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STABILITY OF ROTOR-BEARING SYSTEMS

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ABSTRACT

In various industrial applications there is a need for higher speed, yet reliably operating rotating machinery. A key factor in achieving this type of machinery continues to be the ability to accurately predict the dynamic response and stability of a rotor-bearing system. This paper introduces and explains the nature of rotordynamic phenomena from comparatively simple analytic models. Starting with the most simple rotor model that is supported in two rigid bearings at its ends, the more realistic and more involved cases are considered by incorporating the effects of flexible bearings. Knowledge of these phenomena is fundamental to an understanding of the behavior of complex models, which corresponds to the real rotors of turbomachines.

Key Words : Stability, Rotor-Bearing, Rotordynamics, Turbomachines

ROTOR- YATAK SİSTEMLERİNİN KARARLILIĞI

ÖZET

Endüstrideki çeşitli uygulamalarda yüksek hızının yanısıra güvenli olarak çalışan dönen rotorlu makinaya ihtiyaç vardır. Bu tür bir makinayı elde etmedeki anahtar faktörün rotor-yatak sisteminin dinamik tepkisini ve kararlılığını doğru tahmin edebilme olduğu geçerliliğini korumaktadır. Bu makale basit analitik modellerle rotor dinamiği kavramının doğasını tanıştırmakta ve açıklamaktadır. İki ucundan sabit yataklarla desteklenen en basit rotor modeli ile başlanarak, daha gerçekçi ve daha detaylı durumlar esnek yatak etkileri de katılarak ele alınmıştır. Bu olguların bilinmesi gerçek turbomakinaların rotorlarını temsil eden karmaşık modellerin davranışını anlamada esastır.

Anahtar Kelimeler : Kararlılık, Rotor-Yatak, Rotor dinamiği, Turbomakinalar

1. INTRODUCTION

Rotating machinery, one of the most important classes of machinery, is used extensively throughout the industrialized world. Its uses are extremely diverse: in power stations, aircraft engines, medical equipment and many other applications. Indeed, it is difficult to think of many types of machine that do not include rotating components in one form or another.

In operation the rotor undergoes bending and torsional vibration. The vibration of a rotor depends upon its geometry and the type of support, as well as on the excitation forces. The vibrating rotor also excites its foundation.

Failure of the machine components in applications such as aeroengines, turbogenerators, military equipment, space satellites and others, may put human life in jeopardy and cost a lot of money to repair. Therefore, in the design of high-speed rotating machinery, the following questions must be addressed:

a) For a given running speed, what are a rotor's natural frequencies?
b) What are the rotor's critical speeds?

c) What are the anticipated steady-state response levels over the rotor's operating range?
d) Will the rotor be stable over its operating range?

In order to address these questions, one must consider a complete rotordynamic model, which accounts for the rotor's structural-dynamic plus fluid-structure-interaction forces, seal generated forces, external time-dependent forces, imbalance, etc.

All rotating machinery is supported by one or more bearings, which play a vital part of the entire system, since it is the component that permits the relative motion between the stationary and moving parts. There are two general types of bearings which are commonly used in rotor-bearing system applications. These are fluid-film bearings and rolling-element bearings.

Bearings can have a significant effect on machine's vibration characteristics. The fluid-film of a fluid-film bearing acts like a spring-damper system and it influences the machine critical speeds and imbalance response. Moreover, bearing fluid-film forces can cause rotor instability that result in serious levels of self-excited vibration. Shaft seals have a similar effect as fluid-film bearings. They influence the critical speeds, can provide damping or on the other hand cause instability. Instability from fluid-film bearings and shaft seals arises from the fact that during radial displacement of the rotor a restoring force is produced, which has a component at right angles to this displacement.

Extensive studies of the rotor-bearing system over a long period of time have resulted in a good understanding of the forces induced by bearings. These studies have been incorporated into codes, which are used to design rotor systems, see for example (Szeri, 1980; Szeri, 1987; Childs, 1993; Kramer, 1993). In addition many theoretical studies, numerical calculations and measurements have been carried out to determine the effect of self-exited vibration in the turbomachinery due to shaft seals, see for example (Childs and Scharrer, 1988; Eser and Kazakia, 1995; Yucel, 2000; Kwanka, 2001;).

Here, we don't intend to consider the subject of the effects of the fluid-film bearings and shaft seals on the stability of rotors. More information about these subjects can be obtained at (Childs, 1993; Kramer, 1993).

In practice, instability must be avoided and one must know as much as possible about the conditions and about behavior during instability. Therefore, in the following sections the stability of rotor-bearing systems is considered. Starting with the most simple rotor model that is supported in two rigid bearings at its ends, more realistic and more involved cases are considered by incorporating the effects of flexible bearings.

2. STABILITY CONSIDERATIONS OF ROTOR-BEARING SYSTEMS

The material in this section is given to introduce and explain the nature of rotordynamic phenomena from comparatively simple analytic models. The phenomena demonstrated by flexible rotors and techniques employed for their analysis is basically similar to other areas of vibrations and structural dynamics.

The vibration problems can be represented by the equation of motion

$$M\ddot{X} + C\dot{X} + KX = F(t) \tag{1}$$

The simple harmonic vibration of the rotor is described with the terms $M\ddot{X}$ and KX of the above equation. Damping, either from the structure of the shaft or from the bearing structure is characterized by the term $C\dot{X}$. Imbalanced rotor effects are described by the forcing term $F(t)$ on the right hand side. Flexible bearings, hydrodynamic bearings and gas seals introduce terms of the form KX and $C\dot{X}$.

The complete solution of Eqn. (1) consists of the solution of the homogeneous equation together with the particular solution corresponding to the right-hand side. Solution of the homogeneous equation requires the eigenvalues of the system to be found. These are conjugate complex or real and characterize the natural vibration. The imaginary part corresponds to the natural frequency in question and the real part gives the stability of the natural vibration. For a negative real part, the vibration decays with time that means the system is stable, and for positive real part it grows which means it is unstable. The stability boundary of the system is reached when the real part of an eigenvalue is zero.

In the next two subsections, we consider simple rotor models to study their stability. The basic model used in this work is the Jeffcott rotor (Kramer, 1993). With this simple model most of the important results can be shown and explained analytically.

2. 1. Jeffcott Rotor With Rigid Bearings

The most simple rotor model consists of a heavy disk of mass m mounted at mid-span of a massless elastic shaft. The shaft is supported in two rigid bearings at its ends as shown in Figure 1. This model is called a Laval shaft or Jeffcott rotor (Rao, 1983; Goodwin, 1989; Childs, 1993; Kramer, 1993). The shaft has a circular cross-section with constant diameter over its whole length and turns with constant angular velocity ω.

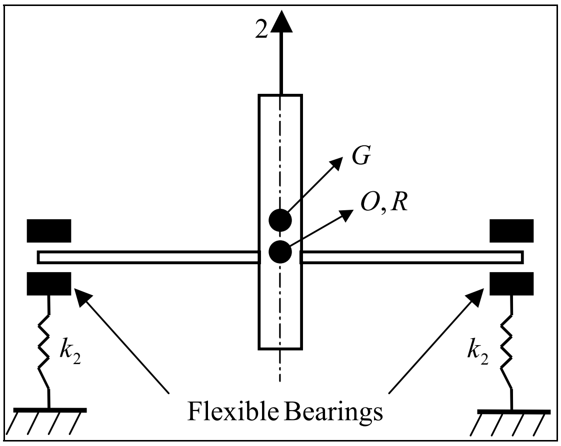

Figure 1. Schematic of Jeffcott rotor with rigid bearings

The mid-span disk has a center of mass that due to unbalance is at a point G, a distance e from the geometrical center (disc center) O; this distance is known as the eccentricity. The disk is assumed to move only in its own plane, more precisely in the plane defined by axis 1, 2 in Figure 2. The coordinates y_1, y_2 give the movement of the shaft center O relative to the unloaded position and the angle turned through by the disk is given by ωt.

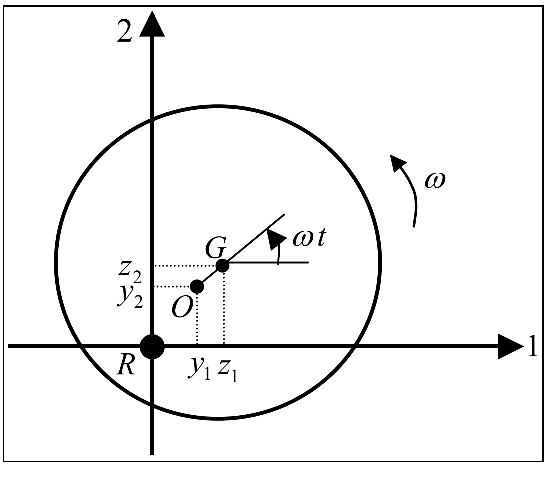

Figure 2. Assumed movement plane for the disk

The position of the midpoint G is determined from the equations

$$\begin{aligned} z_1 &= y_1 + e\cos\omega t \\ z_2 &= y_2 + e\sin\omega t \end{aligned} \tag{2}$$

The equations of motion for the mass-center G can be obtained by using Newton's law as

$$\begin{aligned} m\ddot{z}_1 + d\,\dot{y}_1 + k_r y_1 = 0 \\ m\ddot{z}_2 + d\,\dot{y}_2 + k_r y_2 = 0 \end{aligned} \tag{3}$$

where k_r is the shaft-stiffness coefficient and d is the damping coefficient. Substituting the second derivatives of Eqns. (2) into Eqns. (3), the following equations are obtained

$$\begin{aligned} m\ddot{y}_1 + d\,\dot{y}_1 + k_r y_1 = e\,m\,\omega^2\cos\omega t \\ m\ddot{y}_2 + d\,\dot{y}_2 + k_r y_2 = e\,m\,\omega^2\sin\omega t \end{aligned} \tag{4}$$

We first consider an ideally balanced disk that is for $e = 0$ the problem becomes very simple, as then the angle of rotation of the disk is independent of its displacement. Thus, for free vibration of the Jeffcott rotor Eqns. (4) become

$$\begin{aligned} m\ddot{y}_1 + d\,\dot{y}_1 + k_r y_1 = 0 \\ m\ddot{y}_2 + d\,\dot{y}_2 + k_r y_2 = 0 \end{aligned} . \tag{5}$$

The solutions to Eqns. (5) can be obtained as

$$\begin{aligned} y_1(t) = e^{-\delta t}\left(A_1\cos\omega_d t + B_1\sin\omega_d t\right) \\ y_2(t) = e^{-\delta t}\left(A_2\cos\omega_d t + B_2\sin\omega_d t\right) \end{aligned} \tag{6}$$

where $\delta = d/2m$ and $\omega_d = \sqrt{\omega_n^2 - \delta^2}$ with the natural frequency $\omega_n = \sqrt{k_r/m}$. In Eqns. (6), the coefficients A_1, A_2, B_1 and B_2 are real constants.

Equations (6) describe the path of the shaft center O in the 1-2 plane during free vibration, which is called natural motion. With zero damping this natural motion consists of harmonic vibrations in directions 1 and 2 with natural frequency ω_n. With damping the natural motion is similar to that without damping, except that the amplitudes decrease with time by the factor $e^{-\delta t}$.

If we consider the case of unbalance excitation that is $e \neq 0$, the particular solutions of the Eqns. (4) become

$$y_1(t) = r\cos(\omega t - \varepsilon)$$
$$y_2(t) = r\sin(\omega t - \varepsilon) \quad (7)$$

Where the whirl amplitude r at the disk is defined by $r = e\,V'(\eta)$ with

$$V'(\eta) = \frac{\eta^2}{\sqrt{(1-\eta^2)^2 + (2D\eta)^2}},$$
$$\eta = \omega/\omega_n, \quad D = d/(2m\omega_n) \quad (8)$$

and the phase angle ε is defined by $\varepsilon = \arctan\left(\frac{2D\eta}{1-\eta^2}\right)$. From Eqns. (7), it is seen that the shaft center moves with angular velocity ω in a circle of radius r. The angle between r and e remains constant.

The characteristic of the whirl amplitude $V'(\eta) = r/e$ versus excitation frequency ratio η for various values of the damping ratio D is shown in Figure 3. For small damping ($D<<1$), the maximum whirl amplitude is given to a good approximation by $V'_{max} \approx 1/(2D)$. The angular velocity at the maximum value is called critical and is usually assumed to be simplified to $\omega_c = \omega_n$. Correspondingly, the critical speed of the Jeffcott rotor is $n_c = \omega_n/(2\pi)$.

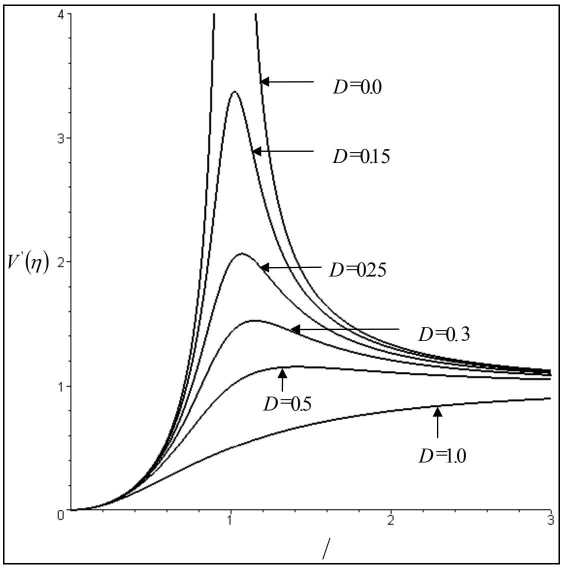

Figure 3. Dynamic magnification of rotor whirl amplitude as a function of speed for rigid bearings

For an undamped rotor, we notice from Figure 3 that resonance occurs when $\omega = \omega_n$. Hence, the rotor whirls with large amplitudes at resonance with the natural frequency of the stationary shaft in lateral vibration and hence this speed is called the "critical speed" of the rotor. For the Jeffcott rotor model, the rotor's critical speed is indistinguishable from its natural frequency; however, this is not generally the case.

In summary, the following conclusions can be drawn from Figure 3: First of all, at driving force of lower or higher speed, the amplitude response is much smaller. But, when the running speed is equal to the natural frequency of the system, the amplitude is magnified. Theoretically, the amplitude can build up to infinite values. The only thing that prevents this build up is damping. In other words, when operating at or near critical speed, damping is the only way to control the amplitude of vibration.

2. 2. Jeffcott Rotor with Flexible Bearings

In the previous subsection the bearings supporting the rotor have been assumed to be rigid. However, the bearings of real shafts are more or less flexible and have their special dynamic characteristics. But, these will not be investigated here in details. More detailed information about the dynamic characteristics of bearings can be found at (Szeri, 1980; Childs, 1993; Kramer, 1993).

In this subsection it will simply be assumed that a bearing can be replaced by massless springs in two mutually perpendicular radial directions (preferably horizontal and vertical) as shown in Figure 4. The two bearings have equal pairs of stiffnesses in directions 1 and 2, respectively (as shown in Figure 5).

When $k_1 \neq k_2$, the bearing is referred to as anisotropic. Using k_r as the stiffness of the shaft with stiff bearings, the total stiffness of the system of shaft and bearings, in directions 1 and 2, is

$$k'_1 = \frac{k_r}{1 + k_r/(2k_1)}, \quad k'_2 = \frac{k_r}{1 + k_r/(2k_2)} \quad (9)$$

Hence, the equations of motion for the Jeffcott rotor with flexible bearings become

$$m\ddot{y}_1 + d\dot{y}_1 + k'_1 y_1 = e\,m\,\omega^2 \cos\omega t$$
$$m\ddot{y}_2 + d\dot{y}_2 + k'_2 y_2 = e\,m\,\omega^2 \sin\omega t \quad (10)$$

The particular solutions of the above equations can be found as

$$y_1(t) = \hat{y}_1 \cos(\omega t - \varepsilon_1)$$
$$y_2(t) = \hat{y}_2 \sin(\omega t - \varepsilon_2) \quad (11)$$

Where $\hat{y}_i = e\,V_i'(\eta)$ with

$$V_i'(\eta_i) = \frac{\eta_i^2}{\sqrt{(1-\eta_i^2)^2 + (2D_i\eta_i)^2}},$$
$$\eta_i = \omega/\omega_i\,, \quad \omega_i = \sqrt{\frac{2k_i}{k_r + 2k_i}}\omega_n \quad (12)$$

and

$$D_i = \frac{d}{2m\omega_i},$$
$$\varepsilon_i = \arctan\left(\frac{2D_i\eta_i}{1-\eta_i^2}\right), \quad i = 1.2 \quad (13)$$

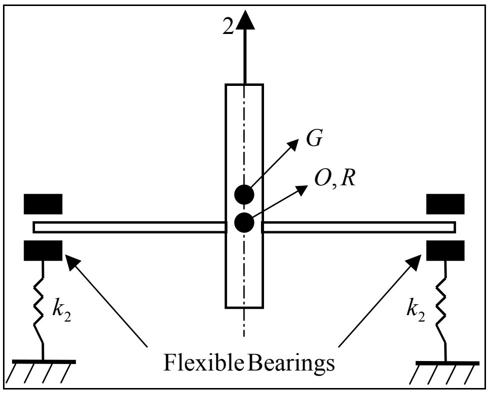

Figure 4. Schematic of Jeffcott rotor with flexible bearings

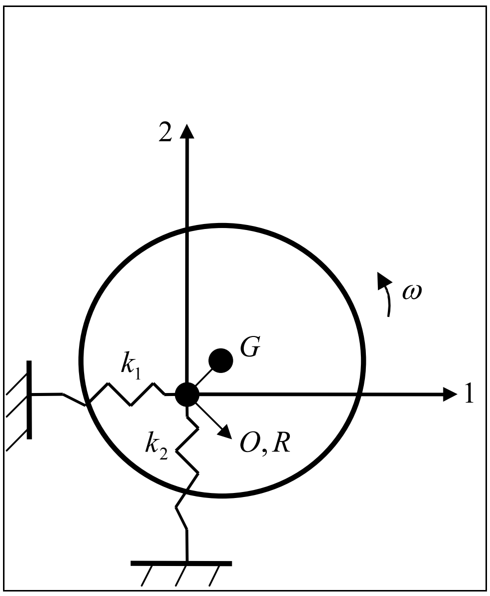

Figure 5. Bearing stiffnesses in directions 1 and 2

Accordingly, the disc center undergoes harmonic motion in directions 1 and 2, under the action of unbalance excitation, with a frequency equal to the shaft frequency and amplitudes $\hat{y}_1$ and $\hat{y}_2$ whose character corresponds to $V'(\eta)$ in Figure 3.

The characteristic of the whirl amplitudes $V_i'(\eta)$ versus excitation frequency ratios η_i for damping ratios $D_i = 0$ is shown in Figure 6. The plot is for $k_1 = 0.5k_r$ and $k_2 = 2k_r$. Because of different stiffnesses the shaft has two natural frequencies ω_1, ω_2 that is two critical speeds n_1, n_2. At the critical speeds the amplitude builds up to infinite values because of zero damping factor. The maximum amplitudes for small damping ($D_i << 1$, $i = 1,2$) are: $V_{i,max}' \approx 1/(2D_i)$.

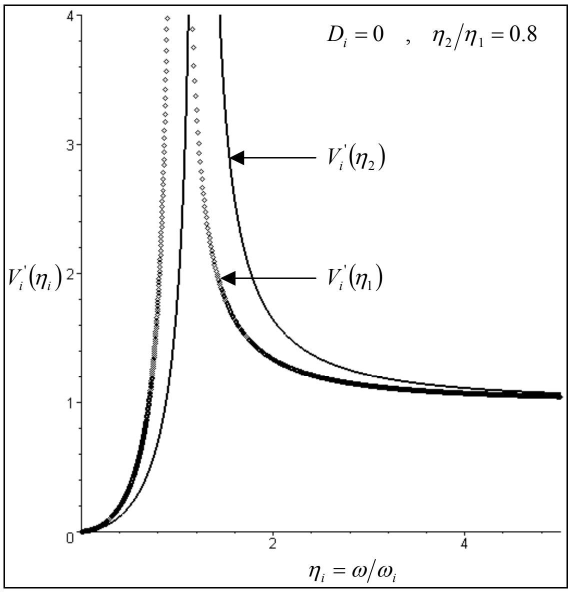

Figure 6. Dynamic magnification of rotor whirl amplitude as a function of speed for flexible bearings

With rigid bearings, the disc center describes a circle in plane 1-2 as discussed before. This is also the case for isotropic flexible bearings with $k_1 = k_2$, where $\hat{y}_2 = \hat{y}_1$ and $\varepsilon_2 = \varepsilon_1$. With anisotropic flexible bearings, that is $k_1 \neq k_2$, the path of the disc center is, in general, an ellipse whose shape and major axis direction depends on the shaft speed. Figure 7 shows the orbits of the shaft center O with some external damping force $D = 0.1$ for $k_1 = 0.5k_r$, $k_2 = 2k_r$. Here the elliptical orbit is roughly horizontal in the region of first critical speed ($n_1 \approx \omega_1/(2\pi)$), and roughly vertical in the region of second critical speed ($n_2 \approx \omega_2/(2\pi)$). For the values of $1.04n_1$ and $0.96n_2$ the orbit degenerates to a straight line. Below and above these speeds the direction of rotation of the orbit is the same as for the rotational speed of the shaft that is forward whirl. Between these values it is in backward whirl.

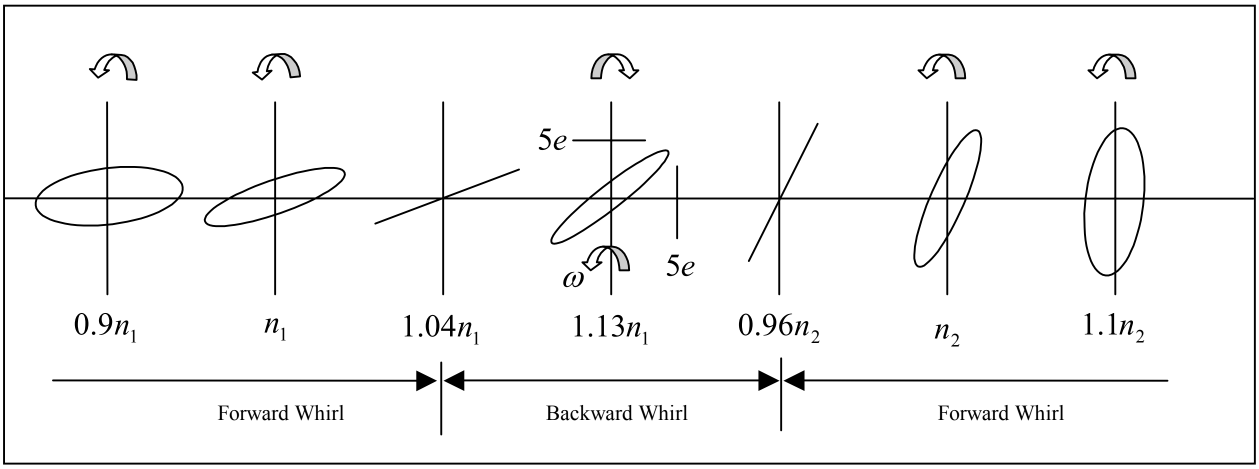

Figure 7. Orbits of the disk center O

3. CONCLUSIONS

In this study the model of Jeffcott rotor was used from which the most important phenomena encountered in rotordynamics were presented. Knowledge of these phenomena is fundamental to an understanding of the behavior of complex models, which corresponds to the real rotors of turbomachines. For a full rotordynamic analysis the forces due to the bearing fluid-structure-interaction, shaft seals, turbines and pump impellers must be included.

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