z_1(x)$ limit values are calculated. According to the Table 1, $h_e^{rest} - 70$ mV, $h_i^{rest} - 70$ mV, $h_e^{rev} 45$ mV, $h_i^{rev} - 90$ mV. The maximum limit value of $\overline{nz}_1 = \frac{1}{\tau_e}\left(\frac{he^{rev} - he(t)}{he^{rev} - he^{rest}}\right) = \frac{1}{0.04}\left(\frac{45\,\mathrm{mV} - (-70)\,\mathrm{mV}}{45\,\mathrm{mV} - (-70)\,\mathrm{mV}}\right) = 25$. In addition, the minimum limit value of $\underline{nz}_1 = \frac{1}{0.04}\left(\frac{45\,\mathrm{mV} - (45)\,\mathrm{mV}}{45\,\mathrm{mV} - (-70)\,\mathrm{mV}}\right) = 0$. The limit values of the other nonlinear design functions $(z_2(x), \ldots, z_7(x))$ are calculated similarly. The designed sector nonlinear-based TS fuzzy system rule-base are given as follows

1. $z_1(x) = \frac{1}{\tau_e}\left(\frac{he^{rev} - he(t)}{he^{rev} - he^{rest}}\right) \in [0, 25]$ where $\underline{nz}_1 = 0$ and $\overline{nz}_1 = 25$ are set. The weighting functions are defined as

$$\begin{aligned} w_1^1 &= \frac{\overline{nz}_1 - z_1(x)}{\overline{nz}_1 - \underline{nz}_1}, \\ w_0^1 &= 1 - w_1^1. \end{aligned} \tag{9}$$

The value of the designed function can be determined using the weighted sum of the functions as

$$z_1(x) = \underline{nz}_1 w_0^1(z_1) - \overline{nz}_1 w_1^1(z_1). \tag{10}$$

2. $z_2(x) = \frac{1}{\tau_e}\left(\frac{hi^{rev} - he(t)}{he^{rest} - hi^{rev}}\right) \in [-1.687 \times 10^2, -25]$ where $\underline{nz}_2 = -1.687 \times 10^2$ and $\overline{nz}_2 = -25$ are set. The weighting functions are defined as

$$w_1^2 = \frac{\overline{nz}_2 - z_2(x)}{\overline{nz}_2 - \underline{nz}_2}, \quad w_0^2 = 1 - w_1^2. \tag{11}$$

The value of the designed function can be determined using the weighted sum of the functions as

$$z_2(x) = \underline{nz}_2 w_0^2(z_2) - \overline{nz}_2 w_1^2(z_2). \tag{12}$$

3. $z_3(x) = \frac{1}{\tau_i}(\frac{he^{rev} - hi(t)}{he^{rev} - hi^{rest}}) \in [25, 29.347]$ where $\underline{nz}_3 = 25$ and $\overline{nz}_3 = 29.347$ are set. The weighting functions are defined as

$$w_1^3 = \frac{\overline{nz}_3 - z_3(x)}{\overline{nz}_3 - \underline{nz}_3}, \quad w_0^3 = 1 - w_1^3. \tag{13}$$

The value of the designed function can be determined using the weighted sum of the functions as

$$z_3(x) = \underline{nz}_3 w_0^3(z_3) - \overline{nz}_3 w_1^3(z_3). \tag{14}$$

4. $z_4(x) = \frac{1}{\tau_i}(\frac{hi^{rev} - hi(t)}{hi^{rest} - hi^{rev}}) \in [-25, 0]$ where $\underline{nz}_4 = -25$ and $\overline{nz}_4 = 0$ are set. The weighting functions are defined as

$$w_1^4 = \frac{\overline{nz}_4 - z_4(x)}{\overline{nz}_4 - \underline{nz}_4}, \quad w_0^4 = 1 - w_1^4. \tag{15}$$

The value of the designed function can be determined using the weighted sum of the functions as

$$z_4(x) = \underline{nz}_4 w_0^4(z_4) - \overline{nz}_4 w_1^4(z_4). \tag{16}$$

5. $z_5(x) = \frac{1}{h_e}(\frac{S_e^{max}}{1+\exp^{[-g_e(h_e-\theta_e)]}}) \in [-0.081, 2.222]$ where $\underline{nz}_5 = -0.081$ and $\overline{nz}_5 = 2.222$ are set. The weighting functions are defined as

$$w_1^5 = \frac{\overline{nz}_5 - z_5(x)}{\overline{nz}_5 - \underline{nz}_5}, \quad w_0^5 = 1 - w_1^5. \tag{17}$$

The value of the designed function can be determined using the weighted sum of the functions as

$$z_5(x) = \underline{nz}_5 w_0^5(z_5) - \overline{nz}_5 w_1^5(z_5). \tag{18}$$

6. $z_6(x) = \frac{1}{h_i}(\frac{S_i^{max}}{1+\exp^{[-g_i(h_i-\theta_i)]}}) \in [-0.282, -0.016]$ where $\underline{nz}_6 = -0.282$ and $\overline{nz}_6 = -0.016$ are set. The weighting functions are defined as

$$w_1^6 = \frac{\overline{nz}_6 - z_6(x)}{\overline{nz}_6 - \underline{nz}_6}, \quad w_0^6 = 1 - w_1^6. \tag{19}$$

The value of the designed function can be determined using the weighted sum of the functions as

$$z_6(x) = \underline{nz}_6 w_0^6(z_6) - \overline{nz}_6 w_1^6(z_6). \tag{20}$$

7. $z_7(x) = \frac{1}{h_e}(\frac{S_e^{max} g_e \exp^{[g_e(\theta_e-h_e)]}}{(\exp^{[g_e(\theta_e-h_e)]}+1)^2}) \in [-0.021, 1.06 \times 10^{-13}]$ where $\underline{nz}_7 = -0.021$ and $\overline{nz}_7 = 1.06 \times 10^{-13}$ are set. The weighting functions are defined as

$$w_1^7 = \frac{\overline{nz}_7 - z_7(x)}{\overline{nz}_7 - \underline{nz}_7}, \quad w_0^7 = 1 - w_1^7. \tag{21}$$

The value of the designed function can be determined using the weighted sum of the functions as

$$z_7(x) = \underline{nz}_7 w_0^7(z_7) - \overline{nz}_7 w_1^7(z_7). \tag{22}$$

Using the above definitions, R_i^j fuzzy sets $(i = 0, 1; j = 1, \ldots, 7)$ and TS fuzzy rule base can be defined according to the weighting functions. There are 7 nonlinear design functions $(p = 7)$ and 128 fuzzy rules $(r = 2^p = 128)$. Some of the rule base as follows

$$
\begin{aligned}
&\text{Rule 1: IF } z_1 \text{ is } R_0^1 \text{ and } z_2 \text{ is } R_0^2 \text{ and } z_3 \text{ is } R_0^3 \text{ and } z_4 \text{ is } R_0^4 \text{ and} \\
&\qquad z_5 \text{ is } R_0^5 \text{ and } z_6 \text{ is } R_0^6 \text{ and } z_7 \text{ is } R_0^7 \\
&\qquad \text{THEN} \quad \dot{\mathbf{x}}(t) = \mathbf{A}_1\mathbf{x}(t) + \mathbf{B}_1 u(t), \quad y(t) = \mathbf{C}_1\mathbf{x}(t) \\
&\qquad \text{The corresponding } \mathbf{A}_1 \text{ matrix depends on } [\underline{nz}_1, \underline{nz}_2, \underline{nz}_3, \underline{nz}_4, \underline{nz}_5, \underline{nz}_6, \underline{nz}_7] \\
&\qquad \mathbf{B}_1 = \begin{bmatrix} \frac{1}{\tau_e} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \\
&\qquad \mathbf{C}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
&\qquad \text{The fuzzy membership function of this rule is } h(1)(\mathbf{z}) = w_0^1 w_0^2 w_0^3 w_0^4 w_0^5 w_0^6 w_0^7 \\
&\text{Rule 2: IF } z_1 \text{ is } R_0^1 \text{ and } z_2 \text{ is } R_0^2 \text{ and } z_3 \text{ is } R_0^3 \text{ and } z_4 \text{ is } R_0^4 \text{ and} \\
&\qquad z_5 \text{ is } R_0^5 \text{ and } z_6 \text{ is } R_0^6 \text{ and } z_7 \text{ is } R_1^7 \\
&\qquad \text{THEN} \quad \dot{\mathbf{x}}(t) = \mathbf{A}_2\mathbf{x}(t) + \mathbf{B}_2 u(t), \quad y(t) = \mathbf{C}_2\mathbf{x}(t) \\
&\qquad \text{The corresponding } \mathbf{A}_2 \text{ matrix depends on } [\underline{nz}_1, \underline{nz}_2, \underline{nz}_3, \underline{nz}_4, \underline{nz}_5, \underline{nz}_6, \overline{nz}_7] \\
&\qquad \mathbf{B}_2 = \begin{bmatrix} \frac{1}{\tau_e} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \\
&\qquad \mathbf{C}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
&\qquad \text{The fuzzy membership function of this rule is } h(2)(\mathbf{z}) = w_0^1 w_0^2 w_0^3 w_0^4 w_0^5 w_0^6 w_1^7 \\
&\qquad \vdots \\
&\text{Rule 127: IF } z_1 \text{ is } R_0^1 \text{ and } z_2 \text{ is } R_1^2 \text{ and } z_3 \text{ is } R_1^3 \text{ and } z_4 \text{ is } R_1^4 \text{ and} \\
&\qquad z_5 \text{ is } R_1^5 \text{ and } z_6 \text{ is } R_1^6 \text{ and } z_7 \text{ is } R_0^7 \\
&\qquad \text{THEN} \quad \dot{\mathbf{x}}(t) = \mathbf{A}_{127}\mathbf{x}(t) + \mathbf{B}_{127} u(t), \quad y(t) = \mathbf{C}_{127}\mathbf{x}(t) \\
&\qquad \text{The corresponding } \mathbf{A}_{127} \text{ matrix depends on } [\underline{nz}_1, \overline{nz}_2, \overline{nz}_3, \overline{nz}_4, \overline{nz}_5, \overline{nz}_6, \underline{nz}_7] \\
&\qquad \mathbf{B}_{127} = \begin{bmatrix} \frac{1}{\tau_e} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \\
&\qquad \mathbf{C}_{127} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
&\qquad \text{The fuzzy membership function of this rule is } h(127)(\mathbf{z}) = w_0^1 w_1^2 w_1^3 w_1^4 w_1^5 w_1^6 w_0^7 \\
&\text{Rule 128: IF } z_1 \text{ is } R_0^1 \text{ and } z_2 \text{ is } R_1^2 \text{ and } z_3 \text{ is } R_1^3 \text{ and } z_4 \text{ is } R_1^4 \text{ and} \\
&\qquad z_5 \text{ is } R_1^5 \text{ and } z_6 \text{ is } R_1^6 \text{ and } z_7 \text{ is } R_1^7 \\
&\qquad \text{THEN} \quad \dot{\mathbf{x}}(t) = \mathbf{A}_{128}\mathbf{x}(t) + \mathbf{B}_{128} u(t), \quad y(t) = \mathbf{C}_{128}\mathbf{x}(t) \\
&\qquad \text{The corresponding } \mathbf{A}_{128} \text{ matrix depends on } [\underline{nz}_1, \overline{nz}_2, \overline{nz}_3, \overline{nz}_4, \overline{nz}_5, \overline{nz}_6, \overline{nz}_7] \\
&\qquad \mathbf{B}_{128} = \begin{bmatrix} \frac{1}{\tau_e} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \\
&\qquad \mathbf{C}_{128} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
&\qquad \text{The fuzzy membership function of this rule is } h(128)(\mathbf{z}) = w_0^1 w_1^2 w_1^3 w_1^4 w_1^5 w_1^6 w_1^7
\end{aligned}
\tag{23}
$$

After that, observer-based controller design using the TS fuzzy model instead of the CM model can explain.

TS fuzzy controller based stabilization

Fuzzy controllers are designed to provide $\mathbf{x}(t) \to 0$ when $t \to \infty$ to stabilization of control systems. For Eq. (3) the following fuzzy controller based stabilization is designed via parallel distributed compensation (PDC) (Wang et al. 1995)

$$\begin{aligned} &\text{IF} \quad z_1(t) \quad \text{is} \quad M_{i1} \quad \text{and}\ldots \quad \text{and} \quad z_p(t) \quad \text{is} \quad M_{ip} \\ &\text{THEN} \quad \mathbf{u}(t) = -\mathbf{F}_i\mathbf{x}(t), \quad i = 1, 2, \ldots, r. \end{aligned} \tag{24}$$

The fuzzy controller is to determine the local feedback gains $\mathbf{F}_i$ with respect to linear state feedback rules as

$$\mathbf{u}(t) = -\frac{\sum_{i=1}^{r} w_i(\mathbf{z}(t))\mathbf{F}_i\mathbf{x}(t)}{\sum_{i=1}^{r} w_i(\mathbf{z}(t))} = -\sum_{i=1}^{r} h_i(\mathbf{z}(t))\mathbf{F}_i\mathbf{x}(t). \tag{25}$$

Using (25) into (4), the continuous fuzzy system is obtained as

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\mathbf{z}(t))h_j(\mathbf{z}(t))\{\mathbf{A}_i - \mathbf{B}_i\mathbf{F}_j\}\mathbf{x}(t) \\ y &= \sum_{i=1}^{r} h_i(\mathbf{z}(t))\mathbf{C}_i\mathbf{x}(t). \end{aligned} \tag{26}$$

If $\mathbf{G}_{ij} = \mathbf{A}_i - \mathbf{B}_i\mathbf{F}_j$, then Eq. (26) can be rewritten as

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \sum_{i=1}^{r} h_i(\mathbf{z}(t))h_i(\mathbf{z}(t))\mathbf{G}_{ii}\mathbf{x}(t) \\ &\quad + 2\sum_{i<j}^{r} h_i(\mathbf{z}(t))h_j(\mathbf{z}(t))\left\{\frac{\mathbf{G}_{ij} + \mathbf{G}_{ji}}{2}\right\}\mathbf{x}(t). \end{aligned} \tag{27}$$

The continuous fuzzy control system defined by (27) is asymptotically stable in the presence of a common positive defined $\mathbf{P}$ matrix such that

$$\begin{aligned} &\mathbf{G}_{ii}^T\mathbf{P} + \mathbf{P}\mathbf{G}_{ii} < 0, \quad i = 1, 2, \ldots, r \\ &\left(\frac{\mathbf{G}_{ij} + \mathbf{G}_{ji}}{2}\right)^T\mathbf{P} + \mathbf{P}\left(\frac{\mathbf{G}_{ij} + \mathbf{G}_{ji}}{2}\right) \le 0, \quad i<j \end{aligned} \tag{28}$$

In addition, if r that fire is less than or equal s where $1 < s \le r$, the continuous fuzzy control system defined by (27) is asymptotically stable in the presence of a common positive defined $\mathbf{P}$ matrix and a common positive semi-definite matrix $\mathbf{Q}$ such that

$$\begin{aligned} &\mathbf{G}_{ii}^T\mathbf{P} + \mathbf{P}\mathbf{G}_{ii} + (s-1)\mathbf{Q} < 0 \\ &\left(\frac{\mathbf{G}_{ij} + \mathbf{G}_{ji}}{2}\right)^T\mathbf{P} + \mathbf{P}\left(\frac{\mathbf{G}_{ij} + \mathbf{G}_{ji}}{2}\right) - \mathbf{Q} \le 0, \quad i<j \end{aligned} \tag{29}$$

for all i and j excepting the pairs (i, j) such that $h_i(\mathbf{z}(t))h_j(\mathbf{z}(t)) = 0, \forall t$ and $s > 1$ (Tanaka et al. 1998). It is specified in Wang et al. (1995, 1996) that the common $\mathbf{P}$ problem for the fuzzy controller design can be solved numerically and the stability conditions of (28) can be expressed by linear matrix inequalities (LMIs) (Boyd et al. 1997). In Tanaka et al. (1998), LMI-based designs for fuzzy controllers/observers were presented for both discrete-time and continuous-time fuzzy control systems. In these designs, nonlinear systems were defined by fuzzy models. LMI-based designs provide system stability, decay rate and constraints on control input/output (Boyd et al. 1997). According to Tanaka et al. (1998), the design problem that determines the $\mathbf{F}_i$ coefficients for CFS can be defined as ($\mathbf{X} > 0, \mathbf{Y} \ge 0$ and $\mathbf{M}_i (i = 1 \sim r)$ satisfying):

$$\begin{aligned} &-\mathbf{X}\mathbf{A}_i^T - \mathbf{A}_i\mathbf{X} + \mathbf{M}_i^T\mathbf{B}_i^T + \mathbf{B}_i\mathbf{M}_i - (s-1)\mathbf{Y} > 0 \\ &2\mathbf{Y} - \mathbf{X}\mathbf{A}_i^T - \mathbf{A}_i\mathbf{X} - \mathbf{X}\mathbf{A}_j^T - \mathbf{A}_j\mathbf{X} \\ &\qquad + \mathbf{M}_j^T\mathbf{B}_i^T + \mathbf{B}_i\mathbf{M}_j + \mathbf{M}_i^T\mathbf{B}_j^T + \mathbf{B}_j\mathbf{M}_i \ge 0, \quad i<j \end{aligned} \tag{30}$$

where $\mathbf{X} = \mathbf{P}^{-1}, \mathbf{M}_i = \mathbf{F}_i\mathbf{X}, \mathbf{Y} = \mathbf{X}\mathbf{Q}\mathbf{X}$. The conditions in (30) are LMI's and a positive definite matrix $\mathbf{X}$, a positive semi-definite matrix $\mathbf{Y}$ and $\mathbf{M}_i$, which satisfy the above conditions, can be found. There are powerful mathematical programming tools available in the literature to solve this feasibility problem (Sturm 1999; Lofberg 2004). Therefore, $\mathbf{F}_i, \mathbf{P}$ and $\mathbf{Q}$ can be obtained as $\mathbf{P} = \mathbf{X}^{-1}, \mathbf{F}_i = \mathbf{M}_i\mathbf{X}^{-1}, \mathbf{Q} = \mathbf{P}\mathbf{Y}\mathbf{P}$ from the solutions $\mathbf{X}, \mathbf{Y}$ and $\mathbf{M}_i$.

TS fuzzy observer design for unmeasurable dynamics

An observer is used to reconstruct or estimate state variables when the state of a system is not fully available. A fuzzy observer, which is designed by the PDC, can be used to estimate the unobservable states of a real-time system. The observer rule for continuous fuzzy system is represented by

$$\begin{aligned} &\text{IF} \quad z_1(t) \quad \text{is} \quad M_{i1} \quad \text{and}\ldots \quad \text{and} \quad z_p(t) \quad \text{is} \quad M_{ip} \\ &\text{THEN} \quad \dot{\hat{\mathbf{x}}}(t) = \mathbf{A}_i\hat{\mathbf{x}}(t) + \mathbf{B}_i\mathbf{u}(t) + \mathbf{K}_i(\mathbf{y}(t) - \hat{\mathbf{y}}(t)), \\ &i = 1, 2, \ldots, r. \end{aligned} \tag{31}$$

where $\hat{\mathbf{y}}(t) = \sum_{i=1}^{r} h_i(\hat{\mathbf{z}}(t))\mathbf{C}_i\hat{\mathbf{x}}(t)$, $\hat{\mathbf{x}}(t)$ is the estimated state vector, $\mathbf{K}_i$ is the observer gain for the ith subsystem and $\mathbf{A}_i$ and $\mathbf{C}_i$ must be observable. The purpose of the TS fuzzy observer design is to provide $\mathbf{x}(t) - \hat{\mathbf{x}}(t) \to 0$ when $t \to \infty$. If $\mathbf{z}(t)$ depends on the estimated state variables, the overall fuzzy observer is represented as follows (Tanaka et al. 1998)

$$\dot{\hat{\mathbf{x}}}(t) = \sum_{i=1}^{r} h_i(\mathbf{z}(t))\{\mathbf{A}_i\hat{\mathbf{x}}(t) + \mathbf{B}_i\mathbf{u}(t) + \mathbf{K}_i(\mathbf{y}(t) - \hat{\mathbf{y}}(t))\}. \tag{32}$$

The observer error dynamics is represented as

$$\dot{\hat{\mathbf{e}}}(t) = \sum_{i=1}^{r} h_i(\mathbf{z}(t))\{\mathbf{A}_i + \mathbf{B}_i\mathbf{u}(t) - \mathbf{K}_i\mathbf{C}_i\}\hat{\mathbf{e}}. \tag{33}$$

The Lyapunov function ($V = \hat{\mathbf{e}}^T\mathbf{P}\hat{\mathbf{e}}$) is used to prove the stability conditions of (33). The observer error dynamics converges to the zero with designed local gains, asymptotically.

Theorem 1 (Tanaka et al. 1998) *The given dynamics in* (33) *are asymptotically stable, if there exists a common* $\mathbf{P} = \mathbf{P}^T > 0$ *such that*

$$\mathbf{P}(\mathbf{A}_i - \mathbf{K}_i\mathbf{C}) + (\mathbf{A}_i - \mathbf{K}_i\mathbf{C})^T\mathbf{P} < 0 \quad i = 1, \ldots, r \tag{34}$$

where $\mathbf{M}_i = \mathbf{P}\mathbf{K}_i$ (34) *is turn into*

$$(\mathbf{P}\mathbf{A}_i - \mathbf{M}_i\mathbf{C}) + (\mathbf{P}\mathbf{A}_i - \mathbf{M}_i\mathbf{C})^T < 0 \quad i = 1, \ldots, r \tag{35}$$

This LMI can be numerically solved using mathematical programming tools (Lofberg 2004). *In addition, the LMIs for desired decay rate* (α) *can be defined in the TS fuzzy observer design.*

Theorem 2 (Tanaka et al. 1998) *The desired decay rate of* (33) *is at least* $\alpha > 0$, *if there exists a common* $\mathbf{P} = \mathbf{P}^T > 0$ *such that*

$$\mathbf{P}(\mathbf{A}_i - \mathbf{K}_i\mathbf{C}) + (\mathbf{P}(\mathbf{A}_i - \mathbf{K}_i\mathbf{C}))^T + 2\alpha\mathbf{P} < 0 \quad i = 1, \ldots, r \tag{36}$$

The design of the fuzzy observer turns into the determination of local gains $\mathbf{K}_i = \mathbf{M}_i^{-1}\mathbf{P}$ *with solving LMIs in* (36).

Embedded observer–controller design

In the presence of unmeasurable states, the main purpose of the observer-based control strategy is to find the common solution in full compliance with all inequalities. Thus, the ideal behavior of system dynamics can be stabilized. LMI designs can also be used with TS fuzzy observer based controller. It is not easy to obtain observer and controller gains directly when the problem is not convex. $\mathbf{x}(t) \to 0$ for the regulator design and $\mathbf{x}(t) - \hat{\mathbf{x}}(t) \to 0$ for the observer design are required to satisfy when $t \to \infty$. There are two possible case where $z_1(t) \sim z_p(t)$ depend on estimated states by a fuzzy observer or do not depend.

If $z_1(t) \sim z_p(t)$ depends on estimated state, Eq. (37) is used instead of Eq. (25) in the use of the fuzzy observer as follows

$$\mathbf{u}(t) = -\frac{\sum_{i=1}^{r} w_i(\mathbf{z}(t))\mathbf{F}_i\hat{\mathbf{x}}(t)}{\sum_{i=1}^{r} w_i(\mathbf{z}(t))} = -\sum_{i=1}^{r} h_i(\mathbf{z}(t))\mathbf{F}_i\hat{\mathbf{x}}(t). \tag{37}$$

From (37) and (32), we obtain these equations, where $\mathbf{e}(t) = \mathbf{x}(t) - \hat{\mathbf{x}}(t)$

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\mathbf{z}(t))h_j(\mathbf{z}(t))\{(\mathbf{A}_i - \mathbf{B}_i\mathbf{F}_j)\mathbf{x}(t) + \mathbf{B}_i\mathbf{F}_j\mathbf{e}(t)\}\\ \dot{\mathbf{e}}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\mathbf{z}(t))h_j(\mathbf{z}(t))\{\mathbf{A}_i - \mathbf{K}_i\mathbf{C}_j\}\mathbf{e}(t)\end{aligned} \tag{38}$$

The TS fuzzy observer-based controller is represented for a continuous system as

$$\begin{aligned}\dot{\mathbf{x}}_a(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\mathbf{z}(t))h_j(\mathbf{z}(t))\mathbf{G}_{ij}\mathbf{x}_a(t)\\ &= \sum_{i=1}^{r} h_i(\mathbf{z}(t))h_i(\mathbf{z}(t))\mathbf{G}_{ii}\mathbf{x}_a(t)\\ &\quad + 2\sum_{i<j}^{r} h_i(\mathbf{z}(t))h_j(\mathbf{z}(t))\frac{\mathbf{G}_{ij} + \mathbf{G}_{ji}}{2}\mathbf{x}_a(t)\end{aligned} \tag{39}$$

The equilibrium point of the system defined by (39) is asymptotically stable if there is a definite positive P matrix such that

$$\begin{aligned}&\mathbf{G}_{ii}^T\mathbf{P} + \mathbf{P}\mathbf{G}_{ii} < 0\\ &\frac{(\mathbf{G}_{ij} + \mathbf{G}_{ji})^T}{2}\mathbf{P} + \mathbf{P}\left(\frac{\mathbf{G}_{ij} + \mathbf{G}_{ji}}{2}\right) < 0, \quad i < j\end{aligned} \tag{40}$$

In addition, the continuous fuzzy control system defined by (39) is asymptotically stable in the presence of a common positive defined $\mathbf{P}$ matrix and a common positive semi-definite matrix $\mathbf{Q}$ such that

$$\begin{aligned}&\mathbf{G}_{ii}^T\mathbf{P} + \mathbf{P}\mathbf{G}_{ii} + (s-1)\mathbf{Q} < 0\\ &\frac{(\mathbf{G}_{ij} + \mathbf{G}_{ji})^T}{2}\mathbf{P} + \mathbf{P}\frac{(\mathbf{G}_{ij} + \mathbf{G}_{ji})}{2} - \mathbf{Q} \le 0, \quad i < j\end{aligned} \tag{41}$$

for all i and j excepting the pairs (i, j) such that $h_i(\mathbf{z}(t))h_j(\mathbf{z}(t)) = 0, \forall t$ and $s > 1$ (Tanaka et al. 1998). The LMI's for decay rate can be defined in the TS fuzzy observer-based system as follows

$$\mathbf{G}_{ii}^T\mathbf{P} + \mathbf{P}\mathbf{G}_{ii} + (s-1)\mathbf{Q} + 2\alpha\mathbf{P} < 0$$
$$\frac{(\mathbf{G}_{ij}+\mathbf{G}_{ji})^T}{2}\mathbf{P} + \mathbf{P}\frac{(\mathbf{G}_{ij}+\mathbf{G}_{ji})}{2} - \mathbf{Q} + 2\alpha\mathbf{P} \le 0, \quad i<j \tag{42}$$

where $\alpha > 0$. The conditions (40), (41) and (42) can be converted to LMI to find feedback gains $\mathbf{F}_i$ and observer gains $\mathbf{K}_i$.

To present the proposed controller more clearly, schematic diagram of the TS fuzzy observer-based controller is illustrated in Fig. 2.

Computational results

In this section, the application results of the embedded observer–controller are presented to suppress the epileptic seizures. The numerical computations were performed with the fourth order Runge–Kutta integration routine and sampling period was selected as $T = 10^{-4}$ s. Using Eqs. (36) and (42), the decay rates for observer/controller design were determined by a grid-search of a reasonable interval then selected as $\alpha_1 = 10^5, \alpha_2 = 10^3$, respectively. Those parameters affect the convergence of the estimation/stabilization errors. There exist seven sector nonlinearity functions in the cortex model therefore 128 TS fuzzy rules are constructed. To estimate and stabilize each TS fuzzy subsystem, 128 observer and controller feedback gain vector are calculated. Below, the first and last feedback gains of the observer ($\mathbf{K}_i$) and controller ($\mathbf{F}_i$) are given.

$$K_1 = \begin{bmatrix} 4.45\times10^{7} \\ 2.61\times10^{-9} \\ -9.11\times10^{-6} \\ -1.02\times10^{-9} \\ -2.40\times10^{-11} \\ 2.87\times10^{-11} \\ 4.33\times10^{-5} \\ -4.12\times10^{-9} \\ -1.87\times10^{-9} \\ 5.62\times10^{-10} \\ -7.53\times10^{-9} \\ 8.34\times10^{-9} \\ 8.30\times10^{-9} \\ -6.60\times10^{-9} \end{bmatrix}, \ldots, K_{128} = \begin{bmatrix} 4.05\times10^{7} \\ 2.37\times10^{-9} \\ -8.27\times10^{-6} \\ -9.33\times10^{-10} \\ -2.18\times10^{-11} \\ 2.61\times10^{-11} \\ 3.94\times10^{-5} \\ -3.74\times10^{-9} \\ -1.70\times10^{-9} \\ 5.11\times10^{-10} \\ -6.84\times10^{-9} \\ 7.58\times10^{-9} \\ 7.54\times10^{-9} \\ -5.99\times10^{-9} \end{bmatrix}. \tag{43}$$

$$F_1 = \begin{bmatrix} 3.96\times10^{3} \\ -6.37\times10^{-5} \\ -0.92\times10^{2} \\ -5.62\times10^{2} \\ -2.60\times10^{-4} \\ 41.01\times10^{-2} \\ -0.94\times10^{2} \\ 6.99\times10^{3} \\ -1.02\times10^{-5} \\ 1.70\times10^{-3} \\ -0.12\times10^{1} \\ 5.03\times10^{2} \\ 0.16\times10^{2} \\ 5.245\times10^{1} \end{bmatrix}, \ldots, F_{128} = \begin{bmatrix} 5.69\times10^{3} \\ -3.92\times10^{-5} \\ -0.12\times10^{2} \\ -9.70\times10^{3} \\ -4.29\times10^{-4} \\ 74.01\times10^{-2} \\ -0.11\times10^{2} \\ 8.95\times10^{2} \\ -6.94\times10^{-6} \\ 2.40\times10^{-3} \\ -0.27\times10^{1} \\ 1.09\times10^{3} \\ 0.17\times10^{1} \\ 0.83\times10^{2} \end{bmatrix}. \tag{44}$$

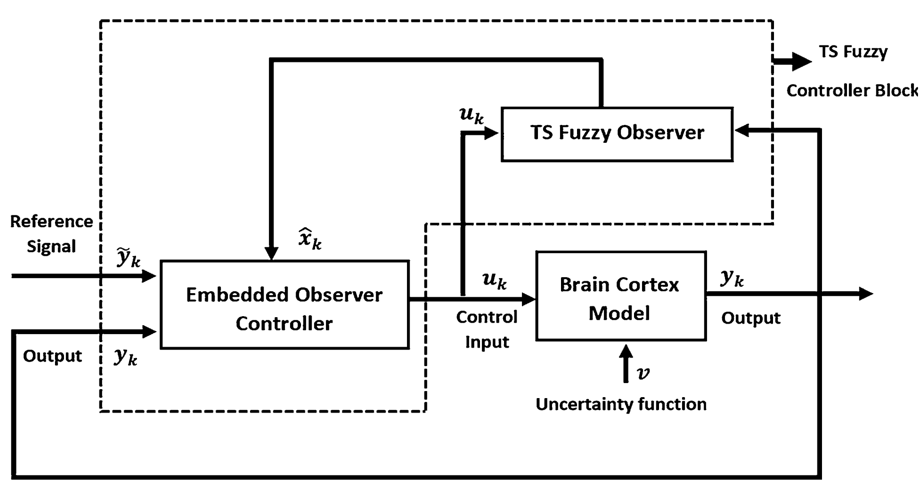
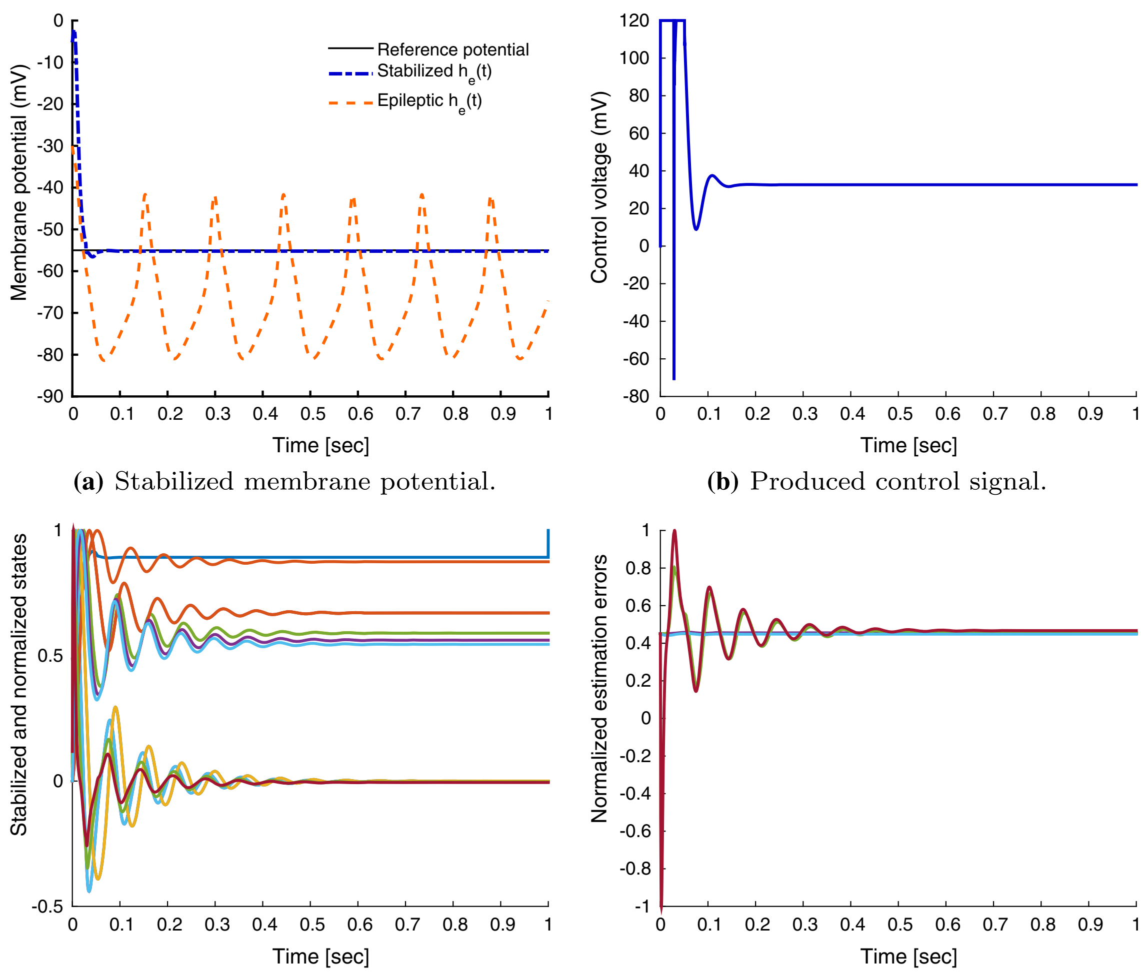


Fig. 2 Block diagram of TS fuzzy observer–controller

It is assumed that the mathematical model of the brain cortex has uncertainties, unknown parameters, unmodeled dynamics, noise or disturbances. Therefore, the observer/controller must suppress these uncertainties when a real-time application is considered. In order to simulate the uncertainty condition, it is applied an artificial uncertainty function to the membrane potential dynamics in epileptic case as $v(t) = 40 + 20sin(h_e(t))$. For the uncertain case of the dynamics, the all stabilization and estimation dynamics of the cortex model are shown in Figs. 3 and 4, respectively. In Fig. 3a, the epileptic and stabilized membrane potential are shown with desired reference membrane potential. The applied control signal to stabilize the membrane potential is given in Fig. 3b. The produced and applied control signal is in the range of the applicable interval. The complete stabilized states are normalized and plotted in Fig. 3c since some of the states have very large values. Remember that these stabilization results are obtained under uncertain case and estimation of the unmeasured states. Therefore, the state estimation errors are normalized and demonstrated in Fig. 3d.

The real stabilized states and their estimates under uncertain case are illustrated in Fig. 4. The estimated states converge to the real values in short periods. At first, there is obtained relatively higher estimation errors, however, their values are to be compensated by the state feedback gains of both observer and controller. These estimation results are based on the system model therefore the uncertainty of the model is also compensated by the observer feedback gains. It also shows the robustness of the designed observer.

The standard PID controller, which can only operate within the linear operating range, is still the most widely used controller in the industrial applications in terms of simple design and efficiency. In this part of the paper, the standard PID control results of the cortex model have been illustrated comparatively to the TS fuzzy observer–controller design. The stabilization and tracking results of the designed controllers for constant membrane potential and spike waveform, including uncertain conditions, are shown



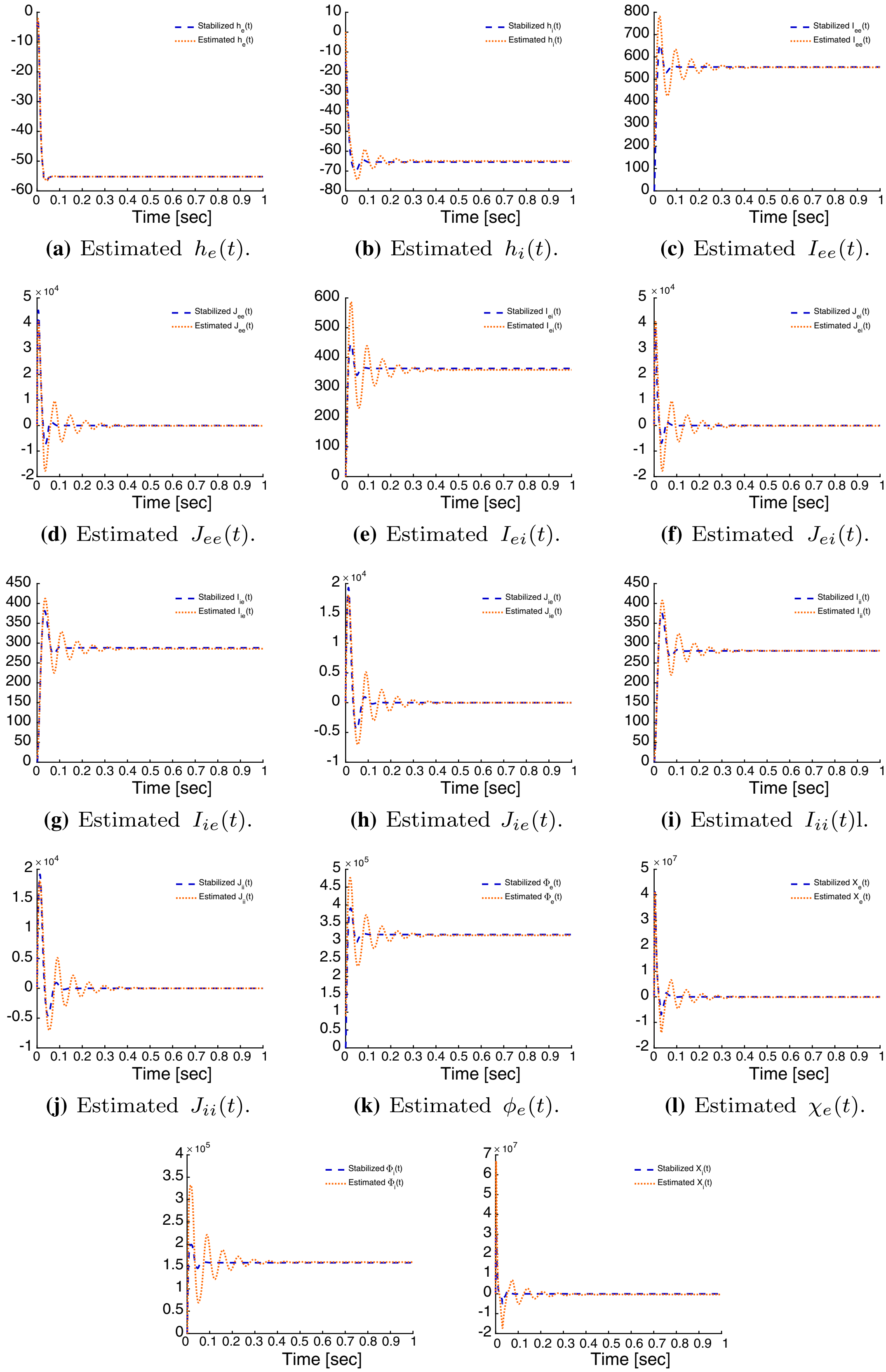
(a) Stabilized membrane potential.

(b) Produced control signal.

(c) Norm of total stabilization errors.

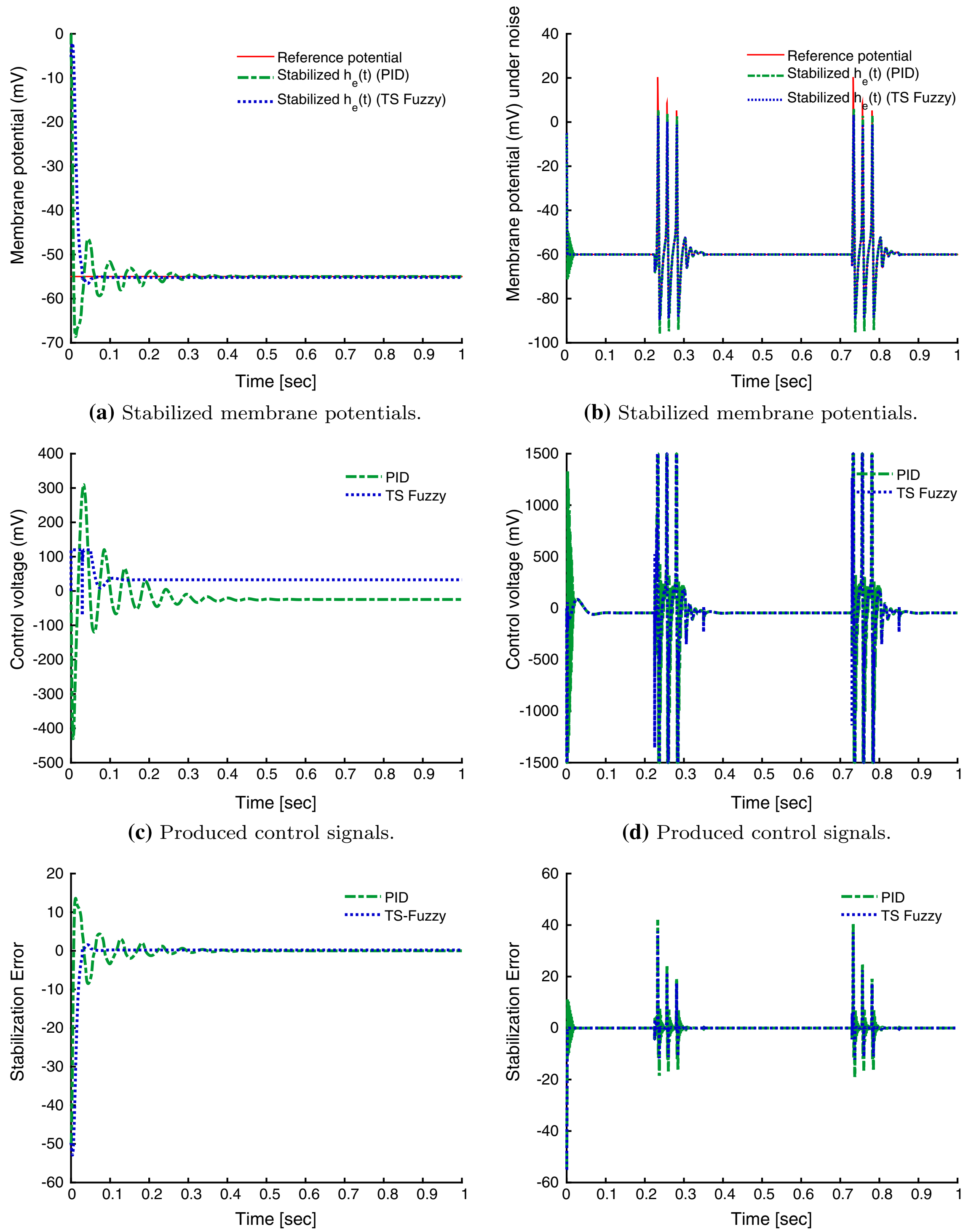
(d) Norm of total estimation errors.

Fig. 3 Observer based stabilization results



(a) Estimated $h_e(t)$. **(b)** Estimated $h_i(t)$. **(c)** Estimated $I_{ee}(t)$. **(d)** Estimated $J_{ee}(t)$. **(e)** Estimated $I_{ei}(t)$. **(f)** Estimated $J_{ei}(t)$. **(g)** Estimated $I_{ie}(t)$. **(h)** Estimated $J_{ie}(t)$. **(i)** Estimated $I_{ii}(t)$l. **(j)** Estimated $J_{ii}(t)$. **(k)** Estimated $\phi_e(t)$. **(l)** Estimated $\chi_e(t)$. **(m)** Estimated $\phi_i(t)$. **(n)** Estimated $\chi_i(t)$.

Fig. 4 TS fuzzy observer estimation results



(a) Stabilized membrane potentials. **(b)** Stabilized membrane potentials. **(c)** Produced control signals. **(d)** Produced control signals. **(e)** Stabilization errors. **(f)** Stabilization errors.

Fig. 5 Comparative stabilization results with different membrane potentials

in Fig. 5. Note that the spike waveform of Hodgkin Huxley neuron is used to show the applicability of the designed controllers. The PID parameters have been calculated as optimum values through grid-search as follows: (i) to stabilize a constant trajectory: $K_P = 4, K_I = 0.3, K_D = 2$. (ii) to stabilize a spike waveform: $K_P = 40, K_I = 35$, $K_D = 0.2$, respectively.

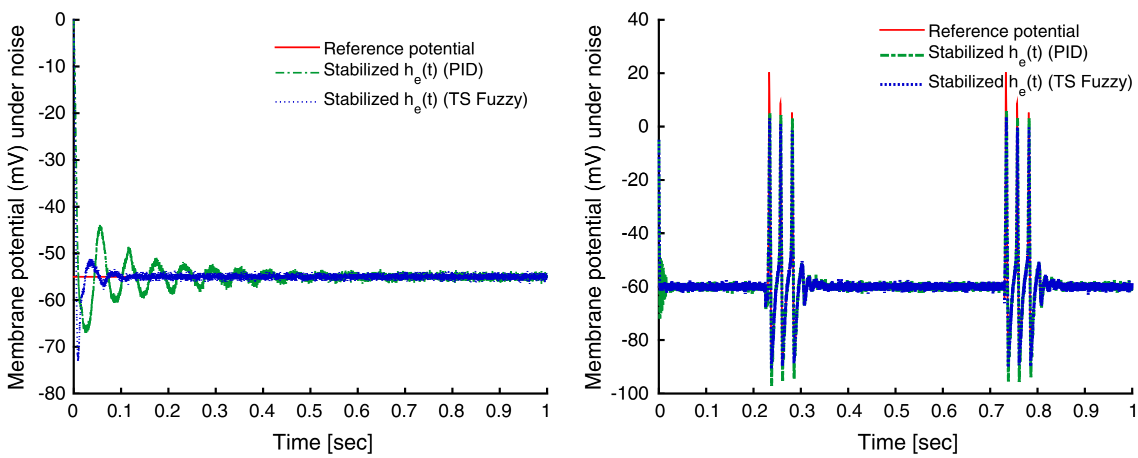
In addition to uncertain case, a Gaussian measurement noise with SNR 20 dB was applied to the measured output for constant membrane potential and spike waveform as shown in Fig. 6. The corresponding performance results for designed controllers in this section are given in Table 2.

According to these results, seizure oscillations are suppressed by both controllers seen in Figs. 5 and 6. The stabilization and tracking of the membrane potential by using the standard PID controller is not as efficient as the TS Fuzzy controller in terms of transient-response characteristics such as rise time, settling time and maximum overshoot.

Discussion and conclusion

The mathematical dynamics of the cortex model has many sector or nonlinearity functions. Therefore, at first, it can be seen difficult to construct a TS fuzzy model of cortex model for an observer/controller designs. However, in the end of a detailed work on the sector functions, a TS fuzzy model is designed for the cortex model.The TS fuzzy controller produces a control signal based on the state feedback so that there is no adapting parameters or online optimization that provides faster generation of the control signal. The feedback gains of controller/observer only depend on the feasible solution of the LMI equations, once a feedback vector is obtained then they are not changed and applied continuously.

Due to the large number of the constructed fuzzy rules, the convergence of the states and production of the control signal can also be expected slow. In contrast, according to the application results, the applied control signal can be produced in an applicable period and the convergence of the epileptic membrane potential needs very-small time in numerical simulation. These are the main motivations of presenting the application results. In fact, in the optimization of the feedback constants, they are designed to



(a) Constant trajectory. **(b)** Spike waveform.

Fig. 6 Stabilization membrane potentials with different trajectories under noise

Table 2 Numerical performances

Performance	PID controller	TS Fuzzy controller
$(RMSE(e) = \sqrt{\frac{1}{T}\int_0^T e(t)^2 dt})$ (for states)		
State estimation err. (for 14 state)	–	8.41×10^2
Stabilization err. (constant ref.)	0.66	0.30
Stabilization err. under noise (constant ref.)	0.77	0.31
Stabilization err. (spike waveform)	0.67	0.44
Stabilization err. under noise (spike waveform)	0.81	0.56
$P(u) = \frac{1}{2T}\int_0^T u(t)^2 dt$ (for control signal)		
Average power (constant ref.)	82.23	53.71
Average power under noise (constant ref.)	84.40	62.33
Average power under (spike waveform)	82.68	52.23
Average power under noise (spike waveform)	85.53	62.21

provide an exponential convergence of states and suppressing the unknown small uncertainties. Noting that the theory of TS fuzzy controller and observer design are well-established and the feedback vectors providing an exponential convergence guarantee is based on the linear system theory and feasible LMI solution.

Although widely used, tuning of the PID parameters is an important problem to be solved to achieve the desired control performance. There are different methods for setting PID parameters in linear time-invariant (LTI) systems. However, these parameters are usually adjusted for local regions using the linearization methods of the nonlinear systems. But linearization method is mostly unsatisfactory for highly nonlinear systems. Furthermore, there may be some uncertainities in the closed-loop control that cause different linearization points when the system contains chaotic dynamics such as cortex model.

As a summary, many sensory device are not needed by using the TS fuzzy observer design, and an exponential convergence of epilepsy stabilization is obtained by using the TS fuzzy controller. Compared with PID controller, it can be concluded that the a TS fuzzy observer–controller can be designed for the complex dynamics of cortex model such that these designs provide a satisfactory level of performances for the application in real-time and production of a portable device.

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