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AN APPROACH TO POWER RATIOS AND PROBABILITIES AND INTERPRETATIONS OF THESE QUANTITIES IN RECTANGULAR QUANTUM WELLS

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ABSTRACT

This study focuses on power and probability expressions belong to normalized frequency and normalized propagation constant of electric field in the rectangular quantum well. The confinement effects of the properties of confined carriers in the energy levels have been studied. Normalized frequency and normalized propagation constant are especially functions of the ordinates and abscissas of the energy eigenvalues for electrons or holes in the rectangular quantum well in the normalized coordinate system (ζ-η). Our calculations also give more accurate results, and present more sensitive comparative examples.

Key Words : Normalize propagation constant, Quantum well lasers

DİKDÖRTGEN KUANTUM ÇUKURLARINDA GÜÇ ORANLARI VE OLASILIKLARINA BİR YAKLAŞIM VE BU NİCELİKLERİN YORUMU

ÖZET

Bu çalışmada, dikdörtgen kuantum çukuru içindeki elektrik alanının normalize propagasyon sabiti ve normalize frekansına ait güç ve olasılık ifadeleri üzerinde durulmuştur. Enerji seviyelerinde bulunan taşıyıcıların özelliklerinden olan hapsedilme etkileri araştırılmıştır. Normalize frekans ve normalize propagasyon sabiti, dikdörtgen kuantum çukurundaki elektron ve deliklerin enerji özdeğerlerinin normalize koordinat sistemindeki (ζ-η) ordinat ve absislerinin fonksiyonudur. Yapılan hesaplamalar ile daha kesin sonuçlar elde edilmiş olup bu anlamda daha hassas karşılaştırmalı örnekler sunulmuştur.

Anahtar Kelimeler : Normalize propagasyon sabiti, Kuantum çukurlu lazerler

1. INTRODUCTION

With the rapid growth presently taking place in the aspect of optoelectronics there is considerable interest in finding suitable structure for use in the semiconductor laser devices. Lasers and similar devices have been increasingly using the quantum effect. Especially, discrete behavior of subatomic particles such as electrons and holes has been confined to ultra minute realms in the rectangular quantum wells (RQWs) in fewer than three dimensions.

In recent years the technologies that make a confinement possible by building nanostructures out of individual atomic layers or molecules have been advancing at a remarkable step. By controlling precisely the structure and the composition of layers of material with tens of atoms or even just a few atoms thick, it is provided that we can program the electronic characteristics that we want to put into a compound material.

By making use of clever material design, electrons can be induced to jump from one energy level to another in one organized way, causing them to perform useful trick in some optoelectronic devices such as rectangular quantum well lasers (RQWLs). One of the keys to understand the basic principles that govern the operations of the RQWLs requires a basic comprehension of simple quantum well problem. The working principles of the RQWLs are based on the confinements of the carriers in the single rectangular quantum wells (RQWs). The RQWLs is widely used to read the information stored on the compact disk.

2. PROPERTIES OF MATERIAL USED

The single rectangular quantum well (RQW) is just one of the three basic regions of the quantum devices, as shown in Figure 1 These are made by growing gallium arsenide (GaAs) films less than, for example, 10 nm thick, between barrier layers of aluminum gallium arsenide (AlGaAs) obtained from usually using molecular epitaxy.

When the semiconductors materials GaAs and the AlGaAs with different band gaps are grown on the top of one another to form sub energy levels in the conduction and valance bands in the RQW, a discontinuities at the band edges are instituted. These man-made structures have been revealing for the salient features of quantum confinement. Reducing the number of dimensions forces the carriers to behave in a more atomlike manner (Kapon, 1999).

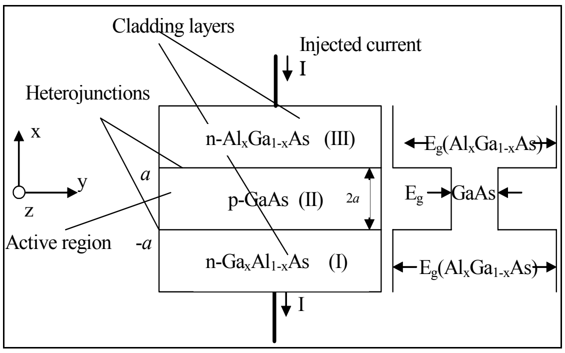

Figure 1. Basic regions of the RQW and the energy-band structure

The only significant difference between an ordinary semiconductor laser and the rectangular quantum well laser (RQWL) is the relative size of each device's active region (AR), where electrons and holes recombine, neutralizing one another and causing a photon to be emit. In a conventional semiconductor laser, when the physical width $2a$ of the AR is comparable to the characteristic lengths such as the Broglie wavelength, the Bhor radius or the mean free path of the electron, a quantum confined size effect (QCSE) occurs which are exhibited as the quantization of carrier's energy in one of its three degrees of freedom, i.e. the energy is restricted to a certain allowed (quantized) values. Electrons are not really confined by physical barrier; instead, researchers must erect the barrier of energy. This physical event introduces new electronic properties, forming new electron and hole energy levels in the GaAs layer which is called AR. Namely, electrons confined within such a layer of thin material permit quantized energy levels between conduction and valance bands, as shown in Figure 2.

For a carrier in the AR with the thickness $2a$ of the RQW having no barrier potential, the energy eigenvalues (EEVs) E_n (Schiff, 1982) at n th energy level are given by

$$E_n = n^2\pi^2\hbar^2/8m^*a^2, \quad n = 1, 2, 3, ... \tag{1}$$

for the optical even electric fields (OEEFs), corresponding to n=1, 3, 5…, and for the optical odd electric fields (OOEFs) corresponding to n=0, 2, 4, …,. In the equation $E_n=n^2E_1$, n=1, 2, 3, …, E_1 is the ground state energy. If there is a confined state with the barrier potential V_o in the RQW with finitely high barriers, the EEVs E_v becomes

$$e_\nu = V_o - n^2\pi^2\hbar^2/8m^*a^2 = V_o - n^2E_1 = \nu^2 e_1 \qquad \nu = n = 0,1,2,3,... \tag{2}$$

Where $\hbar$ is normalized Planck constant as $\hbar = h/2\pi$ and m* effective mass for a carrier. (Schiff, 1982; Verdeyen, 1989).

The barrier potential V_o in equation (2) is determined by the construction of the semiconductor material used (Chow, 1999). In Figure 2, three levels have predicted for electrons and 5 for heavy holes, arising the difference from the large discrepancy between the effective masses of these carriers (Syms and Cozens, 1992).

Motion of the carriers in the single RQWs is quantized in the energy levels perpendicular to the well as shown in Figure 2. The band gap of the structure of the AR is determined primarily by the well length of *a* and the composition of the barrier and well layers, as they determine the energy of the confined states and therefore the transition energies in the structure. However, the band gap structure may be modified by changing the confinement profile of the structures of the AR since its form determines the energy of allowed transitions in the structures of the RQWs. Referring to equation (2), the number of allowed quantised bound states is given by $\nu = a\sqrt{8m^*(V_o - E_n)}/\hbar\pi$. This equation shows that the existence of quantised energy levels depends on the barrier potential V_o (energy depth of the single RQW), the length *a* of the AR as well as the effective mass m* of the carrier in the single RQW and the EEVs E_n with mode number n. As the width 2*a* and energy depth V_o of the single RQW are reduced, the number of quantised states is reduced to the minimum value one. Thus, the photon energy depends on the well thickness and increases as the thickness decreases.

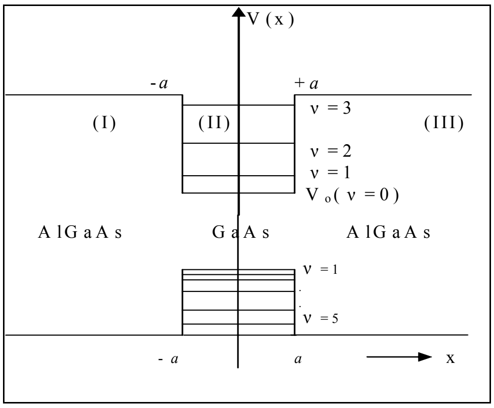

Figure 2. Band diagram for a AlGaAs /GaAs/AlGaAs RQW

3. PRELIMINARIES

Electric field which is generally used in the electrical engineering is a special potential per unit length. Therefore, we shall use the electric field in this work, instead of the potential. The particle is allowed to exist in a certain confined (bound) states which can be described by a wave function such as electric field.

The electric field and its total energy of an electron can be obtained by solving the Schrödinger equation

$$\left[-\nabla^2 + \frac{\hbar^2}{2m^*}V_o\right]E(x,y,z) = \frac{2m^*E_n}{\hbar^2}E(x,y,z), \qquad (3)$$

n=1, 2, 3, ...

Where E(x,y,z) is the electric field describing an electron or a hole in the conduction or valance band. The EEV E_n of the electron/hole in three-dimensional (Schiff, 1982) can be given by

$$E_n(n,k_y,k_z) = \hbar^2/2m^*[\pi^2 n^2/4a^2 + k_y{}^2 + k_z{}^2] \quad (4)$$

Where k_y and k_z are wave numbers in the directions y and z, respectively. The evanescent the electric fields (Buck, 1994) in the cladding layers (CLs) can be respectively expressed as

$$E_{yI,III} = A_{I,III}\exp\left[\pm\alpha_{I,III}(x \pm a)\right] \qquad (5)$$

for the symmetric rectangular quantum wells (SRQWs), where the minus sign on the exponential expression corresponds to the region III, while positive sign shows the region I. $A_{eI,III} = A\cos\zeta$ and $A_{oI,III} = A\sin\zeta$ are taken for the OEEFs and OOEFs respectively. The field in the AR is given by

$$E_{y_{II}} = A\cos(\alpha_{II}x - \theta) = A\cos(\frac{n\pi x}{2a} - \theta) \qquad (6)$$

where if we take as θ=0 and n=1, 3, 5,... it is obtained the OEEFs (Verdeyen, 1989) as

$$E^e{}_{y_{II}} = A\cos\frac{n\pi x}{2a}, n=1, 3, 5,... \qquad (7)$$

and also we take as θ=90° and n=0, 2, 4,... we obtain the OOEFs as

$$E^o{}_{y_{II}} = A\sin\frac{n\pi x}{2a}, n = 0, 2, 4, ... \qquad (8)$$

$|E_{yII}(x)|^2$ is interpreted as the probability of finding the carriers at the position x in the RQW. By denoting complex conjugate with (*), constant A in equation (6)-(8) is determined by

$$\int_{-a}^{a} E_{yII}E_{yII}{}^*\,dx = \int_{-a}^{a}\left|E_{yII}\right|^2 dx = 1, \qquad (9)$$

Which defines the total probability for all possible events is unity in the AR.

Electron need not to be part of an atom to exhibit the quantum energy effect. It is necessary only for the electron to be confined to region whose dimensions are measured anywhere from a few to a few hundred atoms. These properties for sizes approximate the hypothetical, indistinct cloud consisting of myriad points, each of which represents the probability of the electron occupying that position.

The propagation constants (PCs) α_I, α_{II} and α_{III}, which are the wave numbers in special case for the OEEFs and OOEFs of the carriers in the single asymmetric quantum well (ARQW) (Buck, 1994), are defined (Verdeyen, 1989; Temiz, 2002) as

$$\alpha_I^2 = \beta_z^2 - (\frac{\omega n_I}{c})^2 = \beta_z^2 - k_I^2, k_I = \frac{\omega n_I}{c} = k_o n_I, \quad (10)$$

$$\alpha_{II}^2 = (\frac{\omega n_{II}}{c})^2 - \beta_z^2 = k_{II}^2 - \beta_z^2$$
$$k_{II} = \frac{\omega n_{II}}{c} = k_o n_{II} \quad (11)$$

$$\alpha_{III}^2 = \beta_z^2 - (\frac{\omega n_{III}}{c})^2 = \beta_z^2 - k_{III}^2,$$
$$k_{III} = \frac{\omega n_{III}}{c} = k_o n_{III} \quad (12)$$

$$k = \frac{\omega n}{c}, k_o = |k_o| = \omega / c = 2\pi / \lambda . \quad (13)$$

The letter A in equation (6)-(8) is a constant in terms of propagation constant (PC) α_{II} and the length a of the AR. The symbols n_I, n_{II} and n_{III} shown in Fig.1 are indices of the regions. This usual relationship between the indices in the three regions is $n_{II} \rangle n_I \rangle n_{III}$ which gives inequalities $k_I<\beta_z<k_{II}$ and $k_{III}<\beta_z<k_{II}$ between the waves numbers and β_z. In the SRQW, there are the relations $\alpha_I=\alpha_{II}=\alpha_{I,III}$ and so $n_I=n_{III}=n_{I,III}$. β_z is phase constant and k is the wave number. Where c is the velocity of the light in a vacuum. λ is wavelength and k_o is free space wave number. Every frequency defines a free space wave number according to the formula at the right hand of equation (13) (Temiz, 2002).

The optical evanescent field $E_{yI,III}$ relevant the OEEFs and OOEFs in the SRQWs is respectively extending significantly into the cladding wide-band gap semiconductor layers surrounding the AR. $E_{yI,III}$ stands for $E_{yI,}$ or E_{yIII}.

The PC α_{II} in equation (11) which is also called spatial frequency is given (Temiz, 2001, 2002) by

$$\alpha_{II} = 1/\hbar\sqrt{2m^*[V_o - E_n]} =$$
$$\sqrt{\left[n_{II}^2 k_o^2 - \beta_z^2\right]}, n = 0,1,2,3,... \quad (14)$$

in the another form. Referring to equation (10) and (12) for the single SRQWs, we get $\alpha_{I,III}$ as

$$\alpha_{I,III} = \sqrt{2m^*E_n}/\hbar = \sqrt{\beta_z^2 - (n_{I,III}k_o)^2}, n=0,2,3,... \quad (15)$$

in the CLs for the single SRQWs. By defining parameters $\zeta=\alpha_{II}a$ and $\eta=\alpha_{I,III}a$, which form parametric coordinate variables of the EEVs for carriers, as it will be explained in the below, the variables are obtained (Temiz, 2001; 2002) as

$$\zeta = \frac{a}{\hbar}\sqrt{2m^*[V_o - E_n]} \quad (16)$$

$$\eta = \frac{a}{\hbar}\sqrt{2m^*E_n}, n=0,1,2,3,... \quad (17)$$

As it will be expressed in the future, the normalized propagation constant and the depth of the single RQW strongly depends on the parametric coordinates ζ and η of the EEVs for carriers.

The relation between the PCs α_I, α_{II} and α_{III} in the RQW are generally (Temiz, 2001, 2002) given by

$$\alpha_I / \alpha_{II} = \tan(\alpha_{II}a + \theta), \alpha_{III} / \alpha_{II} = \tan(\alpha_{II}a - \theta) \quad (17)$$

Where θ can be $m\pi/2$, m=0, 2, 4,..., for the OEEFs and m=1, 3, 5,..., for the OOEFs (Buck, 1994).

The normalized frequency (NF) V is given by

$$V = \sqrt{\zeta^2+\eta^2} = 2\pi(a/\lambda)NA = ak_0 NA = \frac{a}{\hbar}\sqrt{2m^*V_o} \quad (18)$$

Which yields a circle with radius $a/\hbar\sqrt{2m^*V_o}$ (Buck, 1994). $NA = \sqrt{n_{II}^2 - n_{I,III}^2} \cong n_{II}\sqrt{2\Delta}$ is called numerical aperture of the SRQW. Here Δ is given by as $\Delta = (n_{II}^2 - n_{I,III}^2)/2n_{II}^2 \cong (n_{II} - n_{I,III})/n_{II}$. The intersections of the tangent equation in (17) with the circles in equation (18) yield the points of the EEVs for carriers in the SRQW as shown in ref. (Temiz, 2001, 2002).

For the single SRQWs, normalized propagation constant (NPC) α is defined as

$$\alpha = \eta^2 / V^2 \quad (19)$$

Which is a real quantity and must be $\alpha<1$ (Bhattacharya, 1998). Just as, one can write,

$$1-\alpha=1-\eta^2/V^2=\zeta^2/V^2\rightarrow\zeta=V\sqrt{1-\alpha} \quad (20)$$

That allows us to determine the abscissas ζ of the EEVs for the given NF V and NPC α. Note that equation (17), (18) and (19) yield

$$\alpha=E_n/V_o, n=1, 2, 3, ... \quad (21)$$

Equation (21) permits us to calculate the NPC α (barrier potential V_o) for given V_o (α) and E_n.

The NPC α can be calculated as follows: $\alpha=(1.1428Ve-0.9960^2)/V^2$, which has approximately a linear feature in the range of $1.5\langle V\langle 2.5$ (Rudolf, 1976).

4. POWER QUANTITIES AND CONFINEMENT FACTOR

The ratio of power, which is the ratio of total evanescent power P_ℓ, (P_I+P_{III}) in the region I and III to the active field power (P_{II}) in the AR, denoting the complex conjugate with (*), can generally be expressed as

$$\frac{P_{I+III}}{P_{II}}=\frac{P_\ell}{P_{II}}=R$$

$$P_\ell=\int_{-\infty}^{-a}\left[E_{xI}(x)H_{yI}(x)^*-E_{yI}(x)H_{xI}(x)^*\right]dx+ \quad (22)$$

$$\int_a^\infty[\left[E_{xIII}(x)H_{yIII}(x)^*-E_{yIII}(x)H_{xIII}(x)^*\right]dx]$$

$$P_{II}=\int_{-a}^{a}\left[E_{xII}(x)H_{yII}(x)^*-E_{yII}(x)H_{xII}(x)^*\right]dx$$

or by taking $E_x=0$ for only E_y component

$$\frac{P_\ell}{P_{II}}=R=\frac{\int_{-\infty}^{-a}\left[E_{yI}(x)H_{xI}(x)^*\right]dx+\int_a^\infty\left[E_{yIII}(x)H_{xIII}(x)^*\right]dx}{\int_{-a}^{a}\left[E_{yII}(x)H_{xII}(x)^*\right]dx} \quad (23)$$

Taking the impedances of the fields $Z=-E_y/H_x\rightarrow H_x=-E_y/Z$ into account in the optical medium (Pozar, 1998), the power ratio R becomes

$$R=\frac{\int_{-\infty}^{-a}\left[E_{yI}(x)E_{yI}(x)^*\right]dx+\int_{-a}^{-\infty}\left[E_{yIII}(x)E_{yIII}(x)^*\right]dx}{\int_{-a}^{-a}\left[E_{yII}(x)E_{yII}(x)^*\right]dx} \quad (24)$$

$$=\frac{\int_{-\infty}^{-a}\left|E_{yI}(x)\right|^2dx+\int_{-a}^{-\infty}\left|E_{yIII}(x)\right|^2dx}{\int_{-a}^{-a}\left|E_{yII}(x)\right|^2dx}$$

Only in terms of electric field. The parameter R for the OEEFs (Temiz, 2002) and the parameter r for the OOEFs can be respectively calculated by

$$\frac{P_l}{P_{II}}=R=\frac{1-\alpha}{\eta+\alpha},\frac{P_l'}{P_{II}}=r=\frac{\alpha}{\eta-\alpha} \quad (25)$$

The parameter K (Temiz, 2002) for the OEEFs and the parameter q for the OOEFs, that is, for the ratio of the loss powers, P_ℓ, (P_ℓ') to the input powers, P_i, (P_i') for the fields are

$$\frac{P_\ell}{P_i}=K=\frac{2\int_a^\infty E_{yIII}(x)E_{yIII}(x)^*dx}{2\int_0^a E_{yII}(x)E_{yII}(x)^*dx+2\int_a^\infty E_{yIII}(x)E_{yIII}(x)^*dx} \quad (26)$$

$$=\frac{1-\alpha}{\eta+1}=\frac{1}{1+1/R},\frac{P'_\ell}{P'_i}=q=\frac{\alpha}{\eta}=\frac{1}{1+1/r}$$

The confinement factors (CFs) Γ_{II} and Λ_{II} (Temiz, 2002) for the OEEFs and OOEFs respectively become:

$$\frac{P_{II}}{P_i}=\frac{P_{II}}{P_{II}+P_\ell}=\Gamma_{II}=\frac{\alpha+\eta}{1+\eta}$$

$$\frac{P'_{II}}{P'_i}=\frac{P'_{II}}{P'_{II}+P'_\ell}=\Lambda_{II}=\frac{\eta-\alpha}{\eta}=\frac{1}{1+r}=1-q=\frac{q}{r} \quad (27)$$

and therefore one can writes $K+\Gamma_{II}=1$, $q+\Lambda_{II}=1$.

P_i, P_{II} and P_ℓ (P'_i, P'_{II} or P'_ℓ,) powers mentioned above are respectively optical input power, active region power and the loss power (the total power of the CLs) (Temiz, 2002) for the OEEFs (OOEFs) in the SRQWs, as shown in Figure 4. The apostrophe denotes the parameters about the OOEF (Temiz, 2002).

The parameters r, q and Λ_{II} for the OOEFs are respectively obtained in terms of the parameter R, K and Γ_{II} for the OEEFs as

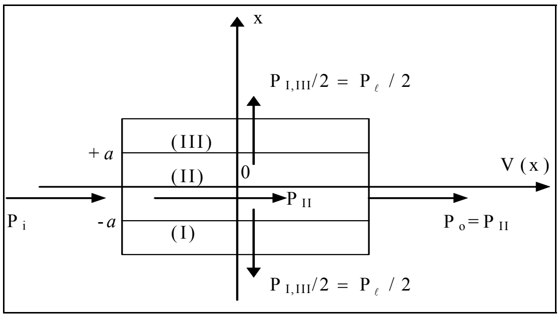

Figure 4. The different powers in the AR and the CLs in the SRQWs

$$r = (1-\eta R)/(\eta - 1),\ q = \frac{1 - K(\eta+1)}{\eta},\ \Lambda_{II} = \frac{(1+\eta)\Gamma_{II}}{\eta} \quad (28)$$

Equation (28) are the transformation relations between some parameters for the OEEFs and the OOEFs. The values n_I=$n_{I,III}$=3.350, n_{II}=3.351, λ=1.55 μm and a=5 A^o give the NF V, the NPC α, the coordinates of the EEVs for the carriers ζ, η, the power ratios R, K and confinement factor Γ_{II}, the input probability I^e_i, the loss probability I^e_ℓ and the active region probability I^e_{II} as V=1.6592x10^{-004} and α=0.0161, ζ=1.6458x10^{-004}, η=2.1058x10^{-005}, R=61.0012, K=0.9839, Γ_{II}=0.0161, I^e_i=62.0012, I^e_ℓ=61.0012 and I^e_{II} =1 for the OEEFs, respectively. In this example, there is only one solution corresponding to the OEEFs for these given values; because there is not a solution for the OOEFs since V<1.57 (Iga, 1994). The transformations between some quantities such as K=1/[1+(1/R)], Γ_{II}=K/R=1-K are current (Temiz, 2002).

5. POWERS AND PROBABILITIES IN THE REGIONS OF THE RQWs

The efficiency of the power of the RQWs, the ratio of the evanescent power (P_{loss}= P_ℓ) in the CLs to the total field power (P_{input}=P_i-input power) in the AR is $K = P_\ell / P_i$. Here, by supposing that the power of the AR (region II) is normalized as unity due to Eq. (9), we obtain the following expressions (Temiz, 2002) for the SRQWs (α_I=α_{III}=$\alpha_{I,III}$). By reminding the modulus t_j=$|E_{yj}|$ of the even field E_{yj}, j=I, II, III, the integrating $I_j = \int t_j^2 dx$ represents physically the probability of finding an electron or a hole at a point x in the j th region of the RQW. We denote the input power, loss power and the active region power, respectively, with $P_i = I_i / Z$, $P_\ell = I_\ell / Z$ and $P_{II} = I_{II} / Z$, for example for the OEEFs, by reminding the obtaining from general power expression $P = \frac{1}{Z}\int_0^a t^2 dx = I / Z$, the powers and probabilities for the regions in the single SRQWs are

$$P_i = \frac{2}{Z}\int_a^\infty (t_{II}^2 + t_{I,III}^2)dx = \frac{2}{Z}(I_{II} + I_{I,III}) \quad (29)$$

and

$$I_i = I_{II} + I_\ell = 1 + I_\ell \quad (30)$$

According to Figure 1. By using L=ζ^2/V^2 which will be interpreted in the future, the powers and the probabilities for the OEEFs in the AR (region II) are

$$P^e_{II} = \frac{2}{Z}\int_0^a \left|E^e_{yII}(x)\right|^2 dx = \frac{A_e \cos^2\zeta}{Z\alpha_{I,III}}\left(\frac{\eta+\alpha}{L}\right) = \frac{A^2_{eI,III}}{Z\alpha_{I,III}}\left(\frac{\eta+\alpha}{L}\right) \quad (31)$$

$$I^e_{II} = 2\int_0^a \left|E^e_{yII}(x)\right|^2 dx = \frac{A_e \cos^2\zeta}{\alpha_{I,III}}\left(\frac{\eta+\alpha}{L}\right) = \frac{A^2_{eI,III}}{\alpha_{I,III}}\left(\frac{\eta+\alpha}{L}\right) \quad (32)$$

and for the OOEFs in equation (8)

$$P^o_{II} = \frac{2}{Z}\int_0^a \left|E^o_{yII}(x)\right|^2 dx = \frac{A^2_o \sin^2\zeta}{Z\alpha_{I,III}}\left(\frac{\eta-\alpha}{\alpha}\right) = \frac{A^2_{oI,III}}{Z\alpha_{I,III}}\left(\frac{\eta-\alpha}{\alpha}\right), \quad (33)$$

$$I^o_{II} = 2\int_0^a \left|E^o_{yII}(x)\right|^2 dx = \frac{A^2_o \sin^2\zeta}{\alpha_{I,III}}\left(\frac{\eta-\alpha}{\alpha}\right) = \frac{A^2_{oI,III}}{\alpha_{I,III}}\left(\frac{\eta-\alpha}{\alpha}\right) \quad (34)$$

Equation (32) and (34) are equal to 1 due to equation (9).

In general, the loss power and the probability in the ARQW become the sum of the powers P_I, P_{III} and the probabilities I_I, I_{III} in the regions I and III, respectively:

$$P_\ell = P_I + P_{III}. \tag{35}$$

$$I_\ell = I_I + I_{III}. \tag{36}$$

We obtain the powers and the probabilities as

$$P^e{}_\ell = \frac{2}{Z}\int_a^\infty \left|E^e{}_{yI,III}(x)\right|^2 dx = I^e{}_\ell / Z = \frac{1}{Z}\frac{L}{\eta+\alpha} \tag{37}$$

$$I^e{}_\ell = 2\int_a^\infty \left|E^e{}_{yI,III}(x)\right|^2 dx = \frac{A^2{}_{eI,III}}{\alpha_{I,III}} = \frac{L}{\eta+\alpha} \tag{38}$$

for the OEEFs and

$$P^o{}_\ell = \frac{2}{Z}\int_a^\infty \left|E^o{}_{yI,III}(x)\right|^2 dx = I^o{}_\ell / Z = \frac{1}{Z}\frac{\alpha}{\eta-\alpha} \tag{39}$$

$$I^o{}_\ell = 2\int_a^\infty \left|E^o{}_{yI,III}(x)\right|^2 dx = \frac{A^2{}_{oI,III}}{\alpha_{I,III}} = \frac{\alpha}{\eta-\alpha} \tag{40}$$

for the OOEFs and Therefore, the equations between equation (32) and (34) give

$$I^e{}_{II} = I^e{}_\ell\left(\frac{\eta+\alpha}{L}\right) = 1,\ I^o{}_{II} = I^o{}_\ell\left(\frac{\eta-\alpha}{\alpha}\right) = 1 \tag{41}$$

Which denote that for the each of OEEFs and the OOEFs the probabilities of the finding carriers in the regions II (in the AR) becomes % 100.

[cf.(36)and (46)] and equation (30) yields

$$\begin{aligned} I^e{}_i &= I^e{}_\ell + I^e{}_{II} = \frac{\alpha+\eta+L}{\alpha+\eta} = I_{II} + \frac{L}{\alpha+\eta} \\ P^e{}_i &= \frac{1}{Z}(I^e{}_\ell + I^e{}_{II}) = \frac{1}{Z}\frac{\alpha+\eta+L}{\alpha+\eta} = \frac{1}{Z}\left(I_{II} + \frac{L}{\alpha+\eta}\right) \end{aligned} \quad ,(42)$$

for the OEEFs and

$$\begin{aligned} I^o{}_i &= I^o{}_\ell + I^o{}_{II} = I^o{}_\ell\frac{\eta}{\alpha} = \frac{\eta}{\eta-\alpha}, \\ P^o{}_i &= \frac{1}{Z}(I^o{}_\ell + I^o{}_{II}) = \frac{1}{Z}\left(I^o{}_\ell\frac{\eta}{\alpha}\right) = \frac{1}{Z}\frac{\eta}{\eta-\alpha} \end{aligned} \quad , \tag{43}$$

for the OOEFs. The loss power and loss probability are given in ref. (Buck, 1994) as

$$\begin{aligned} P_l &= P_{l_{av}} \cong 2\gamma_{av} P_i \\ I_l &= I_{l_{av}} \cong 2\gamma_{av} I_i \end{aligned} \tag{44}$$

Where γ_{av} is approximate (average) loss coefficient. Equation (38)-(43) and (44) give, respectively, as

$$\begin{aligned} &\frac{I^e{}_{\ell av}}{I^e{}_{iav}} = \frac{P^e{}_{\ell av}}{P^e{}_{iav}} = K_{av} = 2\gamma^e{}_{av} \\ &= \frac{L}{\alpha++L+\eta} = \frac{1-\alpha}{1+\eta} = \frac{L}{1+\eta} \end{aligned} \tag{45}$$

for the OEEFs and

$$\frac{I^o{}_{\ell av}}{I^o{}_{iav}} = \frac{P^o{}_{\ell av}}{P^o{}_{iav}} = q_{av} = 2\gamma^o{}_{av} = \frac{\alpha}{\eta} \tag{46}$$

for the OOEFs. Therefore, the output power and probability (Buck, 1994) are given by

$$P_{oav} \cong P_{iav} e^{-2\gamma_{av}}, I_{oav} \cong I_{iav} e^{-2\gamma_{av}} \tag{47}$$

which give

$$\begin{aligned} P^e{}_{oav} &\cong P^e{}_{iav} e^{-2\gamma^e{}_{av}} \\ I^e{}_{oav} &\cong I^e{}_{iav} e^{-2\gamma^e{}_{av}} \end{aligned} \tag{48}$$

for the OEEFs and

$$\begin{aligned} P^o{}_{oav} &\cong P^o{}_{iav} e^{-2\gamma^o{}_{av}} \\ I^o{}_{oav} &\cong I^o{}_{iav} e^{-2\gamma^o{}_{av}} \end{aligned} \tag{49}$$

for the OOEFs, respectively.

For an extract solution, we can get the following applications:

$$I_\ell = I_i - I_o = I_i - I_i e^{-2\gamma}, I_o = I_i e^{-2\gamma}, \tag{50}$$

where I_O is the output probability (Buck, 1994). Therefore, the loss probability and loss power are obtained as

$$I_\ell = I_i(1 - e^{-2\gamma}), P_\ell = P_i(1 - e^{-2\gamma}) \tag{51}$$

which give

$$I^e_\ell = I^e_i(1 - e^{-2\gamma e}), P^e_\ell = P^e_i(1 - e^{-2\gamma e}) \tag{52}$$

for the OEEFs and

$$I^o_\ell = I^o_i(1 - e^{-2\gamma o}), P^o_\ell = P^o_i(1 - e^{-2\gamma o}) \tag{53}$$

for the OOEFs. Equations (52) and (53) yield the power ratios

$$\frac{I^e{}_\ell}{I^e{}_i} = \frac{P^e{}_\ell}{P^e{}_i} = K = 1 - e^{-2\gamma^e} \tag{54}$$

for the OEEFs and

$$\frac{I^o{}_\ell}{I^o{}_i} = \frac{P^o{}_\ell}{P^o{}_i} = q = 1 - e^{-2\gamma^o} \tag{55}$$

for the OOEFs. Equations (54) and (55) give respectively the loss factors as

$$e^{-2\gamma^e} = 1 - K == \frac{\alpha + \eta}{1 + \eta} \tag{56}$$

for the OEEFs and the loss factor

$$e^{-2\gamma^o} = 1 - q = \frac{\eta}{\eta - \alpha} \tag{57}$$

for the OOEFs. Therefore, percentages of sensibilities over the approximation cases of our formulations for the loss coefficients in terms of our parameters for the OEEFs and OOEFs can be respectively given by

$$\Delta\gamma^e = \frac{2\gamma^e - 2\gamma^e{}_{av}}{2\gamma^e} = \frac{\ln\frac{1+\eta}{\alpha+\eta} - K}{\ln\frac{1+\eta}{\alpha+\eta}} = \frac{\ln\frac{1}{1-K} - K}{\ln\frac{1}{1-K}} \tag{58}$$

$$\Delta\gamma^o = \frac{2\gamma^o - 2\gamma^o{}_{av}}{2\gamma^o} = \frac{\ln\frac{\eta}{\eta-\alpha} - q}{\ln\frac{\eta}{\eta-\alpha}} = \frac{\ln\frac{1}{1-q} - q}{\ln\frac{1}{1-q}} \tag{59}$$

For example, we have the error percentage of $\Delta I^e_\ell = (I^e_\ell - I_{\ell av})/I^e_\ell = -2.9508x10^{-005}$ for the values $n_I = n_{I,III} = 3.350$, $n_{II} = 3.351$, $\lambda = 1.55$ µm and $a = 5$ A^o given above example for the OEEFs, so that it is therefore reasonable to prefer our approach over the approximation procedure given in (Buck, 1994). Consequently, equation (50)-(59) are novel expressions about our approach.

6. THE DEPT OF ENERGY LEVELS IN THE RQWs

Using equation (16), (18) and equation (18), (20), (21) and (26) for the OEEFs and referring to the defining $K = P^e_\ell / P^e_i$ in equation (54), we obtain

$$K = (V_o - E_n) / \left[V_o(1 + \eta) \right] = e_\nu \tag{60}$$

e_ν represents the energy levels in the RQW as

$$e_\nu = \nu^2 \hbar^2 \pi^2 / 8m^* a^2 = \nu^2 e_1, \nu = 0, 1, 2, 3, ... \tag{61}$$

Where $e_1 = \hbar^2\pi^2 / 8m^* a^2$ which is the lowest energy level (Schiff, 1982) in existence of carrier confinement in the single RQWs. That is, e_1 is the first energy level in the bottom of the RQW and is equal to the ground energy in equation (1), namely $E_1 = e_1$. Here n denotes the mode number of the field, while ν determines the energy levels in the single RQW from the bottom of the well [cf.(2)]. Note that for $n = \nu = 0$ that equations (2) and (61) give $V_o = E_o = 0$, which points out that there is not induced field in most bottom of the RQW. Referring to equations (60) and (61), we have

$$K = K_n = [\nu^2\hbar^2\pi^2/8m^*a^2]/[V_o(\eta+1)] \\ = \frac{\nu^2 e_1}{V_o(\eta+1)} \quad (62)$$

from which we get the quantum energy level number ν as

$$\nu = \frac{a}{\hbar\pi}\sqrt{KV_o(\eta+1)8m^*}\,. \quad (63)$$

Equation (63) imply that the energy levels in the RQW depend on the parameter K, the barrier potential V_o apart from the ground potential e_1 and ordinates of the EEVs for the carriers. The values n_I=$n_{I,III}$=3.350, n_{II}=3.351, λ=1.55 μm and a=5 A^o in the above example give e_1=21.9241 eV, K=0.9839, V_o=22.2830 eV and $\eta=2.1058x10^{-005}$ for the OEEFs with the single mode and so the quantum energy level number in the RQW is obtained as 1. This is also estimated as $V_o/e_1 \cong 1$ for n=0 and $\nu=1$ in equation (2) approximately. If we define

$$e_\nu = V_o - E_n = L\left|V_o\right| = \nu^2 e_1 \quad (64)$$

where L, $(0\langle L\langle 1)$, represents percent of V_o from bottom edge of the well, equation (60) gives,

$$K = L/\left[1+V\sqrt{1-L}\right] \quad (65)$$

where $L=\zeta^2/V^2$ and by referring to equation (20)

$$L = 1-\alpha\,. \quad (66)$$

The quantity L can be regarded as a quantum well depth parameter in the RQWs, as shown in Figure 5. Equation (64) yields $L=\nu^2 e_1/V_o$, ν=1, 2, 3, …n or from equation (66) we get

$$\alpha = 1-\nu^2 e_1/V_o,\ \nu = 0,1,2,3,\dots\ , \quad (67)$$

Equation (2) yields

$$\alpha=\frac{e_\nu}{V_o}=1-n^2\frac{\hbar^2\pi^2}{8V_o m^* a^2}=1-\frac{n^2E_1}{V_o},n,\nu=0,1,2,\dots \quad (68)$$

or from equations (67) and (68) we have

$$\alpha=1-\frac{\nu^2 e_1}{Vo}=1-\frac{n^2E_1}{Vo},n,\nu=0,1,2,3\dots \quad (69)$$

With the remembering E_1= e_1 for n= ν =1, equation (69) shows that the NPC α does not change in the single mode (n=1) However, in general equation (69) implies that the NPC α depends on the mode number in the condition given the relation $\alpha=E_n/V_o=1-\nu^2E_1/V_o$. Referring to equation (21), equations (67) and (69) are equal to each other in the RQW for n=ν=1, 2, 3, ... with remembering E_o=V_o=0 for n=ν =0, so that e_ν=0 in equations (64) and (66) for E_o=V_o=0 gives that L=0 (See Figure 5). Equations (60)-(69) are also true for the OOEFs. The depth parameter L in equation (66) is a novel result.

If the bottom edge of the single RQW is selected as starting of the dept L of the RQW, as shown in Fig.5, L=0 in equation (66) gives α=1 due to ζ=0 in equations (18) and (19) or according to equation (66) and therefore we obtain the confinement factor Γ_{II} as Γ_{II}=1 (Temiz, 2002), while L=1 yields α as $\alpha = 0$ and so indirectly K=1. In this case we obtain $V_o = n^2\hbar^2\pi^2/8m^*a^2 = n^2E_1$ that gives e_ν=0 in equation (2) which means that quantum confined states are not instituted.

Consequently, the power ratios and the confinement factors are function of the parameter α or L of the RQW. If there is no quantum confined states, then L is unity which means that the quantum well does not institute and therefore the power P_{II} of the AR can not be exist. If L is zero then there is no induced field at the deepest of the well. Therefore, the existence of effectiveness of the AR corresponds to the values between 0 and 1 of the depth parameter L or the NPC α.

7. RESULTS

In this paper we have studied the powers and pobabilities for the regions of the single RQWs to interprete some field parameters affecting confined states in the wells.

The NPC α and the depth parameter L are respectively functions of the NF and the ordinate and the abscissa of the EEV. The depth parameter L is a novel result. That is, the depth of the RQWs depends on the abscissas of the EEVs as shown in equation (66) and the barrier potential V_O influences the NPC α with the respect to equation (21). The indice ν determines the energy levels towards up from the bottom edge of the single RQW. Here, there are some special manners: The parameter K in equation (65) is zero when the depth L of the well becomes 0 (or the NPC α also becomes 1 according to equation (66) for ν=0 at the bottom edge of the well (Temiz, 2002). L=0 (ν=0) gives e_o=0 and V_o=E_n=0 in equation (64), which implies that there is not induced field at the bottom edge of the well as shown in Figure 5.

For the electric field which has the mode n=1, 2, 3, ..., the number of energy levels in the single RQW is described by the ν=n-1, (ν=0, 1, 2,…,) in the EEVs $e_\nu=E_{n-1}$ (Schiff, 1982). That is, in this case, the induced electric field with the mode ν within the single RQW has the number of energy levels ν=n-1. n=0 then (n-1) corresponds to E_{-1}, and ν=-1 respectively, which has no physical significance Even and Odd values of the n are for the OOEFs and OEEFs, respectively. In the case of E^o_{yII}=A=0=*constant* which corresponds to n=0, there is not an induced field and therefore EEV E_n=E_o=0 for n=0 and e_o=0 for ν=0. Consequently, we take the mode indice n as n=1, 2, 3, ..., for the electric field and the energy levels ν in the RQW as ν=0, 1, 2,…, for induced field. The OEEF with the single mode for n=1 has only the EEV E_1 but induced field has EEV e_o=0 in the well. (2) L=1 gives K=1 in equation (65), which means that K is maximum as a unity and the confinement factor in equation (27) is obtained as Γ_{II}=0 and the confinement can not be obtained because of no bound state at the topest edge of the single RQW, as shown in Figure 5. This manner means that there is not an AR, which corresponds to α=0 with respect to equation (66). The larger percentages of the L from the bottom edge of the well is, the larger loss power increases according to equation (37). On the contrary, the losses in the energy levels at the points near to the bottom edge of the single RQW become small. Therefore, for small losses, small quantum energy levels must select at the near to the bottom edge of the single RQW such as ν=1 except for ν=0.

The barrier potential V_o is selected by material process to get the carrier concentration at the certain energy level. When the barrier potential V_o is smaller, then K or the loss P_ℓ is larger Therefore, to reduce the loss, the barrier potential V_o must be increased. Equation (62) estimates the needed barrier potential V_o for given K and the ordinate η of the EEV. Equation (62) yields$\nu^2=K_n/K_1$which implies for the multiple energy levels that the energy increases as proportional to the ν^2. Therefore, for a quantum energy number with ν=1, since carriers are distributed over a smaller range of the energy states, the single RQWs require less pumping to get the population inversion and give greater efficiency. Thus, the injected carriers are less sensitive to the thermal spreading effects. This reduces the temperature dependence of the threshold current density.

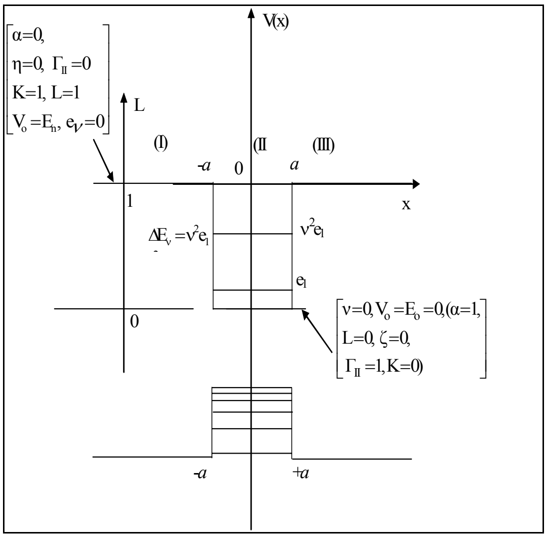

Figure 5. Energy levels and the quantity L within the RQW

The parameters of R, K and Γ_{II} depend on the NPC α. K or R is inversely proportional to the parameter η. To get a good confinement, the NPC α and the NF V should be large according to $\eta = V\sqrt{\alpha}$. But in a single mode, the NF V should be V<π/2 (Iga, 1994). The largest value of η is obtained when ζ=0 and therefore V=η in according to equation (21). For this condition (ζ=0), we obtain α=1 (L=0 in equation (66) in the equation (19) and Γ_{II}=1 in equation (27) and K=0. These results mean that the confinement factor Γ_{II} becomes % 100 on the axis-η. These parameter values correspond to the extreme edge of the bottom of the single RQW.

Another way, the confinement factor Γ_{II} in equation (27) is equal to zero when η=0 gives V=ζ in the equation (18) and for the OOEFs, $\Lambda_{II}=\infty$ in equation (30) These values of the parameters show that there is not confinement on the axis-ζ. This case corresponds to the topest of the single RQW. These extreme values found for η=0 expresses that they are not in agreement for a stable working. Consequently, ζ=0 and η=0 correspond to extreme values of α=1 and α=0, respectively. For the OEEFs and OOEFs; when the width 2*a* of AR is smaller, then, the confinement factors become smaller Therefore, to get a large NPC α according to equation (19) and also to get an ordinate $\eta=\alpha_{I,III}a$ for a certain length *a,* the propagation constant $\alpha_{I,III}$ must be large. The drift velocity v of the carrier according to the

$\alpha_{I,III} = \sqrt{2m^*E_n}/\hbar = p^2/2m^* = v^2m^*/2$ in equation (17) must be large enough. and so it is leaded to select the large drift velocity, which is roughly $v=c/n_{I,III;}$ namely, the velocity v is limited by indices of the CLs. To increase velocity, the indice $n_{I,III}$ must be small. Therefore, propagation constant $\alpha_{I,III}$ or $\eta=\alpha_{I,III}a$ for the CLs can be obtained in a certain value depending on the index $n_{I,III}$. But, if the free space is selected for one of the CLs, then the index $n_{I,III}$ can be taken as 1. This case leads to the using of the surface wave, indicating a wave guided by the dielectric interface (Buck, 1994).

With respect to our formulation, $2\gamma^e = \ln(I^e_i/(I^e_i - I^e_\ell))$ gives 4.12715374 which yields the output probability $I^e_o = I^e_i e^{-2\gamma^e}$ =62.0012xexp(-4.12715374)=1=I^e_{II} for the given example above for the OEEFs. So, we obtain the output probability $I^e_{II} = I^e_o = I^e_i - I^e_\ell = I^e_i e^{-2\gamma^e}$ and the loss probability $I^e_\ell = I^e_i - I^e_o = 62.0012\text{-}1\text{=}61.0012$ and $I^e_{II} = I^e_i - I^e_\ell$ =62.0012-61.0012=1, which is a result of the normalization in the AR. But in the approximation, the loss power coefficient $2\gamma^e_{av}$ belonging to the OEEFs becomes 0.9839 according to $2\gamma^e_{av} = I^e_{\ell av}/I^e_{iav}$ in equation (45), which gives $I_{\ell av}$ as 61.0030. Therefore, the error is obtained as $\Delta I^e_\ell = I^e_{\ell av} - I_{\ell av}$=-0.0018. That is, this approximation given in ref. (Buck, 1994) is erroneous with percent of $-2.9508\text{x}10^{-005}$ according to the our approach. These treatments imply that our work in this paper presents more accurate calculations. Equations (50)-(69) are novel expressions in this paper. In addition, here some quantities such as loss factor, loss coefficient and percentage error have expressed in terms of the parameters K for the OEEFs and q for the OOEFs for the first time.

Consequently, for a convenient confinement of carriers the NPC α and the dept L must satisfy the inequalities 0<α<1 or 1>L>0. Therefore, the important parameters defined above such as R, r, K, q, Γ_{II} and Λ_{II} for the single RQWs can contain stable results for only these inequalities 0<α<1 or 1>L>0.

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