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INVESTIGATION OF ELASTIC PENDULUM OSCILLATIONS BY SIMULATION TECHNIQUE

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ABSTRACT

In this study, elastic spring-mass pendulum oscillations are investigated. In order to solve a nonlinear differential equation system, Simulation Technique based on Modelica Language such as Dymola, SimulationX etc., is used. It's assumed that the spring coefficient in this system is linear and spring mass is neglected. Under these conditions, kinematic behavior of the pendulum was investigated. The governing equation of the system possessing two nonlinear differential equations which interacts each other are solved simultaneously. The obtained results are compared with previous works and seemed good agreements with others.

Keywords : *Elastic pendulum, Nonlinear oscillation, Simulation technique, Modelica, Dymola.*

SİMÜLASYON TEKNİĞİ İLE ELASTİK KÜTLE-YAY SALINIMINLARININ İNCELENMESİ

ÖZET

Bu çalışmada, elastik kütle-yay sarkaç salınımları incelenmiştir. Sistemin lineer olmayan diferansiyel denklemlerini çözmek için Dymola, SimulationX gibi Modelica dili tabanlı Simülasyon Tekniği kullanılmıştır. Sistemdeki yayın direngenliği lineer ve kütlesi ihmal edilmiştir. Bu şartlar altındaki sarkacın kinematik davranışı incelenmiştir. Sistemi ifade eden genel denklem iki tane lineer olmayan ve birbirini etkileyen diferansiyel denklemden oluşmaktadır. Bu denklemler Simülasyon Tekniği ile çözülmüştür. Elde edilen sonuçlar önceki çalışmalarla kıyaslanmış ve uyumlu olduğu görülmüştür.

Anahtar Kelimeler : *Elastik sarkaç, Lineer olmayan salınım, Simülasyon Tekniği, Modelica, Dymola.*

1. INTRODUCTION

Earlier, Nayfeh obtained the solution of the pendulum oscillations analytically and numerically by using Perturbation Techniques (Nayfeh, 1987). Most of nonlinear differential equations can be solved by different numerical methods, such as Finite Element Method (FEM), Finite Difference Method (FDM), Variational Iteration Method (VIM), Homotopy Perturbation Method (HPM) and etc. Most of them use more girds to solve system numerically except Differential Quadrature Method (DQM). Liu and Wu (Liu and Wu, 2000) solved Duffing Equation by using DQM and used Frechet Derivative in order to make system linear. He (He, 1999; 2003) was proposed VIM and HPM to solve nonlinear differential equation (NDE) numerically. In VIM, the problems are initially approximated with possible unknowns. Than a correction function is constructed by a general Lagrange multiplier, which can be identified via the variational theory. In contrast to the traditional perturbation method, HPM does not require a small parameter in an equation.

Author used a Duffing equation with high order of nonlinearity to illustrate it's effectiveness. The differential quadrature method is extended to solve second-order initial value problems by Fung (Fung, 2001). NDE systems, which interacts each other, are not able to be solved easily above methods. For this reason, new numerical techniques such as, Genetic Algorithms (GAs), Artificial Neural Network (ANN), Fuzzy Logic (FL) etc. have been used recently. ST is another alternative method in order to solve NDE. Although the problem construction is very easy, results are more correct when compared with others (Georgiou, 1999). Elastic pendulum oscillation behaviors were also investigated by Chang and Lee, who applied GAs to investigate double pendulum oscillations (Chang and Lee, 2004). Girgin imposed Combining Method to study nonlinear pendulum oscillations (Girgin, 2008). Later, Lynch investigated three dimensional motion of the elastic pendulum assuming that amplitude is small (Lynch, 2002). Lynch and Hougton investigated three dimensional motion of the elastic pendulum in the case of resonance and derived suitable initial conditions using envelope dynamics (Lynch and Hougton, 2004). In addition to, Vetyukov et al. derived the non-linear equation of motion of the 2D floating rectangular frame and the deformation was interpolated by means of polynomial shape functions (Vetyukov et al., 2004).

2. GOVERNING EQUATIONS OF THE SYSTEM

Governing equation of the elastic pendulum is obtained by writing, equilibrium equation of the pendulum shown in Figure 1 at position 2. Coriolis acceleration is added to the system because pendulum mass is connected to spring instead of cord which causes to coriolis acceleration.

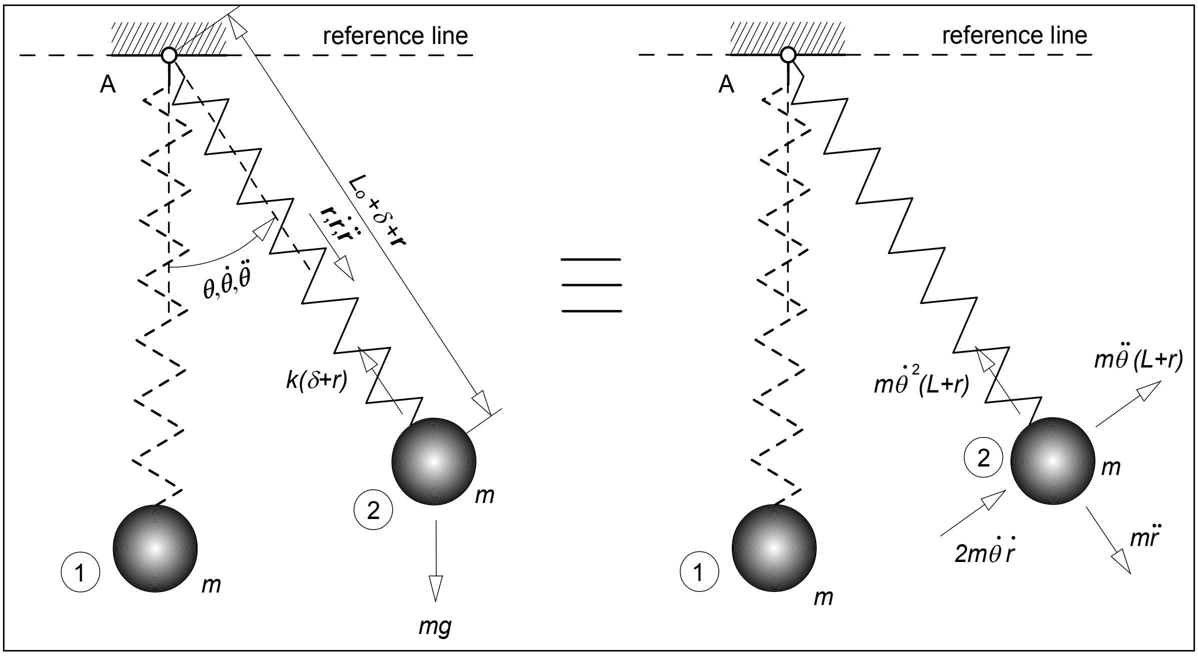

Figure 1. Force equilibrium of the elastic pendulum oscillating about point A.

Writing Newton's principle in r and θ direction give us the governing equations;

$$\frac{d^2r}{dt^2}=\left(\frac{d\theta}{dt}\right)^2(L+r)-\frac{k}{m}r-g(1-\cos\theta) \qquad (1)$$

$$\frac{d^2\theta}{dt^2}=-\frac{2\frac{d\theta}{dt}\frac{dr}{dt}}{L+r}-\frac{g\sin\theta}{L+r} \qquad (2)$$

where;

$L=L_0+\delta$: static stable length of the spring with pendulum.

L_0 : free spring length.

δ : deflection because of mass.

Equation (1) and (2) have two variables (r and θ) and calculated from Figure 1.

The system is conservative because there is no damping. Therefore total energy (kinetic energy and potential energy) of the system is always constant and time invariant (holonomic). Equation (3) and (4) depict total energy of the pendulum at position 1 and 2 as shown in Figure 1.

$$E_1 = \frac{1}{2} m \left(\frac{dr_1}{dt} \right)^2 + \frac{1}{2} k \left(\delta + r_1 \right)^2 - mg\left(L_0 + \delta + r_1 \right) \quad (3)$$

$$E_2 = \frac{1}{2} k \left(\delta + r_2 \right)^2 - mg\left(L_0 + \delta + r_2 \right) \cos\left(\theta \right) \quad (4)$$

where E_i denotes total energy at position i.

In order to investigate behaviors of the elastic pendulum, some parameters must be given. For this reason, natural frequency of spring and natural frequency of pendulum are given in Equation (5) respectively.

$$\omega_s^2 = \frac{k}{m} \quad \text{ve} \quad \omega_p^2 = \frac{g}{L} \quad (5)$$

Furthermore, dimensionless parameter μ is given by,

$$\mu \equiv \frac{\omega_p}{\omega_s} \quad (6)$$

Spring constant (k) and the free length of the spring (L_0) values are calculated by,

$$\mu = \sqrt{1 - \frac{L_0}{L}}, \quad \delta = L - L_0, \quad k = \frac{mg}{\delta} \quad (7)$$

3. NORMALIZED SOLUTION

Results are given in normalized form; hence they can be compared with other works. Following transformations are used for normalization in Equation (1) and (2).

$$r = R \cdot L \quad \text{ve} \quad \tau = \omega_p \cdot t \quad (8)$$

where R and τ are normalized position and time parameters. ω_p is given Equation (5).

When the above transformations are applied, following equations are obtained in dimensionless form.

$$\frac{d^2R}{d\tau^2} - \left(\frac{d\Theta}{d\tau} \right)^2 \left(1 + R\right) + \frac{1}{\mu^2} R + \left(1 - \cos\Theta\right) = 0 \quad (9)$$

$$\frac{d^2\Theta}{d\tau^2} + \frac{2\frac{d\Theta}{d\tau}\frac{dR}{d\tau}}{1+R} + \frac{\sin\Theta}{1+R} = 0 \quad (10)$$

Equation (9) and (10) were solved simultaneously in Dymola and their simulation scheme is depicted in Figure 2.

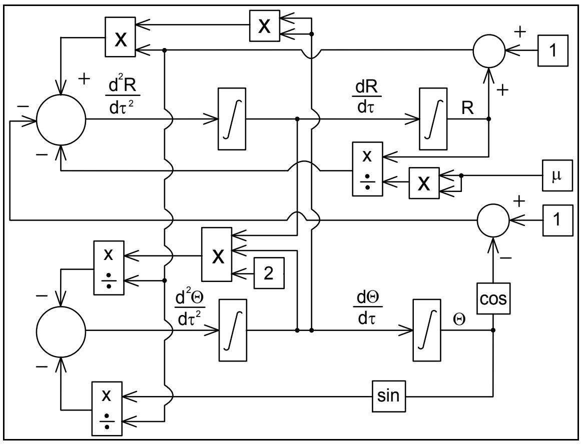

Figure 2. Dimensionless simulation scheme of R and Θ given in Equation (9) and (10).

For stable oscillations, the suitable value of R, depends on initial Θ angle, is obtained with simulation by making feedback. Stable and instable oscillations are given in Figure 3 with continuous and dashed lines respectively.

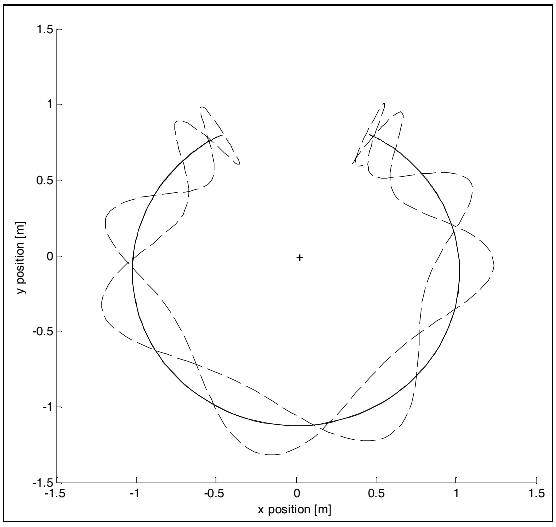

Figure 3. Stable and instable oscillation paths.

Initial angles $\Theta(0)$ are taken from 10^o to 180^o by taking 10^o step size. Initial angular velocity $\frac{d\Theta}{d\tau}(0)$ and relative velocity $\frac{dR}{d\tau}(0)$ are taken 0 and corresponding initial relative position $R(0)$ values depend on initial angle $\Theta(0)$ are taken from Table 1.

Table 1. Computed initial radial position $R(0)$ values depend on initial angle $\Theta(0)$ for dimensionless stable oscillations for $\mu = 0,1$.

$\Theta(0)$	R (0)		$\Theta(0)$	R (0)
10	-0,000161		110	-0,013684
20	-0,000639		120	-0,015220
30	-0,001418		130	-0,016601
40	-0,002468		140	-0,017785
50	-0,003755		150	-0,018740
60	-0,005229		160	-0,019428
70	-0,006851		170	-0,019856
80	-0,008559		180	-0,020000
90	-0,010299		190	-0,019856
100	-0,012030		200	-0,019428

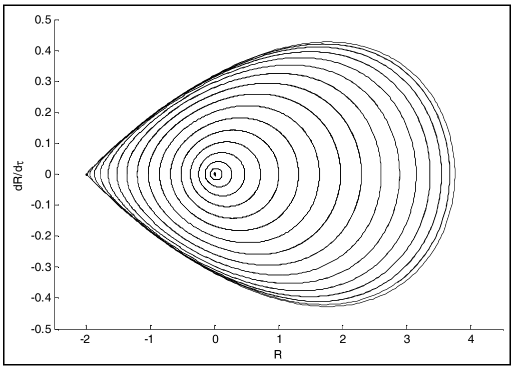

Figure 4. Dimensionless linear velocity diagram versus linear position (R) for $\mu = 0,1$ (present work).

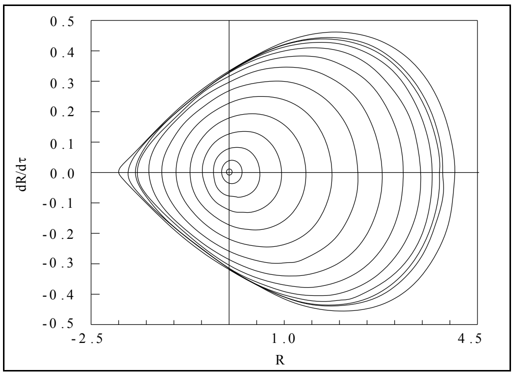

Figure 5. Dimensionless linear velocity diagram versus linear position (R) for $\mu = 0,1$ (Georgiou's work) .

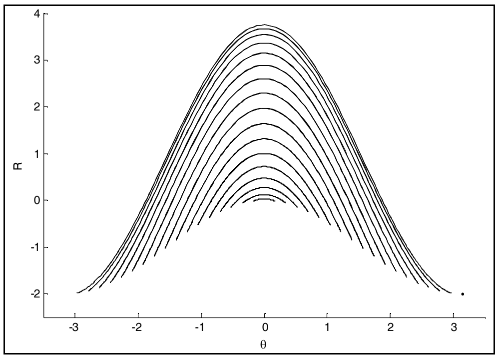

Figure 6. Dimensionless linear position (R) diagram versus angular position (Θ) for $\mu = 0,1$.

This present work shown in Figure 4 was compared with (Georgiou, 1999) shown in Figure 5 and good agreement was seemed.

As shown in Figure 4, one can realize that although it is symmetric about R axis, it is asymmetric about $\dot{R}$ axis. The shape changes from circle to the one sided stretched ellipse like a balloon. Similarly, in Figure 6 is symmetric about R axis and it is asymmetric about Θ axis. In contrast with Figure 7 and 8 are symmetric in all axes.

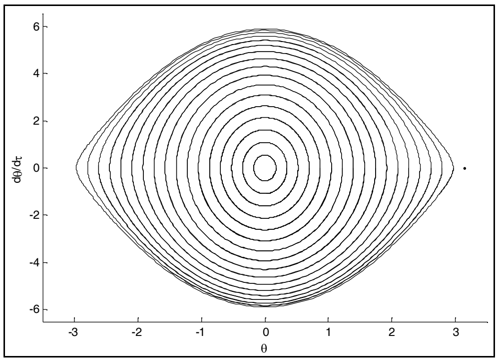

Figure 7. Dimensionless angular velocity diagram versus angular position (Θ) for $\mu = 0,1$.

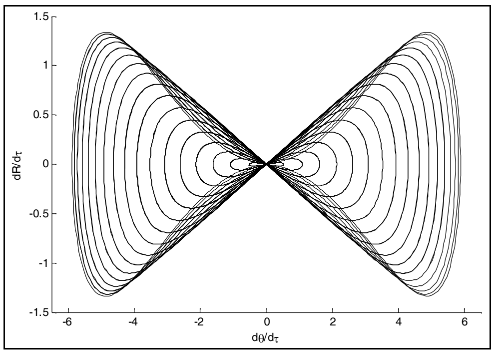

Figure 8. Dimensionless linear velocity diagram versus angular velocity for $\mu = 0,1$.

It's note that pendulum oscillation $\Theta \in [-\pi, \pi]$. If Θ values exceed this interval, these figures changes rapidly because pendulum is rotating about A in Figure 1 instead of oscillations.

4. EFFECT OF μ ON THE PENDULUM OSCILLATION

Oscillation paths in x-y plane are shown in Figures 9,10 and 11 for $\mu = 0.1$, $\mu = 0.3$ and $\mu = 0.4$ values respectively. Although Figure 9 shows smoothness, Figure 11 shows different behavior oscillations for $\Theta(0) \geq 150^o$ because of weak stiffness of the spring. This stage can be investigated later for scientists.

Initial angles $\Theta(0)$ are taken 50^o - 90^o - 110^o - 150^o - 170^o.

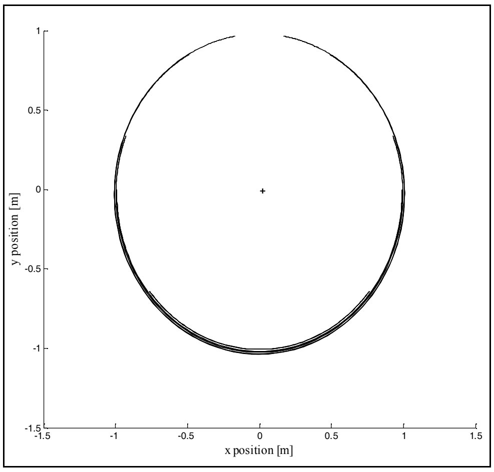

Figure 9. x-y position of the elastic pendulum in Cartesian coordinate system for $\mu = 0,1$.

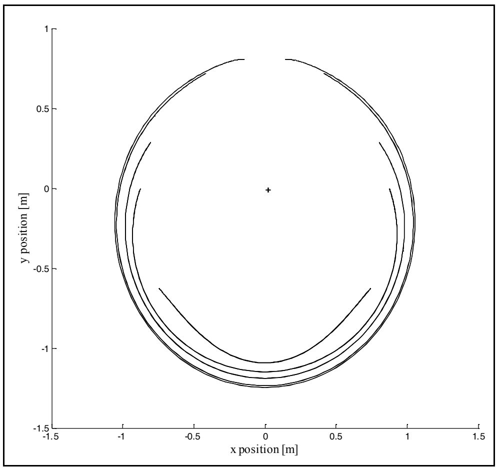

Figure 10. x-y position of the elastic pendulum in Cartesian coordinate system for $\mu = 0,3$.

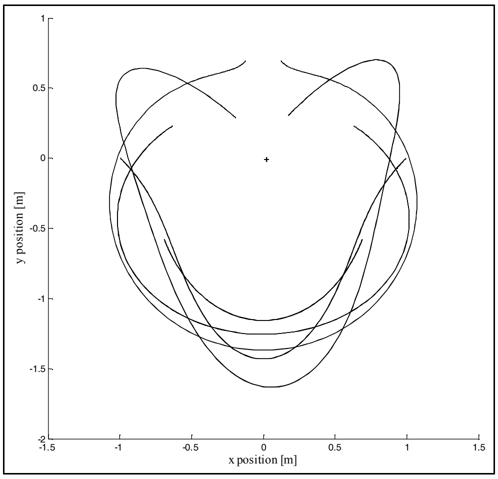

Figure 11. x-y position of the elastic pendulum in Cartesian coordinate system for $\mu = 0,4$.

5. CONCLUSION

In this paper, ST is used to analyze the elastic pendulum motion. It can be clearly seen that initial value NDEs can be easily solved by ST and the more μ values cause to instability in the system. Advantages of ST can be arranged as follows:

1. Problem construction is easy.
2. Results are correct.
3. There is no analytical procedure for a new problem construction.

Disadvantage:

1. It is suitable for only initial value problems.

From now one, it can be extended to two and three dimensional problems for scientists. Also it gives a new technique to solve NDE systems and opens a new area for pendulum oscillations.

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