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APPLICATION OF THE GENERALIZED DIFFERENTIAL QUADRATURE METHOD TO DEFLECTION AND BUCKLING ANALYSIS OF STRUCTURAL COMPONENTS

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ABSTRACT

The method of differential quadrature developed by Richard Bellman in the 1970s is a numerical solution technique for differential systems by means of a polynomial-collocation approach at a finite number of points. In this paper a global method of generalized differential quadrature is presented to solve the problems on deflection, buckling and vibration behaviour of structural components. Furthermore, the applicability of this method to the deflection analysis of beams due to a point load is also demonstrated. An inherent advantage of the approach is its basic simplicity and small computational effort with easy programmability. Results are obtained for various boundary and loading conditions and are compared with existing exact and numerical solutions by other methods. Numerical examples have shown the accuracy, efficiency and great potential of this method for structural analysis.

Key Words: Differential quadrature method, Numerical solution methods, Partial differential equations

GENELLEŞTİRİLMİŞ DİFERANSİYEL QUADRATURE METODUNUN YAPI ELEMANLARININ YER DEĞİŞTİRME VE BURKULMA ANALİZİNE UYGULANMASI

ÖZET

Richard Bellman tarafından 1970’lerde geliştirilen Diferansiyel Quadrature metodu, diferansiyel denklemlerin sonlu sayıdaki düğümlerinin, sayısal çözümüdür. Bu çalışmada Genelleştirilmiş Diferansiyel Quadrature metodunun, yapıların yer değiştirme ve burkulma analizine uygulanışı verildi. Ayrıca, kirişlerin tekil yük altındaki çökmesinin bu metoda uygulanabilirliği de gösterildi. Metodun en önemli bir avantajı, sistemlere basitçe uygulanabilirliği, hesaplama süresinin kısalığı ve programlamanın basitliğidir. Değişik sınır şartları ve farklı yüklemelerde elde edilen sonuçlar, mevcut gerçek ve sayısal sonuçlarla karşılaştırıldı. Elde edilen sonuçların çok ince hassasiyetle doğruluğu gösterildi.

Anahtar Kelimeler : Diferansiyel quadrature metodu, Sayısal çözüm metotları, Kısmi diferansiyel denklem

1. INTRODUCTION

Numerical solution procedures for the solution of partial differential equations have been of extreme importance to progress in many areas of engineering sciences. Admittedly the finite element method, the finite differences technique and the boundary element procedure are the almost universal methodologies, which allow for reliable and prompt solutions for practically and well-posed mathematical models. However, in a large number of practical applications where only reasonably accurate solutions at few specified physical co-ordinates are of interest, the finite element or the finite difference methods becomes inappropriate since they still require a large number of grid points and so large computer capacity. In addition, these methods require considerable skill from the analyst, especially if he is required to prepare the computer code.

In seeking a more efficient numerical method which requires fewer grid points yet achieves acceptable accuracy, the method of differential quadrature, which is based on the assumption that the partial derivatives of a function in one direction can be expressed as a weighted linear sum of all the function values at all mesh points along that direction, was introduced by Bellman (Bellman and Casti, 1971; Bellman et all., 1971). Since then, applications of differential quadrature method to various engineering problems have been investigated and their successes have demonstrated the potential of the method as an attractive numerical analysis technique (Bellman and Roth, 1979; Civan and Sliepcevich, 1986).

However, there exist some major difficulties, which are explained in section (2), in the application of the original method of differential quadrature proposed by Bellman (Bellman and Casti, 1971; Bellman et all., 1971). In order to overcome these difficulties, a method of generalized differential quadrature (GDQ) was developed by Shu and Richard (Shu and Richards, 1992) and has been applied to solve some problems in fluid dynamics. Preliminary results have shown the effectiveness and efficiency of the method.

This paper presents the generalized differential quadrature method and investigates its applications to the problems of deflection, buckling and vibration of structural elements. The applicability of this method to deflection analysis of beams due to a point load is also demonstrated. The weighting coefficients for the approximation of derivatives required in differential quadrature formulation are calculated in a very simple way without any restriction on the choice of grid points. Numerical implementation of the method is straightforward and different boundary conditions can be easily incorporated. This paper is also to explore the potential of the GDQ method as an accurate, efficient and simple numerical method for structural analysis.

2. GENERALIZED DIFFERENTIAL QUADRATURE

The method of differential quadrature is developed based on the assumption that the partial derivative of a function with respect to a space variable of a given discrete point can be expressed as a weighted linear sum of the function values at all discrete points in the domain of that variable. To illustrate the concept, let us consider the first derivative of a one-dimensional function f(x). Suppose x_i (i = 1, 2,..., N) are the grid points obtained by subdividing the x-variable into N discrete values, then the first derivative $\partial f(x)/\partial x$ at $x=x_i$ can be written as;

$$\frac{\partial f(x_i)}{\partial x} = \sum_{j=1}^{N} c_{ij}^{(1)} \cdot f(x_i) \text{ ; for i=1,2,..., N} \quad (1)$$

which $c_{ij}^{(1)}$ are the weighting coefficients of the first derivative.

Two extensively decisive factors in the accuracy of the differential quadrature solutions are; the accuracy of the weighting coefficients and the choice of grid points. In the original formulation of differential quadrature (Bellman and Casti, 1971; Bellman et all., 1971), two approaches were proposed. One assumes that the test functions $g_k(x)$ to be $g_k(x) = x^k$ (k = 1, 2,..., N-1), leading to a set of linear algebraic equations, which are called Vandermonde system of equations, from which the weighting coefficients can be determined. However Vandermonde matrices are known to be inherently ill conditioned and in fact, it is experienced that the weighting coefficients obtained by a direct solution of the Vandermonde equations become increasingly in accurate with an increasing number of grid points. The other approach assumes the test function to be the Nth order Legendre polynomial, leading to simple algebraic expression for the weighting coefficients. However, it requires that x_i (i = 1, 2, ..., N) have to be the roots of the shifted Legendre polynomial. This means that once the number of grid points N is specified, the roots of the shifted Legendre polynomial are given, thus the distribution of the grid points is fixed regardless of the physical problems being considered.

In order to find simple algebraic expressions for the weighting coefficients without restricting the choice of grid meshes, the generalized differential quadrature method was developed by Shu and Richards (Shu and Richards, 1992). In generalized differential quadrature, the test functions are assumed to be the Lagrange interpolated polynomial as;

$$g_k(x) = \frac{M(x)}{(x - x_k).M^{(1)}(x_k)} \text{ ; For k=1, 2, ..., N} \quad (2)$$

where $M(x)=\prod_{j=1}^{N}(x-x_j)$

and

$$M^{(1)}(x_i)=\frac{\partial M(x_i)}{\partial x}=\prod_{j=1,j\neq i}^{N}(x_i-x_j)\,.$$

Upon substitution of (2) into (1), the following relationship can be established (Forray and Newman, 1962).

$$c_{ij}^{(1)}=\frac{M^{(1)}(x_i)}{(x_i-x_j).M^{(1)}(x_j)}\;;\text{for } i\neq j,\ i=1,2,...,N$$

and

j = 1, 2,..., N (3)

$$c_{ii}^{(1)}=\frac{M^{(2)}(x_i)}{2.M^{(1)}(x_i)}\;;\text{ for } i=j,\ i=1,2,...,N \tag{4}$$

Equations (3) and (4) are very simple algebraic expressions for computing $c_{ij}^{(1)}$. There is no restriction in the choice of grid co-ordinates. However, the determination of $c_{ii}^{(1)}$ requires the availability of the second order derivative of M(x) which is more difficult to obtain. Instead of using (4) to calculate $c_{ii}^{(1)}$, a more convenient relationship can be established. It can be shown from Taylor series expansion that the following relationship holds for $c_{ij}^{(1)}$.

$$\sum_{j=1}^{N}c_{ij}^{(1)}=0\;;\text{for } i=1,2,...,N \tag{5}$$

Thus the coefficients $c_{ii}^{(1)}$ can be calculated as;

$$c_{ii}^{(1)}=-\sum_{j=1,j\neq i}^{N}c_{ij}^{(1)}\;;\text{ for } i=1,2,...,N \tag{6}$$

Similarly the weighting coefficients for the second and higher order derivatives can be computed. Again assume the mth order derivative can be expressed as;

$$\frac{\partial^m f(x_i)}{\partial x^m}=\sum_{j=1}^{N}c_{ij}^{(m)}\cdot f(x_i)\;;\text{ for } i=1,2,...,N \tag{7}$$

Then an amazing recurrence relationship can be established for the mth order weighting coefficients $c_{ij}^{(m)}$ when Lagrange interpolated polynomials are used as test functions.

$$c_{ij}^{(m)}=m\cdot\left(c_{ii}^{(m-1)}\cdot c_{ij}^{(1)}-\frac{c_{ij}^{(m-1)}}{x_i-x_j}\right);\text{for } i\neq j,\ m=$$
2, 3,...,N-1, i ,j = 1, 2,..., N (8)

The value of $c_{ii}^{(m)}$ can be obtained from the relationship similar to equation (6),

$$c_{ii}^{(m)}=-\sum_{j=1,j\neq i}^{N}c_{ij}^{(m)}\;;\text{for } i=1,2,...,N \tag{9}$$

To summarise, the recurrence relationships (8) and (9) together with the formulations for the coefficients of first derivatives (3) and (6) constitute complete formulae for the determination of the weighting coefficients from the first to as high as the (N-1)th order derivatives. This set of expressions for the determination of the weighting coefficients is so compact and simple and very easy to be implemented in formulating and programming because of the recurrence feature. All these features give a great convenience to the GDQ for solving practical problems in structural analysis.

Also, it should pointed out that the GDQ method can be used in structural analysis for solving both ordinary and partial differential equations The application of this method to static problems leads to a set of algebraic equations with the function values at grid points as unknowns, while its application to time dependent dynamic problems results in a set of ordinary differential equations with time dependent function values at grid points as unknowns, which can then be solved by an existing integration scheme. Finally, once the function values at all grids are obtained, it is very easy to determine the function values in the overall domain in terms of polynomial approximation,

$$f(x)=\sum_{i=1}^{N}f(x_i)\cdot g_i(x) \tag{10}$$

where $g_i(x)$ are Lagrange interpolated polynomials as expressed in (2).

A natural and often convenient choice for the grid points is that of the equally spaced sampling points. These are given in the normalized form in the x-direction as;

Type-1: Equally spaced sampling points.

$$X_i=\frac{i-1}{N_x-1}\;;\ i=1,2,...,N_x \tag{11}$$

Second type of selection for the grid points is the equally spaced sampling points with adjacent δ-

points. The adjacent points are necessary in some cases to implement the boundary conditions to the general equations. The closeness between the adjacent points could of the order $\delta \cong 10^{14}$ or $\delta = 10^{-5}$; then these points would virtually correspond to a single point, i. e., the boundary point itself. These points can be expressed in the normalized form as;

Type 2: Equally spaced sampling points with adjacent $\delta -$points.

$$X_1=0, \quad X_2=\delta, \quad X_{N-1}=1-\delta \quad X_N=1 \tag{12}$$

$$X_i = \frac{i-2}{N-3}\ ; i = 3, 4,..., N\text{-}2 \tag{13}$$

3. APPLICATIONS

The method of GDQ is used in the following for analyzing some static and dynamic structural problems. Firstly deflection of a beam due to a point load is considered. Secondly buckling behaviours of columns for different boundary conditions are analysed.

3. 1. Static Deflection of a Beam Due to a Point Load

For a Bernoulli-Euler beam in bending, the general form of the governing equation can be written in terms of deflection as;

$$\frac{d^2y}{dx^2} - \frac{M(x)}{E.I} = 0\ ;\ \text{constitutive relation} \tag{14}$$

where $EI \equiv$ flexural rigidity, $x \equiv$ position along the beam.

Now, lets consider the beam shown in Figure 1. The moment value along the beam is:

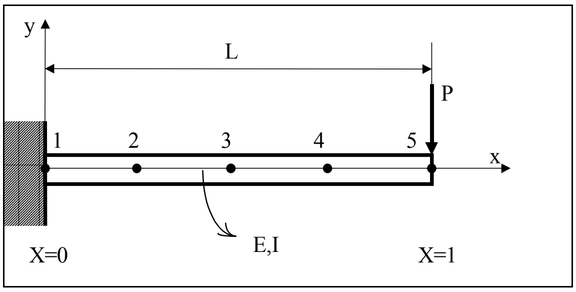

Figure 1. A cantilevered beam loaded at the free end with a point force

$$M(x) = P.(L-x) \tag{15}$$

From equation (14) and (15) the following equation can be obtained.

$$\frac{d^2y}{dx^2} = \frac{M(x)}{E.I} = \frac{P}{E.I}\cdot(L-x) \tag{16}$$

If the variables x and y are normalized as;

$$X = \frac{x}{L},\ \ Y = \frac{y}{\alpha}$$

with $\alpha =$ reference length and substitute in equation (16), the following equation can be obtained.

$$\frac{\alpha}{L^2}\cdot\frac{d^2Y}{dX^2} = \frac{P.L}{E.I}\cdot(1-X) \tag{17}$$

If α is selected as $\alpha = (P.L^3)/(E.I)$, equation (17) becomes,

$$\frac{d^2Y}{dX^2} = 1 - X \tag{18}$$

and then if this equation is differenciated once, the following equation can be obtained;

$$\frac{d^3Y}{dX^3} = -1 \tag{19}$$

The boundary conditions for the beam are;

$$Y=0,\ \frac{dY}{dX} = 0 \quad ; \quad \text{for} \quad X = 0 \tag{20}$$

$$\frac{d^2Y}{dX^2} = 0\ ;\ \text{for } X = 1 \tag{21}$$

Noting that $Y_1 = 0$ and choosing Type-1 grid points with N = 5 the equation (19) can be expressed in the GDQ form as;

$$\sum_{j=2}^{N} c_{ij}^{(3)}.Y_j = -1\ ;\text{for } i = 2,..., N \tag{22}$$

The solution of equation (22) with the boundary conditions gives the results. The analytical solution for this system is $Y = \frac{X^2}{6}\cdot(X-3)$. The obtained results are given in Table 1 with the exact deflection values. As can be seen, the numerical results are very accurate even using five grid points.

Table 1. Deflection Values of the Beam Shown in Figure 1

X	W(exact)	W(GDQ, N = 5)
0.0	0.0	0.0
0.25	-0.0286458333	-0.0286458333
0.5	-0.1041666667	-0.1041666667
0.75	-0.2109375000	-0.2109375000
1.0	-0.3333333333	-0.3333333333

3. 2. Column Buckling

Consider the problem of determining the critical buckling load of a slender elastic column shown in Figure 2.

For the buckling behaviour of a slender elastic column, the governing differential equation can be expressed as;

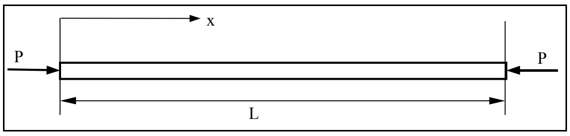

Figure 2a. Buckling of a slender elastic column

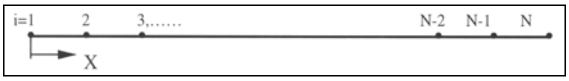

Figure-2b. First type selection of grid points

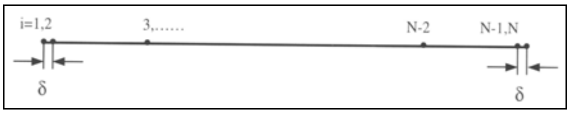

Figure 2c. Second type selection of grid points

$$\frac{d^2}{dx^2}\left(E\cdot I(x)\cdot\frac{d^2w}{dx^2}\right)+P\cdot\frac{d^2w}{dx^2}=0 \qquad (23)$$

Introducing the dimensionless variables, $X = x/L$, $W = w/a$, $a=\frac{f_0.L^4}{E.I}$ and substituting in (23) one obtains

$$\frac{d^2}{dX^2}\left(E\cdot I(X)\cdot\frac{d^2W}{dX^2}\right)=-P\cdot L^2\cdot\frac{d^2W}{dX^2} \qquad (24)$$

For a prismatic column this equation becomes,

$$W^{''''}=-\frac{P.L^2}{E.I}.W^{''} \qquad (25)$$

and GDQ analog of this equation is;

$$\sum_{j=1}^{N}c_{ij}^{(4)}W_j=-\frac{P.L^2}{E.I}\sum_{j=1}^{N}c_{ij}^{(2)}.W_j\ ;\ \text{for } i = 1, 2,..., N \qquad (26)$$

For a tapered (non-prismatic) column (Figure 3), the equation (24) can be written as;

$$I(X)\cdot W''''+2\cdot\frac{dI}{dX}\cdot W'''+\frac{d^2I}{dX^2}\cdot W''=-\frac{P\cdot L^2}{E}\cdot W'' \qquad (27)$$

The GDQ analog of equation (27) is,

$$I\cdot\sum_{j=1}^{N}c_{ij}^{(4)}\cdot W_j+2\cdot\frac{dI}{dX}\sum_{j=1}^{N}c_{ij}^{(3)}\cdot W_j+\frac{d^2I}{dX^2}\sum_{j=1}^{N}c_{ij}^{(2)}\cdot W_j=-\frac{P\cdot L^2}{E}\sum_{j=1}^{N}c_{ij}^{(2)}\cdot W_j \qquad (28)$$

for $i = 1, 2,..., N$ where I can be a given as a function of X.

Equation (26) and (28) can be written in matrix form as;

$$[A].\{W\}=\lambda\ .[B].\{W\} \qquad (29)$$

where $\lambda=-P.L^2/E.I_0$, $\{W\}=[W_1,W_2,...,W_N]^T$.

If the cross-sectional area is defined as follows ;

$$b(x) = b(0);\ \forall \qquad x\in[0,L]$$

$$h(x)=h(0)\cdot(\alpha\cdot x/L+1)\ ;\ \alpha\geq 0 \qquad (30)$$

Then the equation (28) can be written as follows.

$$\sum_{j=1}^{N}[(\alpha.X_i+1)^3.c_{ij}^{(4)}+6.\alpha.(\alpha.X_i+1)^2.c_{ij}^{(3)}+6.\alpha^2.(\alpha.X_i+1).c_{ij}^{(2)}].W_j=\lambda.\sum_{j=1}^{N}c_{ij}^{(2)}\cdot W_j \qquad (31)$$

for i = 1, 2,...,N. The equation (31) is the general equation for a tapered beam.

From equation (26) also the general equation for a prismatic beam can be obtained as:

$$\sum_{j=1}^{N} c_{ij}^{(4)}.W_j = \lambda.\sum_{j=1}^{N} c_{ij}^{(2)}.W_j \text{ ;for i=1,2,..., N} \qquad (32)$$

The buckling load can be obtained by solving the above eigen-value problems together with appropriate boundary conditions.

Three different sets of boundary conditions are considered for both prismatic and non-prismatic columns. For the grid selection of Type 1, boundary conditions for each case can be expressed as follows:

1) Simply supported at X = 0 and X = 1

$$W(0) = W''(0) = 0$$

$$W(1) = W''(1) = 0 \qquad (33)$$

Taking N nodes, the method of GDQ yields;

$$\sum_{j=2}^{N-1} c_{1j}^{(2)}.W_j = 0$$

$$\sum_{j=2}^{N-1} c_{Nj}^{(2)}.W_j = 0 \qquad (34)$$

2) Clamped at X = 0 and X = 1

$$W(0) = W'(0) = 0$$

$$W(1) = W'(1) = 0 \qquad (35)$$

Applying the method of GDQ, the following expressions are obtained.

$$\sum_{j=2}^{N-1} c_{1j}^{(1)}.W_j = 0$$

$$\sum_{j=2}^{N-1} c_{Nj}^{(1)}.W_j = 0 \qquad (36)$$

3) Clamped at X=0 and simply supported at X = 1

$$W(0) = W'(0) = 0$$

$$W(1) = W''(1) = 0 \qquad (37)$$

Applying the method of GDQ, the following expressions are obtained.

$$\sum_{j=2}^{N-1} c_{1j}^{(1)}.W_j = 0$$

$$\sum_{j=2}^{N-1} c_{Nj}^{(1)}.W_j = 0$$

(38)

The general equations for prismatic and tapered beams can be written for this 3 type boundary conditions from equations (31) and (32) as;

$$\sum_{j=2}^{N-1} c_{ij}^{(4)}.W_j = \lambda.\sum_{j=2}^{N-1} c_{ij}^{(2)}.W_j \text{ ; for i = 3,...,N-2 (for a prismatic beam)} \qquad (39)$$

$$\sum_{j=2}^{N-1} [(\alpha.X_i + 1)^3.c_{ij}^{(4)} + 6.\alpha.(\alpha.X_i + 1)^2.c_{ij}^{(3)} + 6.\alpha^2.(\alpha.X_i + 1).c_{ij}^{(2)}].W_j = \lambda.\sum_{j=2}^{N-1} c_{ij}^{(2)} \cdot W_j \qquad (40)$$

for i = 3,..., N-2 (for a tapered beam)

For the second type selection of grid points the general equations are same but the boundary conditions are a little different. For the same (3) cases they can be expressed as follows.

1-) Simply supported at X = 0 and X = 1

$$\sum_{j=2}^{N-1} c_{2j}^{(2)}.W_j = 0$$

$$\sum_{j=2}^{N-1} c_{(N-1)j}^{(2)}.W_j = 0 \qquad (41)$$

2-) Clamped at X = 0 and X = 1

$$\sum_{j=2}^{N-1} c_{2j}^{(1)}.W_j = 0$$

$$\sum_{j=2}^{N-1} c_{(N-1)j}^{(1)}.W_j = 0 \qquad (42)$$

3-) Clamped at X=0 and simply supported at X = 1

$$\sum_{j=2}^{N-1} c_{2j}^{(1)}.W_j = 0$$

$$\sum_{j=2}^{N-1} c_{(N-1)j}^{(1)}.W_j = 0 \qquad (43)$$

where the nodes i = 2 and i=N-1 are taken at a very small distance from the beam ends (see Figure 2 and Figure 3).

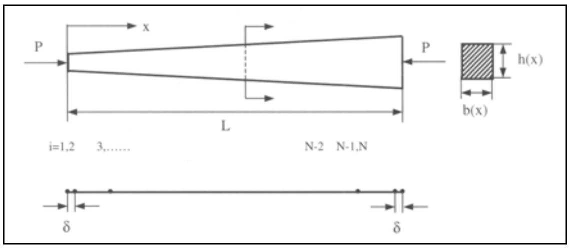

Figure 3. Buckling of a tapered slender elastic column

The solutions for the critical buckling loads can be obtained solving the boundary conditions with the general equations, individually.

The obtained buckling loads of the prismatic columns using the first and second type of grid points are listed in Table 2 and the solutions for the non-prismatic columns are given in Table 3, for different values of α .

For both type of columns, exact analytical results are given wherever available. The obtained results for tapered beams are compared with the finite element results. The results are in excellent agreement with the eigenvalues obtained using the finite element method.

For prismatic columns, calculations are performed for various number of grid points. As observed, the convergence of the solutions using GDQ is excellent. Comparison of the present results with the exact ones shows that the GDQ is a very accurate numerical technique.

Table 2. Critical Buckling Load Coefficients of Prismatic Columns

Boundary cond.	P_{cr} (Exact)	P_{cr} (FEM) Ref.10	Error (%)	Grid points	P_{cr} (GDQ) Type-1	Error (%)	P_{cr} (GDQ) Type-2 ($\delta=10^{-4}$)	Error (%)	P_{cr} (GDQ) Type-2 ($\delta=10^{-5}$)	Error (%)
Pinned-pinned	9.8696	9.9438	0.75	N=7	10.060718	1.936	9.97169	1.034	9.96816	0.998
				N=9	9.8641905	-0.055	9.871428	0.018	9.86787	-0.339
				N=11	9.8697017	0.001	9.87358	0.040	9.87003	0.004
Fixed-fixed	39.4784	39.9730	1.25	N=7	49.090909	24.34	42.44441	7.513	42.43486	7.489
				N=9	38.847825	-1.5957	39.35976	-0.300	39.34453	-0.339
				N=11	39.516455	0.096	39.50049	0.056	39.48629	0.02
Fixed-pinned	20.1421	20.4972	1.76	N=7	19.778356	-1.805	20.08122	-0.302	20.07296	-0.344
				N=9	20.254631	0.558	20.22468	0.410	20.21641	0.369
				N=11	20.186532	0.220	20.1986	0.280	20.19032	0.239

Table 3. The Critical Buckling Load Coefficients Of Non-Prismatic Columns ($\delta = 10^{-4}$, N = 11, Finite Element Results are Obtained Using 40 Elements)

GDQ Method					Finite Element Method		
Boundary conditions	α	0.1	0.2	0.3	0.1	0.2	0.3
Pinned-pinned		11.399	13.015	14.716	-	-	-
Fixed-fixed		45.403	51.968	58.661	-	-	-
Fixed-pinned		23.335	26.632	30.060	23.309	26.603	30.070

The generalized differential quadrature anologs of the boundary conditions and general equation can be written as

$$W_1 = 0$$

$$\sum_{j=2}^{N} c_{ij}^{2}.W_j = -\varpi^2.W_i \text{ ; for i = 2, 3,..., (N-1)}$$

$$\sum_{j=2}^{N} c_{Nj}^{1} W_j = 0 \qquad (46)$$

From this set of eigenvalue equations using the grid point selection of Type-1, the fundamental frequency can easily be evaulated. The obtained results are given in Table 4 from which the convergence of the method can easily be seen.

4. CONCLUSIONS

The numerical technique of generalized differential quadrature method for the solution of partial differential equations was introduced and used to solve some problems in structural analysis for various boundary conditions. The main advatages of the method are its inherent conceptual simplicity and the fact that easily programmable algoritmic expressions are obtained. The present method is seen to yield excellent results for the cases treated even when only a small number of grid points are used for the evaluation. And also boundary conditions are easy to be incorporated in the GDQM. The superb accuracy, efficieny and convenience of this method have shown the great potential of this method for being used in structural analysis.

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