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UKF-SVM Based Generalized Predictive Control of Multicompartment Lung Mechanics Model ⋆

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Abstract: In this paper, least-squares support vector machine (LS-SVM), whose parameters are updated by unscented Kalman filter (UKF), is adopted in the generalized predictive control (GPC) of a system with general multicompartment lung mechanics. Gaussian kernel function is employed since it presents a good approximation to the inner product of nonlinear mapping possessed in the SVM formulation. In the SVM literature, it is well known that the width parameter σ of the Gaussian kernel function has an important effect on the performance. However, it is not possible to train that parameter together with the other parameters of SVM when using linear least squares. This is why we use UKF for parameter adaptation in the SVM formulation. At each time instant of the control task, all parameters of the LS-SVM model, including σ, are tuned simultaneously. Another reason to employ UKF is; it avoids the suboptimal solutions caused by linearization based filters, e.g., extended Kalman filter. Due to these facts, we train the SVM model using UKF and it will be referred to as the UKF-SVM model. Simulation results concerning the application of UKF-SVM based GPC to a multicompartment lung mechanics model yields plausible performance using small amount of support vectors even when there are time-varying lung parameters and disturbance of high level affecting the system. The adopted approach can also be useful when there is not any knowledge of the system dynamics, i.e., black box. Note that, multicompartment lung mechanics system is a stand-in model that can mimic the behavior of human lung. Thus, it is appropriate for hardware-in-the-loop simulation which opens a path to the real-patient-tests of mechanical respiratory systems in the future.

Keywords: Multicompartment lung mechanics, robust control, LS-SVM, UKF, generalized predictive control

1. INTRODUCTION

Generalized predictive control (GPC) is a subclass of model predictive control. In this control scheme, unkown dynamics of the underlying system should first be approximated in the regression framework. The regressor model is utilized to both predict future behavior of the system and to provide necessary gradient information to update the control input signal vector. At each time instant, first one of the produced control signals is applied to the system (Clarke et al., 1987).

The conventional support vector machine (SVM) performs quadratic programming (QP) to minimize a convex cost function with inequality constraints. (Suykens et al., 2002) proposed least squares SVM (LS-SVM) which is based on linear equations with equality constraints. This is an alternative to the conventional QP-based one and it has a simple algebraic expression. Moreover, the optimal solution can be obtained simply by least squares.

It is a critical issue to determine the optimum parameters of the kernel function used in SVM. Because, classification or regression performance is highly effected by the kernel parameter selection. Simultaneous adaptation of all model parameters (including the kernel parameter) is performed in the studies (Dilmen and Beyhan, 2017, 2018) which were published by the authors and included online classification and regression. In those studies, LS-SVM formulation is treated as a multi-input multi-output (MIMO) optimization problem where the parameters $\boldsymbol{\alpha}$, b and the Gaussian kernel width parameter σ are tuned by unscented Kalman filter (UKF) at each time instant when a new sample arrives.

Multicompartment lung mechanics models are used for hardware-in-the-loop simulation of human lung. It is useful in especially for developing mechanical respiratory systems which are used in cases of respiratory system failures in human beings. Mechanical ventilation systems arise as a solution to that problem. The primary goal of mechanical respiratory systems is, by applying a limited input pressure, to maintain adequate minute ventilation which is the tidal volume multiplied by the number of breaths per minute. Modelling of human lung tackles subtle problems such as determination of the lung compliance and air resistance parameters over the entire range of the lung volume. Also, the variable patient physiology affects the

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modelling adversely. As a result, parametric uncertainity or disturbance is inevitable. Thus, robust control algorithms are of great importance.

This study introduces a remedy with significant quality to the literature and researchers of mechanical ventilation. In this paper, UKF-SVM is adopted responsible for online approximation of the dynamics of multicompartment lung mechanics model in the GPC scheme. UKF-SVM is both utilized for predicting the future outputs of that model and to update the control signal applied to that model. Hence, in this study, previous work of the authors by (Dilmen and Beyhan, 2017) which is about online system identification is extended so as to include an adaptive control performance. Simulations include time-varying parameters and external disturbance of high level affecting the system.

The remainder of the paper is organized as follows. Sect. 2 details the conventional LS-SVM regressor, UKF algorithm, UKF-SVM regressor and adaptive windowing algorithm. Sect. 3 reviews the GPC and UKF-SVM based GPC. Sect. 4 presents the simulation of UKF-SVM based GPC on multicompartment lung mechanics. Finally, Sect. 5 summarizes this paper.

2. ONLINE SUPPORT VECTOR MACHINE

2.1 Batch LS-SVM Regressor

Conventional batch LS-SVR (LS support vector regressor) (Suykens et al., 2002) will be briefly given in this subsection. Consider that N data pairs $\{\mathbf{x}_n, y_n\}_{n=1}^N$ where $\mathbf{x}_n \in \Re^d$ and $y_n \in \Re$ exist. Equality constraint based quadratic programming problem is given by

$$\begin{aligned} \min_{\mathbf{w},b,\mathbf{e}} \mathrm{L} = \quad & \frac{1}{2}\mathbf{w}^T\mathbf{w} + \lambda\frac{1}{2}\sum_{n=1}^{N} e_n^2 \\ \text{Const.}: \quad & y_n = \mathbf{w}^T\varphi(\mathbf{x}_n) + b + e_n, n = 1,\ldots,N. \end{aligned} \tag{1}$$

where e_n is the error variable and λ is the regularization parameter which penalizes the error. $\varphi(\cdot)$ is a nonlinear mapping from the input space to a higher dimensional feature space. $\mathbf{w}$ is the weight vector in the dimension of feature space and b is the bias term. Lagrangian equation is obtained as follows.

$$\pounds(\mathbf{w}, b, \mathbf{e}, \boldsymbol{\alpha}) = L(\mathbf{w}, b, \mathbf{e}) - \sum_{n=1}^{N} \alpha_n\{\mathbf{w}^T\varphi(\mathbf{x}_n) + b + e_n - y_n\} \tag{2}$$

where α_n's are the Lagrange multipliers. The Karush-Kuhn-Tucker (KKT) conditions for optimality are as follows.

$$\begin{cases} \frac{\partial \pounds}{\partial \mathbf{w}} = 0, \mathbf{w} = \sum_{n=1}^{N} \alpha_n \varphi(\mathbf{x}_n) \\ \frac{\partial \pounds}{\partial b} = 0, \sum_{n=1}^{N} \alpha_n = 0 \\ \frac{\partial \pounds}{\partial e_n} = 0, \alpha_n = \lambda e_n, n = 1,\ldots,N \\ \frac{\partial \pounds}{\partial \alpha_n} = 0, \mathbf{w}^T\varphi(\mathbf{x}_n) + b + e_n - y_n = 0, n = 1,\ldots,N. \end{cases} \tag{3}$$

When the Lagrangian equation and optimality conditions are combined, a set of linear equations is obtained as follows.

$$\left[\begin{array}{c|c} 0 & \mathbf{1}^T \\ \hline \mathbf{1} & \boldsymbol{\Omega} + \lambda^{-1}\mathbf{I} \end{array}\right] \left[\begin{array}{c} b \\ \hline \boldsymbol{\alpha} \end{array}\right] = \left[\begin{array}{c} 0 \\ \hline \mathbf{y} \end{array}\right] \tag{4}$$

where $\mathbf{y} = [y_1 \quad y_2 \quad \ldots \quad y_N]^T$, $\mathbf{1} = [1 \quad 1 \quad \ldots \quad 1]^T \in \Re^N$ and

$$\Omega_{ji} = K(\mathbf{x}_j, \mathbf{x}_i), \quad j = 1,\ldots,N, \quad i = 1,\ldots,N \tag{5}$$

$$K(\mathbf{x}_j, \mathbf{x}_i) = \varphi(\mathbf{x}_j)^T\varphi(\mathbf{x}_i) \tag{6}$$

$K(\cdot,\cdot)$ is a kernel function which is an alternative to the inner product of mapping function $\varphi(\cdot)$. It avoids the necessity of exact knowledge about $\varphi(\cdot)$. Several kernel functions exist, e.g. Gauss, polynomial. They must satisfy Mercer conditions and must be positive semi-definite. $\boldsymbol{\alpha}$ and b are the LS solution to (4) and LS-SVR output is obtained as follows.

$$y(\mathbf{x}) = \sum_{n=1}^{N} \alpha_n K(\mathbf{x}_n, \mathbf{x}) + b \tag{7}$$

2.2 UKF

UKF provides a solution to the suboptimal estimations of extended Kalman filter due to linearization. For a random variable whose first two moments (expected value and covariance) of its probability distribution are known, sigma points generated around the expected value with the same covariance can yield the real values of first three moments via a nonlinear transformation. This is called unscented transformation (UT), (Wan and Van Der Merwe, 2000). Let us have a random variable $\mathbf{x} \in R^d$ with expected value and covariance $\bar{\mathbf{x}}$ and $\boldsymbol{P_x}$ respectively. It is transformed via a nonlinear transformation $\mathbf{y} = \mathbf{G}(\mathbf{x})$ such that the statistics of $\mathbf{y}$ are calculated by generating a matrix $\mathbf{X} \in R^{d\times(2d+1)}$ consisting of $\mathbf{X}_i$ sigma vectors.

$$\begin{aligned} &\mathbf{X}_0 = \bar{\mathbf{x}} \\ &\mathbf{X}_i = \bar{\mathbf{x}} + (\sqrt{(d+\psi)\boldsymbol{P_x}})_i, && i = 1,\ldots,d \\ &\mathbf{X}_i = \bar{\mathbf{x}} - (\sqrt{(d+\psi)\boldsymbol{P_x}})_i, && i = d+1,\ldots,2d \\ &Wm_0 = \frac{\psi}{d+\psi} \\ &Wc_0 = \frac{\psi}{d+\psi} + 1 - \eta^2 + \theta \\ &Wc_i = Wm_i = \frac{1}{2(d+\psi)}, && i = 1,\ldots,2d \end{aligned} \tag{8}$$

In (8) $\psi = \eta^2(d+\kappa) - d$ is a scaling parameter. η determines the prorogation of sigma points around $\bar{\mathbf{x}}$ and is usually set to a small number. κ is the second scaling parameter and is usually set to zero. θ is the a priori information about distribution of random variable $\mathbf{x}$ and its optimal value for Gaussian distribution is 2. $(\sqrt{(d+\psi)\boldsymbol{P_x}})_i$ is the i_{th} row of the matrix square root (Cholesky factorization can be employed). Process and observation covariance matrices must be involved to advance from UT to UKF as a recursive filter. Due to the page limitations we omit the UKF algorithm here, but the reader may refer to Algorithm 1 in (Dilmen and Beyhan, 2017). Note that, in that paper, $\mathbf{F}$ and $\mathbf{G}$ are process and measurement functios while $\mathbf{P}_w$ and $\mathbf{P}_v$ denote the process and measurement noise covariance matrices respectively.

2.3 UKF Based SVM Parameter Adaptation

UKF-SVM model adapts its parameters via unscented Kalman filter. Since it is used as the regressor in the GPC scheme, details of the model training is given for regression case. When we rearrange (4) and write explicitly, we obtain the corresponding measurement function of the model.

$$\mathbf{Y}_{SVM} = \mathbf{G}_{SVM}(\mathbf{X}_{SV}, \mathbf{Y}_{SV}, b, \boldsymbol{\alpha}, \sigma)$$

$$\begin{pmatrix} 0 \\ y_1 \\ y_2 \\ \vdots \\ y_s \end{pmatrix} = \begin{pmatrix} \sum_{k=1}^{s} \alpha_k \\ b + K(\mathbf{x}_1, \mathbf{x}_1)\alpha_1 + \ldots + K(\mathbf{x}_1, \mathbf{x}_s)\alpha_s + \lambda^{-1}\alpha_1 \\ b + K(\mathbf{x}_2, \mathbf{x}_1)\alpha_1 + \ldots + K(\mathbf{x}_2, \mathbf{x}_s)\alpha_s + \lambda^{-1}\alpha_2 \\ \ddots \\ b + K(\mathbf{x}_s, \mathbf{x}_1)\alpha_1 + \ldots + K(\mathbf{x}_s, \mathbf{x}_s)\alpha_s + \lambda^{-1}\alpha_s \end{pmatrix} \quad (9)$$

In (9), $\mathbf{X}_{SV}$ and $\mathbf{Y}_{SV}$ are the input-output sample pairs in the support vector set. They are the observed variables. b, $\boldsymbol{\alpha}$ and σ parameters constitute a multidimensional parameter vector, $\mathbf{p}_{SVM} = [b \quad \alpha_1 \quad \alpha_2 \quad \ldots \quad \alpha_n \quad \sigma]^T \in R^{s+2}$. Output $\mathbf{Y}_{SVM}$ is also a multidimensional vector so the model is MIMO type. (9) presents the measurement function of the model. Process function is needed to estimate the parameters optimally by UKF and it is actually identity transition matrix.

$$\begin{aligned} \mathbf{p}_{SVM_{n|n-1}} &= \mathbf{F}_{SVM}(\mathbf{p}_{SVM_{n-1}}) \\ \mathbf{F}_{SVM} &= \mathbf{I}_{(s+2)\times(s+2)} \end{aligned} \quad (10)$$

Let $\mathbf{w}$ and $\mathbf{v}$ be the process and measurement noises while $\mathbf{Q}$ and $\mathbf{R}$ denote the corresponding covariance matrices which have small value (e.g., 1e-6). (9) and (10) can be combined implicitly considering the noises.

$$\begin{aligned} \mathbf{p}_{SVM_{n|n-1}} &= \mathbf{F}_{SVM}(\mathbf{p}_{SVM_{n-1}}) + \mathbf{w}_n \\ \mathbf{Y}_{SVM_n} &= \mathbf{G}_{SVM}(\mathbf{X}_{SV_n}, \mathbf{Y}_{SV_n}, b_n, \boldsymbol{\alpha}_n, \sigma_n) + \mathbf{v}_n \end{aligned} \quad (11)$$

Now it turned to be a parameter estimation problem and after the substitutions in (12) are done, parameter estimation can be performed via UKF.

$$\begin{aligned} &\mathbf{F} \leftarrow \mathbf{F}_{SVM}, \quad \mathbf{G} \leftarrow \mathbf{G}_{SVM}, \quad \mathbf{x} \leftarrow \mathbf{p}_{SVM} \\ &\mathbf{y} \leftarrow \mathbf{Y}_{SVM}, \quad \mathbf{P}_w \leftarrow \mathbf{Q}, \quad \mathbf{P}_v \leftarrow \mathbf{R} \end{aligned} \quad (12)$$

2.4 Update of the Support Vector Set

In this paper, the regressor model will process the data sequentially. A suitable support vector set should be maintained during the operation. It should be small enough to provide fast convergence by keeping the computational load low while providing the set with the capability of well representing the incoming data. This is possible by an adaptive support vector set strategy (Dilmen and Beyhan, 2017) which provides both single update (only decremental or incremental) and sequential updates. It also can determine whether there is no need for any of these updates. Let SV denote the support vector set and s_{max} the maximum number of support vectors allowed. The UKF-SVM model starts with an empty support vector set, i.e., $SV = \emptyset$ and as the new input-output data sample pairs are observed from the system, adaptation of that set is performed as depicted in Algorithm 2 in (Dilmen and Beyhan, 2017), which is omitted here due to the limited space. In the following, *incupd* and *decupd* are incremental and decremental updates of the parameter vector $\mathbf{p}_{SVM,n}$ and the parameter estimation error covariance matrix $\mathbf{P}_n$ while the n_{th} sample is being processed. How $\mathbf{p}_{SVM}$ and $\mathbf{P}$ are updated incrementally/decrementally are explained as follows.

- *incupd*: Let us have the current parameter vector as

$$\mathbf{p}_{SVM} = \begin{bmatrix} b \\ \boldsymbol{\alpha} \\ \sigma \end{bmatrix}_{(s+2)\times 1}, \boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_s \end{bmatrix}_{s\times 1} \quad (13)$$

When a new sample is added, corresponding α parameter (initially 0) will be added to the top of the parameters $\boldsymbol{\alpha}$.

$$\boldsymbol{\alpha}_+ = \begin{bmatrix} \alpha_{new} = 0 \\ \boldsymbol{\alpha} \end{bmatrix}_{(s+1)\times 1}, \mathbf{p}_{SVM_+} = \begin{bmatrix} b \\ \boldsymbol{\alpha}_+ \\ \sigma \end{bmatrix}_{(s+3)\times 1} \quad (14)$$

Let the current parameter estimation error covariance matrix $\mathbf{P} \in R^{(s+2)\times(s+2)}$ be as follows.

$$\mathbf{P} = \begin{bmatrix} P_{1,1} & P_{1,2} & \ldots & P_{1,s+1} & P_{1,s+2} \\ P_{2,1} & P_{2,2} & \ldots & P_{2,s+1} & P_{2,s+2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ P_{s+1,1} & P_{s+1,2} & \ldots & P_{s+1,s+1} & P_{s+1,s+2} \\ P_{s+2,1} & P_{s+2,2} & \ldots & P_{s+2,s+1} & P_{s+2,s+2} \end{bmatrix} \quad (15)$$

Corresponding rows and columns to the new α parameter will be added (initially 1 on the diagonal and 0 other) and $\mathbf{P}_+ \in R^{(s+3)\times(s+3)}$ will be obtained.

$$\mathbf{P}_+ = \begin{bmatrix} P_{1,1} & 0 & P_{1,2} & \ldots & P_{1,s+1} & P_{1,s+2} \\ 0 & 1 & 0 & \ldots & 0 & 0 \\ P_{2,1} & 0 & P_{2,2} & \ldots & P_{2,s+1} & P_{2,s+2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ P_{s+1,1} & 0 & P_{s+1,2} & \ldots & P_{s+1,s+1} & P_{s+1,s+2} \\ P_{s+2,1} & 0 & P_{s+2,2} & \ldots & P_{s+2,s+1} & P_{s+2,s+2} \end{bmatrix} \quad (16)$$

- *decupd*: Removing the support vector with the smallest α value or the oldest one may not yield good results in every case. On the other hand, leave-one-out (LOO) cross validation is proven to be a standard criterion for comparing the generalization power of statistical models. Therefore, LOO is used to determine which support vector to be removed from the set. It is aimed to choose the support vector which will provide the UKF-SVM model with the smallest approximation error after its removal. Let us assume the l_{th} vector has been determined to be removed. It will be pushed to the end of the SV set and then will be deleted. Corresponding α_l parameter will be pushed to the end of $\mathbf{p}_{SVM}$ and then will be deleted. Corresponding row and column to parameter α_l will be pushed to the last row and column in the matrix $\mathbf{P}$ and then will be deleted as well.

3. UKF-SVM BASED GENERALIZED PREDICTIVE CONTROL

Section 3.1 explains the GPC scheme while Section 3.2 details the UKF-SVM based GPC controller. Note that, the GPC method employed in this paper is adopted from (Iplikci, 2010) for the case of SISO systems.

3.1 Generalized Predictive Control

Let us represent a single-input single-output (SISO) nonlinear system by a nonlinear autoregressive with exogenous input (NARX) model.

$$y_n = g(u_n, u_{n-1}, \ldots, u_{n-n_u}, y_{n-1}, \ldots, y_{n-n_y}) \quad (17)$$

g is assumed to be unknown and will be parameterized by a function approximator model. Let $\tilde{y}_n$ denote the desired reference signal and $\hat{y}_n$ denote the model output for the n_{th} time instant. GPC aims to track the reference signal as close as possible while keeping the changes in the control signal as low as possible. Hence, it produces a control input vector $\mathbf{u}_n = [u_n \dots u_{n+K_u}]^T \in \Re^{K_u+1}$ at each time instant and applies the first element to the system. Also, constraints on the control signals are taken into consideration, which bound the control signal and contributes to stability.

$$\begin{aligned}\min_{\mathbf{u}_n} \mathrm{f}(\mathbf{u}_n) =& \sum_{k=1}^{K_y} (\tilde{y}[n+k] - \hat{y}[n+k])^2 + \\ & \zeta \sum_{k=0}^{K_u} (u[n+k] - u[n+k-1])^2 \\ =& (\tilde{\mathbf{Y}}_n - \hat{\mathbf{Y}}_n)^T (\tilde{\mathbf{Y}}_n - \hat{\mathbf{Y}}_n) + \zeta \mathbf{u}_n^T \mathbf{L} \mathbf{u}_n \\ & - 2\zeta u[n]u[n-1] + \delta u^2[n-1] \\ \text{Const.} \ :& u_{min} \le u[n+k] \le u_{max}, k = 0, 1, \dots, K_u \\ & \mid u[n+k] - u[n+k-1] \mid \le \triangle u_{max}, \\ & k = 0, \dots, K_u \end{aligned} \tag{18}$$

$$\mathbf{L} = \begin{bmatrix} 2 & -1 & 0 & \dots & 0 & 0 \\ -1 & 2 & -1 & \dots & 0 & 0 \\ 0 & -1 & 2 & \ddots & 0 & 0 \\ 0 & 0 & -1 & \ddots & -1 & 0 \\ 0 & 0 & 0 & \ddots & 2 & -1 \\ 0 & 0 & 0 & \ddots & -1 & 1 \end{bmatrix} \tag{19}$$

ζ, penalizes the sudden changes in the control signal. K_u and K_y are future horizon values for input and output. Always $K_u \le K_y$ should be satisfied beause, an output cannot depend on any input which occurs in the future, i.e, later than when the output occurs (due to causality). At each time instant, it is aimed to track the reference signal for the following K_y time instants. First element of the produced optimal control signal vector is applied to the system. $\tilde{\mathbf{Y}} = [\tilde{y}_{n+1} \dots \tilde{y}_{n+K_y}]^T \in \Re^{K_y}$ and $\hat{\mathbf{Y}} = [\hat{y}_{n+1} \dots \hat{y}_{n+K_y}]^T \in \Re^{K_y}$. A function approximator model is used to predict the future behavior of the system whose output expression is unknown. Also, that model is utilized to provide the necessary gradient information to update the control input vector at each time instant. The update is as follows.

$$\mathbf{u}_{n+1} = \mathbf{u}_n + \triangle \mathbf{u} \tag{20}$$

Modified Newton optimization can be employed for this update.

$$\triangle \mathbf{u} = -\left(\frac{\partial^2 f(\mathbf{u}_n)}{\partial \mathbf{u}_n^2}\right)^{-1} \frac{\partial f(\mathbf{u}_n)}{\partial \mathbf{u}_n} \tag{21}$$

Gradient vector is obtained as follows.

$$\frac{\partial f(\mathbf{u}_n)}{\partial \mathbf{u}_n} = -2\left(\frac{\partial \hat{\mathbf{Y}}_n}{\partial \mathbf{u}_n}\right)^T (\tilde{\mathbf{Y}}_n - \hat{\mathbf{Y}}_n) + 2\zeta \mathbf{L}\mathbf{u}_n - 2\begin{bmatrix} \zeta u[n-1] \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{22}$$

$\left(\frac{\partial^2 \hat{\mathbf{Y}}_n}{\partial \mathbf{u}_n^2}\right)$ term which would be produced in the Hessian expression has a very small value so it can be ignored (obsevred emprically in the experiments). Thus, Hessian can be written approximately as

$$\frac{\partial^2 f(\mathbf{u}_n)}{\partial \mathbf{u}_n^2} \cong 2\left(\frac{\partial \hat{\mathbf{Y}}_n}{\partial \mathbf{u}_n}\right)^T \left(\frac{\partial \hat{\mathbf{Y}}_n}{\partial \mathbf{u}_n}\right) + 2\zeta \mathbf{L} \tag{23}$$

Considering (17), model output depends only on the control signals with a time index equal to or smaller than that of itself. Therefore, Jacobian matrix $\left(\frac{\partial \hat{\mathbf{Y}}_n}{\partial \mathbf{u}_n}\right)$ is expressed as follows.

$$\frac{\partial \hat{\mathbf{Y}}_n}{\partial \mathbf{u}_n} = \begin{bmatrix} \frac{\partial \hat{y}[n+1]}{\partial u[n]} & \frac{\partial \hat{y}[n+1]}{\partial u[n+1]} & 0 & \dots & 0 \\ \frac{\partial \hat{y}[n+2]}{\partial u[n]} & \frac{\partial \hat{y}[n+2]}{\partial u[n+1]} & \frac{\partial \hat{y}[n+2]}{\partial u[n+2]} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \hat{y}[n+K_y]}{\partial u[n]} & \frac{\partial \hat{y}[n+K_y]}{\partial u[n+1]} & \frac{\partial \hat{y}[n+K_y]}{\partial u[n+2]} & \dots & \frac{\partial \hat{y}[n+K_y]}{\partial u[n+K_u]} \end{bmatrix} \tag{24}$$

Computational load of the derivatives depend on the approximator model chosen.

3.2 UKF-SVM Based GPC Controller

$\hat{\mathbf{Y}}_n$ and $\frac{\partial \hat{\mathbf{Y}}_n}{\partial \mathbf{u}_n}$ are obtained utilizing the UKF-SVM model. Let $\mathbf{x}_n$ be the support vector generated at the n_{th} time instant after observing the input-output data pair $\{u_n, y_n\}$ from the system.

$$\mathbf{x}_n = [u_n \quad u_{n-1} \quad \dots \quad u_{n-n_u} \quad y_{n-1} \quad \dots \quad y_{n-n_y}]^T \tag{25}$$

where n_u and n_y are the past horizons for the input and output samples used to construct the support vector form the new incoming data pair and from the past samples. The model output at the n_{th} time instant is obtained as

$$\hat{y}_n = \sum_{j=1}^{s} \alpha_s K(\mathbf{x}_j, \mathbf{x}_n) + b \tag{26}$$

Let us write the Gaussian kernel function

$$K_{jn} = exp\left(-\frac{d_{jn}}{2\sigma^2}\right) \tag{27}$$

where

$$\begin{aligned} d_{jn} =& (\mathbf{x}_j - \mathbf{x}_n)^T (\mathbf{x}_j - \mathbf{x}_n) \\ =& \sum_{i=0}^{n_u} (x_{j,i+1} - u_{n-i})^2 + \sum_{i=1}^{n_y} (x_{j,n_u+1+i} - y_{n-i})^2 \end{aligned} \tag{28}$$

Combining (26), (27) and (28), the model output can be rewritten as

$$\hat{y}_n = \sum_{j=1}^{s} \alpha_s exp\left(-\frac{d_{jn}}{2\sigma^2}\right) + b \tag{29}$$

We predict the future outputs of the system using the UKF-SVM model as

$$\hat{y}_{n+l} = \sum_{j=1}^{s} \alpha_s exp\left(-\frac{d_{j,n+l}}{2\sigma^2}\right) + b, \quad l = 1, \dots, K_y \tag{30}$$

where

$$
\begin{aligned}
d_{j,n+l} = & \sum_{i=0}^{n_u} \begin{cases} (x_{j,i+1} - u_{n+l-i})^2, & l-i < K_u \\ (x_{j,i+1} - u_{n+K_u})^2, & l-i \geq K_u \end{cases} \\
& + \sum_{i=1}^{min(l,n_y)} (x_{j,n_u+1+i} - \hat{y}_{n+l-i})^2 \\
& + \sum_{i=l+1}^{n_y} (x_{j,n_u+1+i} - y_{n+l-i})^2
\end{aligned} \tag{31}
$$

We see that, to obtain the gradient vector in (22) and Hessian matrix in (23), Jacobian matrix in (24) which includes the first order derivative of model future outputs wrt the future inputs is crucial. Thus, utilizing (30) and (31), first order derivatives are obtained in the following way.

$$
\begin{aligned}
\frac{\partial \hat{y}_{n+l}}{\partial u_{n+h}} &= \sum_{j=1}^{s} \alpha_s \frac{\partial exp\left(-\frac{d_{j,n+l}}{2\sigma^2}\right)}{\partial u_{n+h}}, \quad h = 0, \ldots, K_u \\
&= \sum_{j=1}^{s} -\frac{1}{2\sigma^2} exp\left(-\frac{d_{j,n+l}}{2\sigma^2}\right) \frac{\partial d_{j,n+l}}{\partial u_{n+h}}
\end{aligned} \tag{32}
$$

where

$$
\begin{aligned}
\frac{\partial d_{j,n+l}}{\partial u_{n+h}} = & \sum_{i=0}^{n_u} \begin{cases} -2(x_{j,i+1} - u_{n+l-i})\delta(l-i,h), & l-i < K_u \\ -2(x_{j,i+1} - u_{n+K_u})\delta(K_u,h), & l-i \geq K_u \end{cases} \\
& + \sum_{i=1}^{n_y} -2(x_{j,n_u+1+i} - \hat{y}_{n+l-i}) \frac{\partial \hat{y}_{n+l-i}}{\partial u_{n+h}} \delta_1(l-i,h)
\end{aligned} \tag{33}
$$

Note that $\delta_1(\cdot)$ stands for the unit step function while $\delta(\cdot)$ is the Dirac delta function. Equation (32) and (33) can be combined and substituted in (22), (23) and (24) to obtain the gradient vector and Hessian matrix necessary for the control input vector update.

4. SIMULATION RESULTS

In this study, we adopt a linear model (Hou et al., 2014) of multicompartment lung mechanics as the nominal model in purpose. It involves the lung compliances and air resistances as constant system parameters. However, during control, i) we assume that, system dynamics are unknown to us and ii) actual system has time-varying compliances (parametric uncertainity). In addition, we apply an artifical external disturbance to the underlying system as well. Let us define the parametric uncertainity in the system as internal disturbance. We actually test UKF-SVM in the GPC framework to assess its performance in an online black box modelling based control scheme with internal and external disturbances.

4.1 Nominal System Model

Let us briefly explain the multicompartment lung mechanics model. It has a dichotomoy that is inspired by human lung where at each generation of a new airway, the airway opens to a subsequent two-branch airway, and so on. If we have a system of γ generations, then we have 2^γ lung compartments. A lung mechanics model for $\gamma = 2$ is given in Fig. 1. In Fig. 1, $R^{in}_{j,i}$ denotes the airway resistance for the i_{th} airway of the j_{th} generation for $j = 0, \ldots, \gamma$ and $i = 1, \ldots, 2^j$. Note that, the superscript 'in' stands for the inhalation period and its exhalation counterpart is denoted by $R^{ex}_{j,i}$. Also, x_i and c_i for $i = 1, \ldots, 2^\gamma$, denote the air volume and the associated compliance of the compartment i. Let us write the state space equations for the model. Note that, system has switched dynamics since the parameters associated with the inhalation and exhalation periods need not be the same.

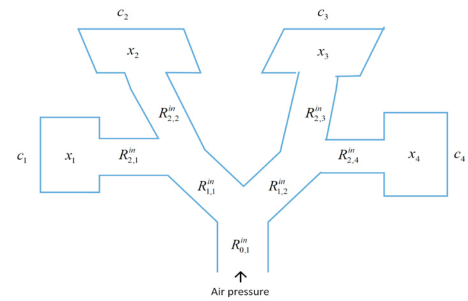

Fig. 1. Lung mechanics model with 4 compartments, $\gamma = 2$.

$$
\begin{aligned}
\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) \\
y(t) &= \mathbf{1}^T \mathbf{x}(t)
\end{aligned} \tag{34}
$$

where $\mathbf{1} \in \Re^{d_x}$ is a vector of ones, $\mathbf{x} = [x_1 \ldots x_{d_x}]^T \in \Re^{d_x}$ is the state vector, $d_x = 2^\gamma$, $\mathbf{x}(0) = \mathbf{x}_0$ and

$$
\begin{cases} \mathbf{A} = -\mathbf{R}_{in}^{-1}\mathbf{C}, \quad \mathbf{B} = \mathbf{R}_{in}^{-1}\mathbf{1}, & 0 \leq t \leq T_{in} \\ \mathbf{A} = -\mathbf{R}_{ex}^{-1}\mathbf{C}, \quad \mathbf{B} = \mathbf{R}_{ex}^{-1}\mathbf{1}, & T_{in} < t \leq T_{in} + T_{ex} \end{cases} \tag{35}
$$

T_{in} denotes the inhalation period while T_{ex} denotes the exhalation period. Thus, a breathing period is computed as $T = T_{in} + T_{ex}$. System (35) is periodic with the period T and the output $y(t)$ is actually sum of the states, which means, we measure the total lung volume as the system output. Input $u(t)$ to the system is the air pressure applied at the initial airway whose resistance is denoted by $R^{in}_{0,1}$. $\mathbf{C} = diag\,[1/c_1 \ldots 1/c_{d_x}]$. Note that, $\mathbf{R}_{in} = \sum_{j=0}^{\gamma} \sum_{i=1}^{2^j} R^{in}_{j,i} \mathbf{Z}_{j,i} \mathbf{Z}^T_{j,i}$ and $\mathbf{R}_{ex} = \sum_{j=0}^{\gamma} \sum_{i=1}^{2^j} R^{ex}_{j,i} \mathbf{Z}_{j,i} \mathbf{Z}^T_{j,i}$ where the l_{th} element of $\mathbf{Z}_{j,i} \in \Re^{d_x}$ is 1 for all $l = (i-1)2^{\gamma-j+1}, (i-1)2^{\gamma-j+2}, \ldots, i2^{\gamma-j}$ and 0 otherwise. Last, let us give the parameters for nominal model of the system. We should indicate that we gave a general representation of the respective system by Fig. 1. However, we use a two-compartment lung mechanics model in the simulations ($\gamma = 1$). $R^{in}_{0,1} = 9$ cm H_2O/l/s, $R^{in}_{1,1} = 16$ cm H_2O/l/s and $R^{in}_{1,2} = 16$ cm H_2O/l/s. $R^{ex}_{j,i}$ counterpart is twice the $R^{in}_{j,i}$, $j = 0, 1$ and $i = 1, 2$. Lung compartment compliance nominal values are $c_i = 0.11$/cm H_2O, $i = 1, 2$.

4.2 Illustrative Example

Initial compartment air volumes are set $\mathbf{x}_0 = [0.5, 0]^T$ liters. Also, input air pressure has an upper bound $u_{max} = 19$ cm H_2O. Inhalation and exhalation periods are $T_{in} = 2$ s and $T_{ex} = 3$ s respectively while the sampling period is $T_s = 0.1$ s. In the simulation, in contrast to the nominal model, the lung compliance parameters are time-varying and denoted by c^{tv}_i, $i = 1, 2$. A time-varying compliance by (36) has a profile which varies between half and full value of the nominal compliance within any of the inhalation or exhalation periods.

$$\begin{aligned} &c_i^{tv}(t) = c_i + \bar{c}_i(t) \\ &\bar{c}_i(t) = -0.5c_i + 0.5c_i sin(2\pi f t') \\ &\begin{cases} t' = mod(t,T), f = 0.5/T_{in} & , mod(t,T) \leq T_{in} \\ t' = mod(t,T) - T_{in}, f = 0.5/T_{ex} & , mod(t,T) > T_{in} \end{cases} \end{aligned} \quad (36)$$

The term $\bar{c}_i$ in (36) corresponds to a hard parametric uncertainity which is created artificially. It causes an internal disturbance on the system. In the simulation, in addition to the internal disturbance, an external disturbance given by (37) is applied to the underlying system.

$$\mathrm{d}_{i,ext}(t) = d_o + \sum_{k=1}^{3} d_{m_k} sin\big(2\pi(1/T_{d_k})t\big), i = 1,2 \quad (37)$$

where $d_o = -0.2$, $T_d = [2, 5, 10]$ s, $d_m = [0.03, 0.02, 0.01]$ and $\mathbf{d}_{ext} = [\mathrm{d}_{1,ext}, \mathrm{d}_{2,ext}]^T$.

GPC parameters are chosen as $K_y = 10$, $K_u = 2$, $u_{min} = 0$, $u_{max} = 19$, $\triangle u_{max} = 5$, $\delta = 1e-6$. Note that, in the planned future real time control, $\triangle u_{max}$ will be determined experimentally. Maximum number of support vectors allowd is $s_{max} = 5$ and $n_u = 2$, $n_y = 4$ are found to be suitable experimentally. Fig. 2 shows the tracking result

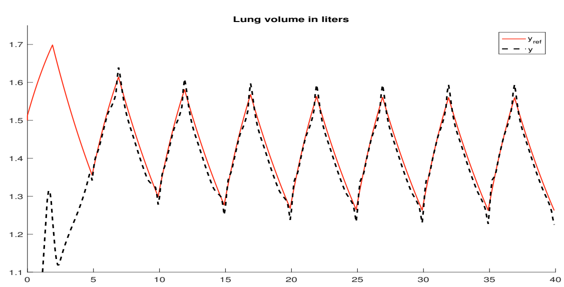

Fig. 2. Tracking result.

for a reference respiratory pattern y_{ref} where y denotes the system output obtained when UKF-SVM based GPC is performed. The root mean squarred tracking error is 0.1799. It is seen from Fig. 2 that, even in such challenging case as i) dynamics of the underlying system is assumed unknown and ii) it is under both hard internal (parametric uncertainity) and external disturbances given by (36) and (37), proposed UKF-SVM based GPC approach can achieve an acceptable tracking performance. Fig. 3 shows the produced control input signal. Fig. 4 shows us the

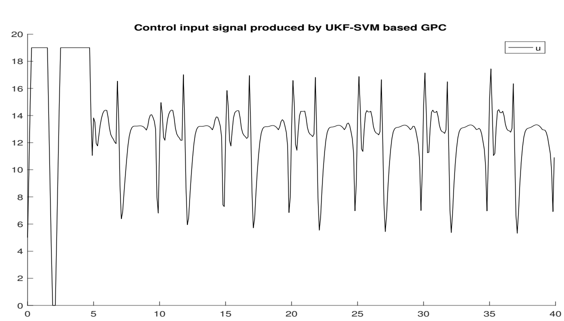

Fig. 3. Produced control input signal.

time evolution of b, $\boldsymbol{\alpha}$ and σ parameters of UKF-SVM in addition to the number of support vectors used during control.

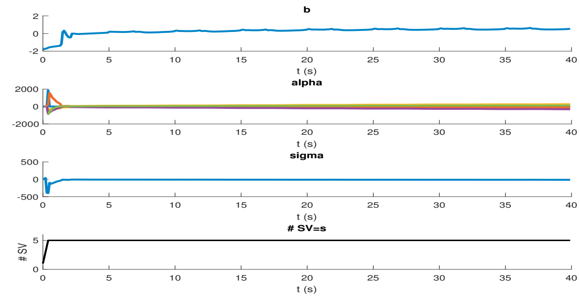

Fig. 4. Parameters b, $\boldsymbol{\alpha}$, σ of UKF-SVM and # support vectors used during control where $s_{max} = 5$.

5. CONCLUSION

In this paper, UKF-SVM model is successfully employed in the generalized predictive control scheme, where multicompartment lung mechanics model, whose dynamics are assumed unknown, is controlled in the simulation. The generalized predictive control formulations using UKF-SVM model are derived rigorously and demonstrated successfully in the GPC scheme. Small number of support vectors, such as $s_{max} = 5$ is enough for an admissible performance. It manages to capture well the time-varying dynamics of the underlying system. Also, it seems to be a useful tool in robust control even at high levels of disturbance. In the future, we plan to employ it for control of multicompartment lung mechanics in real time where the real system is assumed to be a new design. Note that, since UKF-SVM based GPC performs control based on online black box modelling strategy, it is expected to be a reliable adaptive controller for that task. Several disturbance types will be considered on the real system in the future study.

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