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Full Length Article

A proposal for measuring efficiency losses of asset management companies: Frontier-based approach

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Abstract

The performance of funds by asset management companies needs to be based on an objective benchmark. Studies related to the topic focus on measuring performance using a discrete form that cannot capture precise total efficiency losses. In this study, we propose a continuous approach to compare the performance of funds by taking advantage of the mean-variance efficient frontier with consideration of multiple risk levels. In an empirical analysis, our proposed method is applied to asset management companies with respect to open-end funds. For comparison, we use the output of an averaged Sharpe index. Because averaging the Sharpe index is inevitable for multiple risk levels, this method of calculation causes a loss of efficiency information. Hence, the proposed method has a continuous form of measuring performance, and the results of the two methods demonstrate significantly different patterns.

JEL classification: G11; G23; O16

Keywords: Efficiency loss; Financial performance; Mean-variance; Open-end funds

1. Introduction

Fund managers face several types of risks in financial markets, such as currency, legal and regulatory, and country risks that affect the performance of portfolio returns. Well-diversified funds can reduce those risks and are likely to have less volatility on average (Tkac, 2001, p. 2). The accomplishment of managers is directly related to how well they manage those risks. In general, fund managers receive compensation based on the returns of the funds that they manage. Therefore, managers with lower performance tend to increase portfolio risk in order to obtain higher compensation (Kempf et al., 2009). However, how well they manage the trade-off between risk and return is also important. Compensation contracts based on performance relative to a benchmark explicitly create incentives for managers (Elton et al., 2003; Ingersoll et al., 2007). Moreover, the relationship between fund inflows and performance can be seen implicitly as stimulus for those incentives (Alexander & Baptista, 2010; Basak et al., 2007; Khorana, 2001; Sensoy, 2009). The measurement of fund manager performance is one of the topics in the finance literature that receives the most attention (Han et al., 2021). These studies gained momentum in particular after the establishment of frameworks of modern portfolio theory (MPT) and capital asset pricing model (CAPM). In the light of these developments, researchers mainly focus on excess returns and the elimination of unsystematic risks as well as measuring managerial performance (Yan & Wu, 2020, p. 257).

Studies on measuring portfolio performance in the finance literature date to the 1960s and 1990s (Jensen, 1968; Modigliani & Modigliani, 1997; Sharpe, 1966; Sortino & Price, 1994; Treynor, 1965), using traditional performance

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measurement techniques. These techniques are intensively used in measuring the performance of individual funds. Many empirical studies evaluate the performance of managers depending upon individual fund performance (Carlson, 1970; Filip et al., 2015; Gjerde & Sættem, 1991; Khang & Miller, 2021; Lee & Rahman, 1990; Mains, 1977; Omag, 2010). The traditional performance measurement techniques have progressively improved, those techniques have been subject to several critiques, such as the inability to capture the managers' total excess returns, the presumption of completely well-diversified portfolios, their inadequacy for considering upside and downside risk, their ignoring the relationship among securities, the assumption of linearity, and the lack of assessment of multiple risk levels. Moreover, all traditional performance measurement techniques focus on the performance of individual securities and portfolios, rather than the performance of an entire asset management company.

The goal of this study is to measure the overall performance of asset management companies (i.e., managerial performance), rather than measuring individual fund performance. By considering an asset management company as a portfolio, we can assess the efficiency loss for each risk level. Furthermore, total efficiency loss for an asset management company is the sum of efficiency losses over all risk levels in terms of managerial performance, and a methodology is needed to reflect this kind of performance measurement. We believe that the paper makes two contributions to the literature. First, we established a benchmark frontier benefiting from the mean-variance (MV) approach to measuring the total efficiency loss of a company. Second, we propose a unique performance measurement methodology subject to continuous process unlike the traditional measurement methods.

In this study, we propose a methodology using the MV approach in which efficient frontiers are used to measure the overall performance of asset management companies. To do so, each asset management company (hereafter, "company") is considered a subset of all funds available in the market. Therefore, each company can be viewed as a portfolio. More precisely, the efficient frontier of each company is generated as a suboptimal frontier. In the next step, a benchmark efficient frontier is generated using all funds available in the funds market. In the final step, the company (suboptimal) efficient frontier is compared with the benchmark (fund market) efficient frontier in terms of the total efficiency loss of each company. Then, companies are sorted from the most to the least efficient by calculating efficiency scores using the efficiency loss values. To test the proposed method, in the empirical section of this study, we prefer open-end funds because of their total asset size and the dominant number of funds available in the market. In other words, the universe of empirical analysis is the open-end fund market.

The structure of this study is as follows. In Section 2, we review the literature on performance measurement techniques. In Section 3, we outline the theoretical framework in the proposed approach. In Section 4, we offer the results of the empirical application. The last section concludes with some final remarks.

2. Literature review on performance measurement techniques

Studies on measuring portfolio performances in the finance literature began to emerge in the 1960s. Studies conducted by Treynor (1965), Sharpe (1966), and Jensen (1968) are among pioneering works on the subject. These studies, as well as those by Sortino and Price (1994) and Modigliani and Modigliani (1997) in the 1990s, are called traditional performance measurement techniques. These techniques may differ in terms of how they evaluate risk because of the emphasis on the combination of expected returns, and risk is a key element in portfolio analysis (Sharpe, 1966). The risks in the models, such as systematic risk, standard deviation, and downside risk, are important in interpreting the results.

Another crucial part of performance measurement is a predetermined benchmark. The selection of the accurate benchmark is essential in assessing the performance of individual securities and portfolios. Roll (1992) states that if the benchmark is inefficient, then the managed portfolio will be inefficient as well. The benchmarks in traditional performance measurements are built on the ex-post security market line (SML), for example, the Jensen index and the Treynor index; the ex-post capital market line (CML), for instance, the Sharpe index and M-squared; and the target downside deviation, such as the Sortino ratio.

The Jensen index (1968) uses the ex-post SML as a benchmark. This index is the difference between the expected rate of return on the portfolio and its expected return if the portfolio were positioned on the SML.[1] However, several studies criticize the index as being insensitive to risk and market performance. In practice, the expected returns or beta coefficients should be estimated by the portfolios over several periods. Even if the sample estimates are perfectly accurate, the index may not provide evidence as to which fund has more skillful managers. A portfolio manager's performance is related to the magnitude of excess returns obtained by the manager, and the number of securities for which a manager can obtain excess returns. This index cannot provide evidence for both measures (Haugen, 1997, p. 313). Treynor index (1965), like the Jensen index, uses the ex-post SML as a benchmark. The index postulates that risk is produced by general market fluctuation and risk resulting from unique fluctuations in portfolio securities. To identify the risk due to market fluctuation, the index introduces a characteristic line, which defines the relationship between the rate of return for a portfolio over time and the rate of return for an appropriate market portfolio. To do so, the slope of the characteristic line measures the relative volatility of the portfolio returns, and this slope is the beta coefficient of the portfolio. According to the index, a higher beta characterizes a portfolio that is more sensitive to market returns and has higher market risk.[2] Nevertheless, the index has been criticized in some papers for having given no consideration to

[1] For the model and detailed information see Jensen (1968).

[2] For the model and detailed information, see Treynor (1965).

diversification measures. Using the beta coefficient as a risk measure says nothing about diversification of the portfolio and the performance of portfolio managers. Furthermore, the index implicitly assumes a completely diversified portfolio (Reilly & Brown, 2000, p. 1137). Even though the Treynor index has some advantages over the Jensen index, it is also insensitive to excess returns obtained by the manager.

In contrast, the Sharpe index (1966) uses the ex-post CML as a benchmark. The index is calculated by dividing the risk premium for the portfolio by its standard deviation. It measures the risk premium earned per unit of risk exposure.[3] According to the capital market theory, the Sharpe index uses total risk to compare assets to the CML, unlike the Treynor and Jensen indexes. Thus, the Sharpe index evaluates a portfolio manager based on the rate of return performance and diversification (Reilly & Brown, 2000, p. 1140). The critiques of the index in the literature are as follows. First, the index produces relative, but not absolute, rankings of portfolio performance. A return distribution may produce a string of very small but consistent gains, which produce a very high Sharpe index with very little return. Although that return distribution would not indicate a good investment opportunity, a high Sharpe index can be misleading. Second, another fundamental problem with the index is that although the return is a definite, observable, and meaningful characteristic, risk is not. It is true that the standard deviation can be calculated with any time series of return data, but its meaning will not be the same for all time series. For the standard deviation to be a meaningful statistic, all the time series must be generated with a process that is both stationary and parametric. Even if stationarity and parametricity criteria are met, the Sharpe index can have some negative characteristics. The standard deviation takes into account the distance of each return from the mean, positive or negative. Hence, large positive returns increase the perception of risk, though they could just as easily be negative. However, this might not be the case for a dynamic investment strategy. A manager who follows the Sharpe index in portfolio selection might omit large positive returns from the portfolio because of the perception of high risk and can increase the Sharpe index (Harding, 2002). Furthermore, the Sharpe index can measure performance only at one risk level. Different Sharpe index values may need to be generated and averaged when it is necessary to calculate the index for multiple risk levels. However, taking the average leads to a loss of information on outliers and causes misevaluation.

Like the Sharpe index, M-squared (1997) uses the ex-post CML as a benchmark. This measure simply takes a portfolio's average return and determines what it would have been if the portfolio had had the same degree of total risk as the market portfolio.[4] In this way, M-squared is an adjusted version of the Sharpe index. The Sharpe index is awkward to interpret when it is negative, however, the outcome of M-squared is in percentage returns and easy to interpret. But the M-squared measure is a linear function of the Sharpe index and therefore shares its disadvantages. Indeed, the measure strengthens linearity by producing rankings based on the risk-free rate (Cogneau & Hubner, 2009).

Unlike the techniques discussed above, the Sortino ratio (1994) uses the target downside deviation as a benchmark. In many ways, this ratio is a better technique for measuring and comparing the performance of managers whose investment exhibit skewness in their return distribution. In fact, the ratio is a modified version of the Sharpe index. It uses downside deviation, rather than standard deviation, as the measure of risk. To do so, it uses a threshold (or a required rate of return) specified by the investor, called the desired target return. Although the standard deviation is a measure of the dispersion of data around its mean, both above and below, the target downside deviation is a measure of the dispersion of data below some investor-selected target return (Rollinger & Hoffman, 2013). This adjustment in the Sortino ratio creates an important difference with the Sharpe index.[5] Some critics of the Sortino ratio claim that eliminating the upside returns from the risk calculation is incorrect because strongly positive returns somehow imply the inevitability of correspondingly strong negative returns. However, Sortino et al. (1999) took these critiques into account and updated the technique, proposing a new ratio in which the return is replaced with the upside potential. Another criticism of the ratio is that it is difficult to specify a single target return for investors with multiple targets. Moreover, some studies claim that the ratio does not accurately capture the risk of assets when there are few observed returns below the target (Rom & Ferguson, 1994). Furthermore, although the Sortino ratio is an adjusted version of the Sharpe index, it has the same problem in that it can measure performance at only one risk level. When several risk levels of investors are calculated, different ratio values may need to be determined and then averaged. Taking the average, however, results in the loss of outlier information and misevaluation.

To sum up, even though the traditional performance measurement techniques have progressive improvements, the criticism of them can be divided into six groups: (1) there is a problem with capturing the total excess return of the managers, (2) assumption of completely well-diversified portfolios, (3) inadequacy in considering the upside and downside risk, except for the Sortino ratio, (4) problem of ignoring the relationship among securities, that is, covariances, (5) assumption of linearity, and (6) lack of assessment of multiple risk levels. Furthermore, all traditional performance measurement techniques focus on the performance of individual securities and portfolios, not that of an asset management company overall.

Other studies (Alexander & Baptista, 2010; Basak et al., 2007; Jorion, 2003; Roll, 1992) use the MV efficient frontier

[3] For the model and detailed information, see Sharpe (1966).

[4] For the model and detailed information see Modigliani and Modigliani (1997).

[5] For the model and detailed information see Sortino and Hopelain (1980), Sortino and Van Der Meer (1991), and Sortino and Price (1994).

as a benchmark in measuring the performance of individual securities and portfolios. Roll (1992) and Jorion (2003) focus on tracking error variances (TEV) and create TEV frontiers in order to evaluate the relative performance of fund managers. The theoretical frameworks in these studies are based on the distance between benchmark and TEV efficient frontiers in a discrete manner. These innovative studies solve some of aforementioned problems. However, the difficulty of measuring the overall performance of asset management companies at multiple risk levels still exists. To the best of our knowledge, no study in the finance literature scrutinizes the overall performance of asset management companies at multiple risk levels.

3. Theoretical framework

To form the company and benchmark efficient frontiers, we take advantage of Markowitz's (1952) MV approach. The goal of the MV approach is to create a portfolio with the highest return at a given risk level, considering the relationship between the risks and returns of assets. The MV portfolio optimization model aims to determine the weights (w_i) of the given capital to be invested in each asset (i), where $i = 1, \ldots, m$ and $\sum_{i=1}^{m} w_i = 1$. The purpose of the model is to minimize the risk of returns in the entire portfolio, identified with its variance (standard deviation) while restricting the expected return of the portfolio to attain a specified value. Precisely, E_i is the expected return on the i^{th} asset, $\Omega = \sigma_{ij}$ is the covariance of returns between the i^{th} and j^{th} assets and all assets assumed to be risky ($\sigma_i^2 > 0$). Then, the frontier can be described as the set of portfolios that satisfy the constrained minimization problem (Merton, 1972):

$$min \frac{1}{2}\sigma^2 \tag{1}$$

Subject to

$$\sigma^2 = \sum_{i=1}^{m} \sum_{j=1}^{m} w_i w_j \sigma_{ij} \tag{2a}$$

$$E = \sum_{i=1}^{m} w_i E_i \tag{2b}$$

$$1 = \sum_{i=1}^{m} w_i \tag{2c}$$

Merton (1972) proved that the MV efficient frontier is a parabola in the phase space $[\sigma^2, E]$ and hyperbola in the phase space $[E, \sigma]$. Equations (3) and (4) represent the former and the latter, respectively. Panels A and B in Fig. 1 illustrate the graphs of the efficient frontier in two different phase spaces. In Fig. 1, the solid curves in Panels A and B represent the efficient frontiers.

$$\sigma^2 = f(E) = \frac{C}{D}E^2 - 2\frac{A}{D}E + \frac{B}{D} \tag{3}$$

$$E = f(\sigma) = \frac{A}{C} + \frac{1}{C}\sqrt{D(C\sigma^2 - 1)} \tag{4}$$

The parameters of the parabola (Panel A) and hyperbola (Panel B) are defined as A, B, C, and D. Setting V_{ij} is the inverse of σ_{ij}, *i.e.*, $\Omega^{-1} = V_{ij}$. Then,

$$A \equiv \sum_{i=1}^{n} \sum_{j=1}^{n} V_{ij} E_j \tag{5}$$

$$B \equiv \sum_{i=1}^{n} \sum_{j=1}^{n} V_{ij} E_j E_i \tag{6}$$

$$C \equiv \sum_{i=1}^{n} \sum_{j=1}^{n} V_{ij} \tag{7}$$

$$D \equiv BC - A^2 > 0 \tag{8}$$

Using Equations (5)–(8), the parameters of the parabola and hyperbola can easily be calculated. Then, the function of the efficient frontier is obtained for each phase space. Because the standard deviation is generally used to measure risk, we construct our model in the $[E, \sigma]$ phase space. More specifically, we build our model with hyperbola.

We employ two approaches to determine the performance (i.e., efficiency losses) of the companies. The first approach measures the performance of companies based on the standardized distance; the second approach is based on standardized area between the company and benchmark efficient frontiers. We call the approaches discrete performance measurement (DPM) and continuous performance measurement (CPM), respectively.

3.1. Discrete performance measurement

The DPM approach involves calculating the distance between the company and benchmark efficient frontiers for each specified risk level ($\sigma_c^k; k = 0, \ldots, N; c = 1, \ldots C; \sigma_c^{k=0} = \sigma_{c,min}, \sigma_c^{k=N} = \sigma_{c,max}$), where k and c denote each risk level and company, respectively. N is determined by τ, which is a constant difference in consecutive specified risk levels (Equation (9)). In other words, τ divides the distance between σ_{min} and σ_{max} into N pieces for a specified company, and it is arbitrary based on the sensitivity of the analyst. Fig. 2 illustrates the company $f_c(\sigma_c^k)$ and benchmark $f_m(\sigma_c^k)$ efficient frontiers and distances (Equation (10)) for different levels of specified risks.

$$N = \frac{1}{\tau}(\sigma_{c,max} - \sigma_{c,min}) \tag{9}$$

In the DPM, the distance between $f_c(\sigma_c^k)$ and $f_m(\sigma_c^k)$ is denoted by $d_{c,k}$, which is part of the proxy for firm performance, that is, the efficiency losses at a specified risk level. The efficiency loss here is an excess return that the fund manager cannot capture in the fund market.

$$d_{c,k} = f_m(\sigma_c^k) - f_c(\sigma_c^k) = E_{m,\sigma_c^k} - E_{c,\sigma_c^k} \tag{10}$$

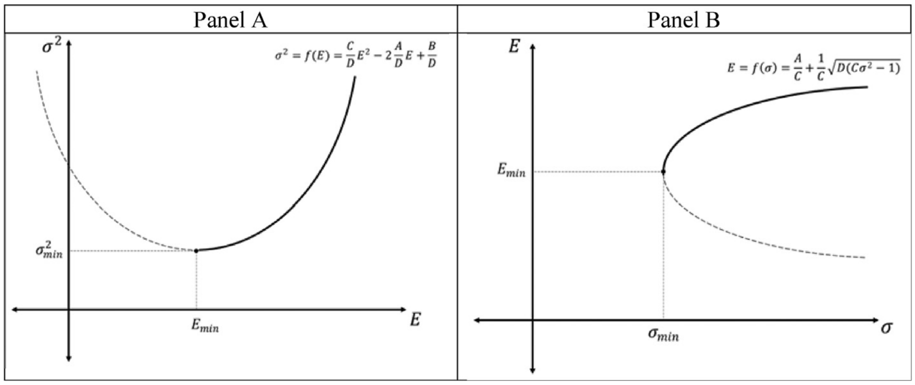

Fig. 1. The graphs of the efficient frontier in different phase spaces.

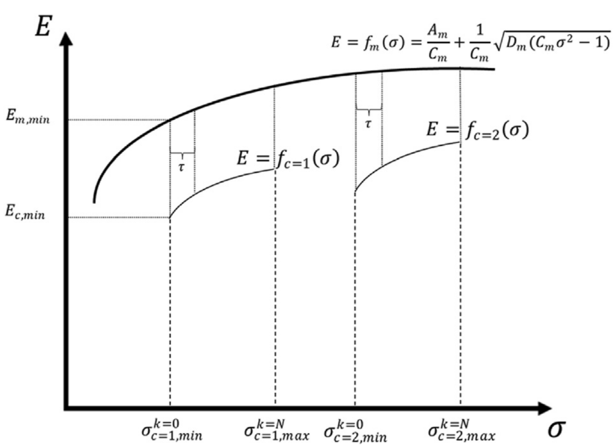

Fig. 2. Illustration of the discrete performance measurement.

Calculating only one $d_{c,k}$ value is not sufficient for measuring performance. To obtain accurate performance measurement, there needs to be $N+1$ values that comprise all risk levels based on τ. In order to assess the relative performance of companies, $N+1$ number of $d_{c,k}$ should be added up, and the sum is denoted by D_c in Equation (11).

$$D_c = \sum_{k=0}^{N} |d_{c,k}| \tag{11}$$

D_c by itself is a necessary assessment but not sufficient. Proper performance measurement is a standardized D_c; in other words, it is an average distance (AD_c), which is calculated with Equation (12).

$$AD_c = \frac{D_c}{N+1} \tag{12}$$

A zero value of AD_c means that the company has the same performance as with the benchmark efficient frontier. To compare the efficiency of the companies, AD_c values should be sorted in ascending order ($AD = \{AD_{min}, \ldots, AD_i, AD_j, \ldots, AD_{max}\}$ where $AD_i \leq AD_j$). In a further step, the AD values are converted into efficiency scores ($0 \leq S \leq 1$) using Equation (13). If AD_i equals AD_{min}, then S will have a value of zero, indicating the most efficient company. But if AD_i equals AD_{max}, then S will have a value of one, implying the least efficient company.

$$S = \frac{AD_i - AD_{min}}{AD_{max} - AD_{min}} \tag{13}$$

As the sensitivity level of the analyst increases, the value of τ approaches zero. In fact, to determine the precise total efficiency loss, it is necessary to make τ approach zero. Consequently, the problem turns into an integration (continuous) problem. The next section introduces the continuous solution to the problem.

3.2. Continuous performance measurement

The CPM is a way to compare relative performance based on areas between the company and benchmark efficient frontiers. Fig. 3 is a simple illustration of this situation. Calculating the areas (total efficiency losses [TEL]) requires a mathematical function of each efficient frontier. By assumption, $f_m(\sigma)$ denotes the function of a benchmark efficient frontier, and $f_c(\sigma)$ denotes the company efficient frontier, then the total efficiency loss of company c (TEL_c) is the integration of the difference between these two functions (Equation (14)), and the standardized area is denoted by $TELS_c$ (Equation (15)).

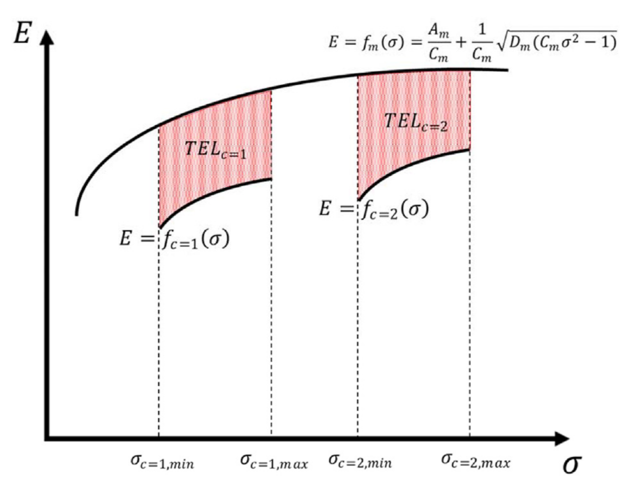

Fig. 3. Illustration of the continuous performance measurement.

$$TEL_c = \int_{\sigma_{c,min}}^{\sigma_{c,max}} \left[f_m(\sigma) - f_c(\sigma)\right] d\sigma \tag{14}$$

$$TELS_c = \frac{TEL_c}{\sigma_{c,max} - \sigma_{c,min}} \tag{15}$$

In Fig. 3, the x-axis represents the risk levels (σ), and the y-axis represents the returns on the portfolios (E) in which each company is considered a suboptimal portfolio. The solution is the integration of Equation (14). Defining $\psi = \frac{A}{C}$, $\xi = \frac{\sqrt{D}}{C}$, and $\gamma = C$, then, Equation (4) can be rearranged as Equation (16).

$$E = f(\sigma) = \psi + \xi\sqrt{(\gamma\sigma^2 - 1)} \tag{16}$$

Integrating Equation (16) with respect to σ and substituting it in Equation (14) results in Equation (17) (see Appendix A). TEL denotes the total efficiency loss of company c ($c = 1, \ldots, C$).[6]

$$TEL_c = \left[\frac{\xi_m\sigma\sqrt{\gamma_m\sigma^2 - 1}}{2} - \frac{\xi_m \operatorname{acosh}\left(\sqrt{\gamma_m}\sigma\right)}{2\sqrt{\gamma_m}}\right] - \left[\frac{\xi_c\sigma\sqrt{\gamma_c\sigma^2 - 1}}{2} - \frac{\xi_c \operatorname{acosh}\left(\sqrt{\gamma_c}\sigma\right)}{2\sqrt{\gamma_c}}\right] + \left[\psi_m\sigma - \psi_c\sigma\right] \Bigg|_{\sigma_{c,min}}^{\sigma_{c,max}} \tag{17}$$

As with the DPM, a zero value of $TELS_c$ means a company has the same performance as with the benchmark efficient frontier. Therefore, company performance can be sorted as $TELS = \{TELS_{min}, \ldots, TELS_i, TELS_j, \ldots, TELS_{max}\}$ where $TELS_i \le TELS_j$. In order to calculate the efficiency scores, Equation (18) can be used ($0 \le S \le 1$). As shown in Equation (18), the calculation of accurate scores requires standardization of $TELS$ values; Equation (15) can be used to do so. If $TELS_i$ equals $TELS_{min}$ then S will have a value of zero, indicating the most efficient company. However, if $TELS_i$ equals $TELS_{max}$ then S will have a value of one, implying the least efficient company.

$$S = \frac{TELS_i - TELS_{min}}{TELS_{max} - TELS_{min}} \tag{18}$$

Based on the theoretical framework, the paper makes two contributions to the literature. First, we established a benchmark frontier benefiting from the MV approach to measuring the total efficiency loss of a company. Second, we propose a unique performance measurement methodology subject to continuous process unlike the traditional measurement methods. More precisely, an accurate assessment of managers' (*i.e.*, companies') performances leads to a continuous summation of efficiency losses at each different risk level. On this point, we questioned how the level of minimum efficiency loss is calculated for a company. Obviously, due to a company has its own universe (sub-optimal) to create its portfolios, there is no way to reach a zero-efficiency loss, theoretically.[7] But it is possible to calculate the minimum level of efficiency loss of a company by using the total efficiency loss function (Equation (17)). Then, managers can position their portfolios to reach the minimum efficiency loss (see Appendix B).

Table 1
The details of narrowing windows.

Windows	Number of companies	Number of funds	Number of observations for each fund
2009–2020	36	178	2915
2010–2020	35	303	2654
2011–2020	34	326	2393
2012–2020	39	402	2133
2013–2020	40	435	1872
2014–2020	38	470	1611
2015–2020	34	454	1350
2016–2020	36	419	1089
2017–2020	32	416	828
2018–2020	31	358	568
2019–2020	28	329	307

4. Empirical application

To illustrate the practical utility of our methodology, we consider an application of the CPM approach using open-end funds data for Turkiye. In this section, after the results of application of the CPM approach are presented, we submit the details on the data. To compare the CPM results, the average Sharpe index is calculated using multiple risk levels.

4.1. Sample data

To test the CPM approach, we use open-end funds because of the total asset size and the dominant number of funds available in the market. The original data comprise 756 open-end funds from 70 asset management companies. However, 79 open-end funds managed by 23 companies were excluded due to missing data. The final data consist of the daily returns on 677 open-end funds for 47 asset management companies. The data gathered from Bloomberg professional data terminals for the period 2009–2020. To see the time effect, we set our sample up in eleven narrowing windows (2009–2020, 2010–2020; …, 2019–2020) (Table 1).

4.2. Evidence from the CPM and the Sharpe index

In this section, in order to provide some comparative evidence, we compare the efficiency scores of the Sharpe index, which is a commonly used technique in the finance literature, and the efficiency scores of the CPM. However, the CPM and

[6] cosh(α) is hyperbolic cosine of α; acosh(α) is the inverse of cosh(α), *i.e.*, acosh(α) = $cosh^{-1}(\alpha)$.

[7] The efficiency loss of a company is zero if and only if the universe of a company manager equals the universe of the fund market, which is impossible in the real world.

Table 2
The performance scores of asset management companies based on the CPM.

2009–2020		2010–2020		2011–2020		2012–2020		2013–2020		2014–2020		2015–2020		2016–2020		2017–2020		2018–2020		2019–2020	
COM25	0.0000	COM25	0.0000	COM25	0.0000	COM65	0.0000	COM65	0.0000	COM25	0.0000	COM25	0.0000	COM28	0.0000	COM17	0.0000	COM23	0.0000	COM23	0.0000
COM20	0.0005	COM65	0.0012	COM63	0.0011	COM63	0.0039	COM20	0.0008	COM64	0.0033	COM28	0.0144	COM67	0.0081	COM31	0.0311	COM10	0.0063	COM66	0.0019
COM63	0.0006	COM63	0.0022	COM37	0.0035	COM37	0.0269	COM63	0.0015	COM20	0.0047	COM70	0.0267	COM62	0.0100	COM62	0.0417	COM33	0.0334	COM13	0.0243
COM65	0.0011	COM37	0.0076	COM34	0.0038	COM64	0.0270	COM37	0.0024	COM49	0.0052	COM67	0.0272	COM58	0.0113	COM34	0.0457	COM45	0.0344	COM62	0.0572
COM64	0.0018	COM34	0.0077	COM38	0.0049	COM34	0.0361	COM64	0.0037	COM24	0.0180	COM62	0.0291	COM5	0.0126	COM58	0.0611	COM58	0.0428	COM33	0.0592
COM37	0.0020	COM31	0.0090	COM8	0.0060	COM20	0.0475	COM61	0.0101	COM33	0.0229	COM58	0.0324	COM69	0.0134	COM5	0.0693	COM34	0.0430	COM10	0.0663
COM53	0.0023	COM58	0.0136	COM24	0.0060	COM53	0.0611	COM24	0.0113	COM2	0.0263	COM21	0.0339	COM17	0.0151	COM66	0.0743	COM70	0.0457	COM58	0.0801
COM31	0.0024	COM67	0.0139	COM58	0.0064	COM61	0.0717	COM58	0.0126	COM67	0.0270	COM69	0.0345	COM24	0.0165	COM32	0.0755	COM62	0.0478	COM52	0.0857
COM34	0.0035	COM10	0.0141	COM67	0.0068	COM38	0.0860	COM35	0.0129	COM62	0.0294	COM8	0.0367	COM33	0.0170	COM55	0.0927	COM26	0.0540	COM40	0.0889
COM58	0.0037	COM33	0.0145	COM33	0.0070	COM35	0.1018	COM25	0.0135	COM58	0.0324	COM24	0.0386	COM32	0.0179	COM45	0.0932	COM31	0.0704	COM28	0.1000
COM10	0.0037	COM8	0.0165	COM31	0.0073	COM58	0.1053	COM33	0.0136	COM69	0.0359	COM32	0.0402	COM38	0.0195	COM10	0.0954	COM5	0.0735	COM7	0.1067
COM33	0.0039	COM69	0.0208	COM70	0.0085	COM31	0.1242	COM67	0.0141	COM8	0.0360	COM38	0.0416	COM45	0.0200	COM15	0.1063	COM40	0.0792	COM31	0.1178
COM67	0.0042	COM45	0.0208	COM62	0.0089	COM67	0.1261	COM31	0.0146	COM45	0.0409	COM45	0.0430	COM10	0.0204	COM61	0.1066	COM9	0.0826	COM34	0.1348
COM38	0.0053	COM62	0.0213	COM69	0.0100	COM8	0.1693	COM8	0.0209	COM32	0.0418	COM68	0.0438	COM55	0.0208	COM68	0.1225	COM55	0.0852	COM26	0.1488
COM35	0.0054	COM32	0.0214	COM45	0.0100	COM23	0.1767	COM45	0.0230	COM7	0.0435	COM17	0.0439	COM7	0.0210	COM21	0.1309	COM32	0.0852	COM42	0.1498
COM8	0.0055	COM7	0.0219	COM32	0.0106	COM45	0.1883	COM69	0.0232	COM66	0.0456	COM31	0.0439	COM66	0.0218	COM56	0.1333	COM15	0.1005	COM3	0.1650
COM45	0.0055	COM28	0.0241	COM7	0.0107	COM69	0.1932	COM32	0.0240	COM15	0.0468	COM61	0.0441	COM61	0.0245	COM9	0.1548	COM61	0.1090	COM35	0.1835
COM69	0.0056	COM66	0.0248	COM66	0.0120	COM62	0.1981	COM55	0.0243	COM35	0.0504	COM7	0.0444	COM26	0.0261	COM28	0.1687	COM68	0.1099	COM61	0.1896
COM62	0.0057	COM61	0.0278	COM10	0.0124	COM32	0.2112	COM28	0.0246	COM68	0.0507	COM35	0.0449	COM52	0.0262	COM35	0.1812	COM56	0.1221	COM70	0.2131
COM7	0.0058	COM68	0.0281	COM40	0.0150	COM7	0.2115	COM2	0.0247	COM28	0.0507	COM33	0.0456	COM15	0.0273	COM38	0.1819	COM67	0.1240	COM32	0.2132
COM24	0.0062	COM35	0.0283	COM5	0.0660	COM2	0.2138	COM68	0.0256	COM31	0.0507	COM66	0.0458	COM70	0.0292	COM67	0.1853	COM21	0.1408	COM15	0.2338
COM70	0.0063	COM38	0.0287	COM61	0.2090	COM33	0.2147	COM7	0.0258	COM17	0.0509	COM34	0.0460	COM35	0.0314	COM3	0.1956	COM35	0.1875	COM2	0.2432
COM28	0.0065	COM17	0.0294	COM68	0.2092	COM55	0.2290	COM38	0.0262	COM61	0.0511	COM15	0.0461	COM21	0.0342	COM70	0.1986	COM38	0.1933	COM68	0.2618
COM40	0.0083	COM40	0.0307	COM28	0.2092	COM28	0.2293	COM34	0.0264	COM38	0.0513	COM10	0.0469	COM34	0.0358	COM26	0.2132	COM28	0.1933	COM67	0.2999
COM21	0.0084	COM70	0.0312	COM35	0.2093	COM66	0.2369	COM17	0.0269	COM34	0.0519	COM40	0.0492	COM9	0.0482	COM33	0.2439	COM2	0.2251	COM21	0.3509
COM17	0.0090	COM24	0.0313	COM17	0.2098	COM10	0.2442	COM10	0.0283	COM70	0.0556	COM3	0.0495	COM40	0.0597	COM52	0.2687	COM52	0.2510	COM38	0.3510
COM55	0.0132	COM55	0.0326	COM3	0.2108	COM68	0.2612	COM15	0.0284	COM40	0.0559	COM5	0.0508	COM23	0.0814	COM40	0.2724	COM43	0.5860	COM9	0.3905
COM61	0.0158	COM3	0.0327	COM55	0.2111	COM3	0.2636	COM66	0.0287	COM3	0.0563	COM55	0.0516	COM31	0.1007	COM23	0.3001	COM12	0.7709	COM43	1.0000
COM68	0.0158	COM21	0.0342	COM21	0.2117	COM17	0.2755	COM40	0.0290	COM10	0.0564	COM52	0.0518	COM12	0.1239	COM43	0.3085	COM3	0.9929		
COM3	0.0170	COM26	0.0343	COM26	0.2117	COM40	0.2922	COM3	0.0292	COM21	0.0583	COM26	0.0520	COM6	0.1458	COM12	0.4213	COM13	0.9989		
COM32	0.0171	COM52	0.0447	COM42	0.2119	COM24	0.2940	COM70	0.0294	COM55	0.0588	COM42	0.0585	COM3	0.2970	COM13	0.9915	COM42	1.0000		
COM26	0.0173	COM42	0.0717	COM52	0.2144	COM70	0.2957	COM62	0.0296	COM26	0.0614	COM9	0.0860	COM68	0.2978	COM42	1.0000				
COM56	0.0196	COM56	0.0734	COM15	0.6685	COM21	0.3035	COM23	0.0300	COM42	0.0643	COM23	0.1283	COM13	0.3032						
COM66	0.0352	COM5	0.1577	COM56	1.0000	COM26	0.3318	COM21	0.0304	COM52	0.0674	COM56	1.0000	COM42	0.3059						
COM5	0.0401	COM15	1.0000			COM42	0.3340	COM26	0.0329	COM5	0.0710			COM46	0.4231						
COM15	1.0000					COM52	0.4445	COM42	0.0343	COM9	0.0892			COM56	1.0000						
						COM9	0.5187	COM52	0.0499	COM23	0.1308										
						COM56	0.9966	COM9	0.0576	COM56	1.0000										
						COM5	1.0000	COM5	0.0959												
								COM56	1.0000												

COM represents asset management companies; COM(c) refers to the *c*th company.

Table 3
The performance scores of asset management companies based on the Sharpe index and the CPM.

2009–2020			2010–2020			2011–2020			2012–2020			2013–2020			2014–2020			2015–2020			2016–2020			2017–2020			2018–2020			2019–2020		
CPM	Avg. Sharpe		CPM	Avg. Sharpe		CPM	Avg. Sharpe		CPM	Avg. Sharpe		CPM	Avg. Sharpe		CPM	Avg. Sharpe		CPM	Avg. Sharpe		CPM	Avg. Sharpe		CPM	Avg. Sharpe		CPM	Avg. Sharpe		CPM	Avg. Sharpe	
COM25	COM31	0.2559	**COM25**	**COM25**	0.3329	**COM25**	COM31	0.2819	**COM65**	COM31	0.3016	**COM65**	COM31	0.4878	**COM25**	**COM25**	0.5006	**COM25**	**COM25**	0.3895	**COM28**	**COM28**	0.1478	**COM17**	**COM31**	0.3006	**COM23**	**COM23**	0.8520	**COM23**	**COM23**	0.9072
COM20	COM61	0.1495	**COM65**	COM31	0.3047	**COM63**	**COM25**	0.2576	**COM63**	COM61	0.2186	**COM20**	COM25	0.1723	**COM64**	COM31	0.2819	**COM28**	COM31	0.3310	**COM67**	COM61	0.1287	**COM31**	**COM62**	0.1876	**COM10**	**COM10**	0.5802	**COM66**	**COM66**	0.8189
COM63	COM35	0.1433	**COM63**	**COM63**	0.1904	**COM37**	**COM63**	0.1737	**COM37**	COM68	0.2032	**COM63**	COM35	0.1707	**COM20**	COM35	0.2236	**COM70**	COM68	0.1769	**COM62**	**COM67**	0.1208	**COM62**	**COM17**	0.1458	**COM33**	COM34	0.3990	**COM13**	COM40	0.5032
COM65	COM28	0.1395	**COM37**	COM61	0.1691	**COM34**	COM61	0.1677	**COM64**	COM35	0.1911	**COM37**	COM68	0.1408	**COM49**	COM68	0.1705	**COM67**	**COM28**	0.1543	**COM58**	**COM62**	0.1190	**COM34**	**COM58**	0.1137	**COM45**	**COM33**	0.3501	**COM62**	**COM33**	0.4041
COM64	COM68	0.1368	**COM34**	**COM37**	0.1661	**COM38**	COM68	0.1517	**COM34**	**COM63**	0.1460	**COM64**	COM61	0.1271	**COM24**	COM28	0.1359	**COM62**	COM35	0.1380	**COM5**	COM17	0.1131	**COM58**	COM28	0.1094	**COM58**	COM26	0.3148	**COM33**	COM61	0.3731
COM37	COM37	0.1367	COM31	COM68	0.1536	COM8	**COM37**	0.1497	COM20	COM28	0.1386	COM61	COM33	0.1133	COM33	COM33	0.1258	COM58	COM17	0.1231	COM69	COM31	0.1104	COM5	COM21	0.1084	COM34	COM40	0.2845	COM10	COM26	0.3670
COM53	**COM25**	0.1296	COM58	COM58	0.1462	COM24	COM35	0.1391	COM53	COM67	0.1316	COM24	COM24	0.1096	COM2	COM67	0.1163	COM21	COM61	0.1223	COM17	**COM58**	0.1096	COM66	**COM34**	0.1053	COM70	COM70	0.2823	COM58	**COM13**	0.3533
COM31	**COM63**	0.1116	COM67	COM35	0.1403	COM58	COM62	0.1263	COM61	**COM64**	0.1305	COM58	**COM64**	0.1079	COM67	COM61	0.1126	COM69	**COM70**	0.1171	COM24	COM35	0.1089	COM32	COM67	0.0992	COM62	COM31	0.2807	COM52	COM10	0.3525
COM34	COM53	0.0955	COM10	COM28	0.1391	COM67	COM70	0.1261	COM38	COM38	0.1282	COM35	**COM63**	0.1068	COM62	COM62	0.1077	COM8	**COM67**	0.1098	COM33	COM21	0.0868	COM55	COM66	0.0992	COM26	**COM45**	0.2724	COM40	COM35	0.3353
COM58	COM40	0.0904	COM33	**COM65**	0.1297	COM33	COM28	0.1189	COM35	**COM37**	0.1192	COM25	COM67	0.1031	COM58	**COM20**	0.1000	COM24	**COM62**	0.1090	COM32	COM33	0.0867	COM45	COM61	0.0971	COM31	COM5	0.2276	COM28	COM34	0.3230
COM10	**COM20**	0.0892	COM8	COM67	0.1211	COM31	COM8	0.1131	COM58	COM58	0.1186	COM33	**COM20**	0.1006	COM69	COM58	0.0985	COM32	COM58	0.1012	COM38	COM38	0.0815	COM10	COM5	0.0849	COM5	COM68	0.2161	COM7	COM31	0.3228
COM33	**COM64**	0.0879	COM69	COM8	0.1125	COM70	**COM38**	0.1112	COM31	COM3	0.1133	COM67	**COM37**	0.0975	COM8	**COM64**	0.0933	COM38	COM38	0.0955	COM45	COM70	0.0807	COM15	COM32	0.0826	COM40	COM61	0.2100	COM31	COM52	0.3119
COM67	**COM65**	0.0875	COM45	COM33	0.1115	COM62	COM67	0.1090	COM67	COM66	0.1122	COM31	COM28	0.0961	COM45	COM17	0.0926	COM45	COM21	0.0947	COM10	**COM5**	0.0776	COM61	COM35	0.0826	COM9	COM55	0.2086	COM34	COM3	0.3041
COM38	COM55	0.0855	COM62	**COM34**	0.1107	COM69	COM40	0.1081	COM8	**COM65**	0.1039	COM8	COM58	0.0925	COM32	COM38	0.0907	COM68	COM7	0.0912	COM55	COM26	0.0775	COM68	COM33	0.0804	COM55	COM32	0.2050	COM26	COM28	0.2946
COM35	COM38	0.0841	COM32	COM10	0.1058	COM45	**COM34**	0.1077	COM23	COM20	0.1031	COM45	COM38	0.0845	COM7	COM8	0.0811	COM17	COM33	0.0879	COM7	COM45	0.0747	COM21	COM68	0.0760	COM32	COM21	0.1953	COM42	COM68	0.2759
COM8	*COM66*	0.0739	COM7	COM66	0.1031	COM32	COM5	0.1053	COM45	COM8	0.0965	COM69	COM7	0.0816	COM66	**COM24**	0.0763	COM31	COM45	0.0857	COM66	COM52	0.0745	COM56	COM38	0.0744	COM15	COM67	0.1830	COM3	COM32	0.2603
COM45	COM33	0.0734	COM28	COM62	0.0981	COM7	COM24	0.1050	COM69	**COM34**	0.0895	COM32	COM40	0.0753	COM15	COM40	0.0754	COM61	COM40	0.0790	COM61	COM24	0.0731	COM9	COM9	0.0674	COM61	COM62	0.1636	COM35	COM58	0.2595
COM69	*COM56*	0.0731	COM66	COM69	0.0911	COM66	COM10	0.0969	COM62	COM23	0.0837	COM55	COM2	0.0728	COM35	COM2	0.0743	COM7	COM66	0.0789	COM26	COM32	0.0697	COM28	COM56	0.0665	COM68	**COM58**	0.1552	COM61	*COM67*	0.2572
COM62	COM34	0.0706	COM61	COM32	0.0900	COM10	COM32	0.0939	COM32	COM53	0.0812	COM28	COM34	0.0714	COM68	COM34	0.0723	COM35	COM24	0.0758	COM52	COM40	0.0660	COM35	COM70	0.0639	COM56	COM35	0.1537	COM70	COM70	0.2559
COM7	COM62	0.0703	COM68	COM45	0.0843	COM40	COM33	0.0928	COM7	COM33	0.0746	COM2	COM3	0.0692	COM28	COM70	0.0710	COM33	COM3	0.0707	COM15	COM34	0.0658	COM38	COM55	0.0612	COM67	COM28	0.1301	COM32	*COM21*	0.2491
COM24	COM58	0.0683	COM35	COM7	0.0819	COM5	COM58	0.0924	COM2	COM10	0.0728	COM68	COM8	0.0686	COM31	COM45	0.0679	COM66	COM34	0.0692	COM70	COM23	0.0652	COM67	COM10	0.0611	COM21	COM38	0.1296	COM15	**COM62**	0.2268
COM70	COM70	0.0680	COM38	COM38	0.0732	COM61	COM69	0.0900	COM33	COM40	0.0713	COM7	COM70	0.0606	COM17	COM3	0.0671	COM34	COM10	0.0620	COM35	COM15	0.0631	COM3	COM45	0.0565	COM35	COM2	0.1053	COM2	*COM38*	0.2053
COM28	COM67	0.0659	COM17	COM40	0.0674	COM68	COM45	0.0866	COM55	COM55	0.0705	COM38	COM10	0.0598	COM61	COM10	0.0612	COM15	COM5	0.0594	COM21	COM10	0.0601	COM70	*COM13*	0.0546	COM38	COM9	0.1035	COM68	COM42	0.1477
COM40	COM17	0.0647	COM40	COM70	0.0456	COM28	COM66	0.0865	COM28	COM45	0.0663	COM34	COM21	0.0572	COM38	COM21	0.0607	COM10	COM32	0.0587	COM34	COM9	0.0535	COM26	COM40	0.0468	COM28	COM52	0.1034	*COM67*	COM7	0.1175
COM21	COM10	0.0643	COM70	COM24	0.0452	COM35	COM7	0.0847	COM66	COM70	0.0577	COM17	**COM65**	0.0569	COM34	**COM49**	0.0576	COM40	COM55	0.0581	COM9	COM55	0.0509	COM33	*COM23*	0.0463	COM2	*COM43*	0.0858	*COM21*	COM2	0.0757
COM17	*COM15*	0.0640	COM24	COM17	0.0417	COM17	COM17	0.0435	COM10	COM21	0.0555	COM10	COM62	0.0531	COM70	COM32	0.0568	COM3	*COM26*	0.0569	COM40	COM66	0.0484	COM52	COM26	0.0449	COM52	*COM12*	0.0776	*COM38*	COM15	0.0746
COM55	COM69	0.0640	COM55	COM55	0.0381	COM3	COM55	0.0423	COM68	COM62	0.0555	COM15	COM32	0.0531	COM40	COM66	0.0530	COM5	COM69	0.0558	COM23	COM69	0.0464	COM40	COM3	0.0433	*COM43*	COM56	0.0679	*COM9*	*COM9*	0.0487
COM61	COM32	0.0618	COM3	COM21	0.0369	COM55	COM21	0.0415	COM3	COM24	0.0551	COM66	COM55	0.0526	COM3	COM69	0.0520	COM55	COM52	0.0541	COM31	COM3	0.0433	*COM23*	COM15	0.0373	*COM12*	*COM13*	0.0507	*COM43*	*COM43*	0.0476
COM68	COM45	0.0614	COM21	COM26	0.0365	COM21	*COM26*	0.0399	COM17	COM17	0.0540	COM40	COM23	0.0519	COM10	COM26	0.0516	COM52	*COM42*	0.0496	COM12	*COM13*	0.0432	*COM43*	COM52	0.0293	*COM3*	*COM3*	0.0487			
COM3	COM24	0.0595	COM26	COM3	0.0336	*COM26*	*COM42*	0.0383	COM40	COM32	0.0509	COM3	COM45	0.0477	COM21	COM55	0.0516	*COM26*	COM8	0.0479	COM6	COM7	0.0431	*COM12*	*COM43*	0.0271	*COM13*	COM15	0.0413			
COM32	COM21	0.0469	*COM52*	*COM52*	0.0299	*COM42*	COM3	0.0372	COM24	COM26	0.0487	COM70	COM26	0.0476	COM55	COM7	0.0487	*COM42*	*COM23*	0.0465	COM3	*COM68*	0.0352	*COM13*	*COM12*	0.0217	*COM42*	*COM42*	0.0169			
COM26	*COM26*	0.0469	*COM42*	*COM56*	0.0293	*COM52*	*COM56*	0.0325	COM70	COM69	0.0476	COM62	COM66	0.0468	COM26	COM42	0.0477	*COM9*	*COM56*	0.0462	*COM68*	COM12	0.0195	*COM42*	*COM42*	0.0134						
COM56	COM8	0.0441	*COM56*	*COM5*	0.0288	*COM15*	*COM52*	0.0325	COM21	*COM42*	0.0475	COM23	*COM42*	0.0459	COM42	*COM52*	0.0474	*COM23*	COM15	0.0451	*COM13*	COM6	0.0185									
COM66	COM3	0.0436	*COM5*	*COM15*	0.0288	*COM56*	*COM15*	0.0324	COM26	COM2	0.0437	COM21	COM69	0.0432	*COM52*	*COM23*	0.0456	*COM56*	*COM9*	0.0450	*COM42*	*COM42*	0.0136									
COM5	COM7	0.0429	*COM15*	*COM42*	0.0277				*COM42*	COM7	0.0424	COM26	COM17	0.0411	*COM5*	*COM56*	0.0453				*COM46*	*COM56*	0.0127									
COM15	*COM5*	0.0427							*COM52*	*COM52*	0.0421	*COM42*	COM15	0.0408	*COM9*	*COM5*	0.0449				*COM56*	*COM46*	0.0126									
									COM9	*COM56*	0.0410	*COM52*	*COM52*	0.0397	*COM23*	*COM9*	0.0443															
									COM56	*COM5*	0.0403	*COM9*	*COM9*	0.0389	*COM56*	COM15	0.0399															
									COM5	*COM9*	0.0402	*COM5*	*COM56*	0.0385																		
												COM56	*COM5*	0.0383																		

*COM represents asset management companies; COM(c) refers to the cth company. The green boldface indicates the five companies with the best performance based on the CPM and emphasizes the same companies in the average Sharpe scores whereas the red italic indicates the five companies with the worst performance. In order to conserve space, the performance scores for the CPM are not included in Table 3, but they are given in Table 2.

the Sharpe index techniques use different approaches in calculating overall performances. The performance scores generated by the CPM and the Sharpe index are illustrated in Tables 2 and 3, respectively.

Table 2 presents the efficiency scores (*S*) of the companies in ascending order based on the proposed CPM approach. COM(c) refers to the *c*th company. Each company has its own efficiency score in the next column. The first row of Table 2 also uses narrowing windows.

Varying time windows enables a dynamic analysis that captures the downturns (bad states, or turbulent periods) and upturns (good states, or good financial environments) in the economy, in which the benchmark frontier reacts to economic conditions (Basak et al., 2007). From this point of view, each time window has its own efficient frontier at which managers have no ability to outperform the benchmark efficient frontier in the state of downturns and upturns. However, the managers have the choice to adjust their policies to the changing conditions in the economy (active portfolio management). In other words, the proposed approach signals a change in the weights of each asset available in a fund to adjust their efficient frontier. No future action can be taken by managers without knowing the changes in the asset allocation of funds over the investment horizon. The CPM can provide information for managers to take ex-ante actions (i.e., altering the weights of each asset available in a fund).

As shown in Table 2, the most efficient companies for different narrowing windows have a zero-score value. The last values of each narrowing window indicate the least efficient companies with a score of one. A detailed analysis of Table 2 indicates that the performance of the companies change according to different narrowing windows, meaning that the performance of fund managers varies across different time horizons. Thus, fund investors should continually monitor the managers, and the managers evaluate their own performance and take steps to maximize their incentives. For instance, COM33 has no place in the top five companies in the first nine windows. We assume that company managers examine their own performance and take positions to decrease its total efficiency loss. Therefore, COM33 participates in the top five companies for the last two windows. By contrast, COM56 pursues a volatile trading strategy, and the company is among the bottom five companies in the first eight windows. For the last window samples, firm performance seems to recover. In this sense, as an objective tool, the CPM approach sheds light on how managers can take the proper position as well as how investors can select the appropriate company, rapidly. Overall, using an accurate and objective performance approach stimulates the efficiency of financial markets.

The Sharpe index is used for a comparison of the CPM results. The Sharpe index equation for a single asset is presented in Equation (19):

$$\text{Sharpe}_i = \frac{r_i - r_f}{\sigma_{r_i}} \tag{19}$$

where r_i is the return on fund *i*, r_f is the risk-free rate in the market, and σ_{ri} is the standard deviation of *i*. The main problem with using Equation (19) is that it does not consider multiple risk levels (i.e., standard deviations) if analysts aim to evaluate the performance of any asset management company. Managers of asset management companies should invest at multiple risk levels in securities and portfolios in order to satisfy the spectrum of investors. However, the Sharpe index focuses on a single risk level, which is insufficient for assessing overall performance. To solve this problem, the average values of the Sharpe index on different risk levels (Atilgan et al., 2013; Aygoren et al., 2017; Tokat & Hayrullahoglu, 2021) should be calculated. The average Sharpe index equation is as follows (Equation (20)):

$$\text{Avg.Sharpe}_i = \frac{1}{K}\sum_{k=1}^{K}\frac{r_{ik} - r_f}{\sigma_{r_{ik}}} \tag{20}$$

where *K* is the number of risk levels in which managers of an asset management company invest to satisfy the spectrum of investors. r_{ik} and $\sigma_{r_{ik}}$ are the fund returns and the standard deviation of fund for the i^{th} company at the k^{th} risk level. *K* overlaps with the risk levels of the DPM in calculating the average Sharpe score of a company for an accurate comparison. In Table 3, the performance scores of asset management companies based on the Sharpe index are illustrated in addition to the performance rankings of the CPM.[8] The green boldface indicates the five companies with the best performance based on the CPM and emphasizes the same companies in the average Sharpe scores whereas the red-italic indicates the five companies with the worst performance.

In Table 3, the results of the two approaches have significantly different patterns. For instance, the five best and worst companies according to the CPM are not same in the ranking of the average Sharpe scores. There are major differences between the windows. Although for the windows 2013–2020 and 2014–2020, COM65 and COM49 rank first and fourth in the CPM, their performance ranks 25th based on the average Sharpe score. Another interesting example is COM34. Although it ranks among the top five companies according to the CPM, it descends to the bottom in its average Sharpe score. The bottom five companies also show major differences in rankings. Especially in the 2009–2020 window, COM66, COM56, and COM15 rank in the bottom five companies with regard to the CPM scores, whereas they are nearly in the top five in their average Sharpe scores. The 2019–2020 window has even more intriguing findings: COM62 ranks in the top five companies, depending on the CPM score, for which it ranks among the bottom five companies depending on the average Sharpe score. This might be due to the lack of continuous summation (integration) in Sharpe index at multiple risk levels. This shortcoming also applies to other traditional performance measurement techniques. As a new approach, the CPM eliminates this problem by calculating total efficiency losses with a

[8] In order to conserve space, the performance scores for the CPM are not included in Table 3, but they are given in Table 2.

continuous summation of efficiency losses using multiple risk levels.

5. Concluding remarks

Asset management companies tend to highlight well-performing funds to attract new investors because fund managers earn incentives based on the returns on the funds that they manage. However, how well the fund managers manage the risk and return trade-off is vital for small investors. The finance literature created its own traditional methods for determining the performance of individual financial assets. Even though these traditional methods are widely used, they have several problems. According to the literature, the critiques of traditional performance measuring methods can be divided into six groups: (1) they cannot accurately capture the total excess return of the managers; (2) nearly all of them assume that all portfolios are well diversified; (3) they have insufficient consideration of upside and downside risk; (4) they ignore covariances; (5) all of them assume linearity; and (6) they do not take into account multiple risk levels. Furthermore, all the methods concentrate on the performance of individual securities and portfolios with a discrete assessment, rather than assessing the performance of an asset management company overall with a continuous assessment. Because of these problems, the traditional performance measuring methods might have misleading results in some cases, thus, an accurate performance measurement method is needed. In terms of portfolio performance comparison, calculation of total efficiency losses is required, rather than the efficiency loss at a specific risk level. To avoid this problem, we propose a continuous-based method.

The goal of this study is to determine companies' efficiency losses at continuous risk levels. In this way, it is possible to calculate total efficiency losses for managers (i.e., asset management companies). In our proposed method, we generate a benchmark portfolio with all assets available in the fund market for all specified risk levels, and the asset management companies are considered suboptimal portfolios. The framework of the MV approach is a tool for comparison in terms of efficiency losses. As a performance criterion, efficiency scores are calculated, and companies are ordered from the most to the least efficient.

To test the CPM method, we used open-end funds for the period 2009–2020, indicating that the universe of empirical analysis is the open-end fund market. Then, to see the time effect, we set up our sample in eleven narrowing windows. Varying time windows provide a dynamic analysis, capturing the upturns and downturns in the economy in which the benchmark and the company frontiers react to economic conditions. After applying the proposed method to the open-end funds market, we calculate average Sharpe ratios to compare the results. Because the Sharpe index can measure fund performance at only one risk level, it needs to be averaged for multiple risk levels to create a comparable score. Consequently, the results of the two methods exhibit significantly different patterns. The CPM method has a continuous way to measure performance; thus, we believe that the method provides more accurate results than traditional methods.

In the finance literature, the traditional performance measurement methods have had progressive improvements, and they try to solve the problems belonging to each other. This study contributes to the literature by providing solutions to all the problems by using a continuous form. Although the CPM method can ensure the capture of the total excess return of managers, it does not assume well-diversified portfolios and linearity. It considers covariances between the assets and uses the upside and downside risk levels. Most important, the CPM measures the performance of asset management companies (or the managers) at multiple risk levels.

We suggest the following topics for further studies. First, individual asset management companies may have the ability to measure their divisions' performances via the CPM. Second, it can also be used to measure individual fund performance if the allocation of assets (the weights of each asset available in a fund) is known. Therefore, the CPM might be an alternative to traditional performance measurement methods, such as the Sharpe ratio. Third, the fund allocation (the weights) could be calculated to determine the minimum total efficiency loss for a company. Managers should emphasize this problem in view of the dynamic nature of the market.

Declaration of competing interest

There is no conflict of interest.

Appendices

A. The Proof of the CPM

In this section, we provide the proof of Equation. (17). We first introduce the following notations: Let $\Omega = \sigma_{ij}$ is the covariance of returns between the i^{th} and j^{th} assets an all assets assumed risky ($\sigma_i^2 > 0$). V_{ij} is the inverse of Ω (*i.e.*, Ω^{-1}). The parameters of the parabola and the hyperbola for the MV efficient frontier are defined as A, B, C, and D. Following equations describe the parameters:

$$A \equiv \sum_{i=1}^{n} \sum_{j=1}^{n} V_{ij} E_j \qquad \text{A.1}$$

$$B \equiv \sum_{i=1}^{n} \sum_{j=1}^{n} V_{ij} E_j E_i C \qquad \text{A.2}$$

$$C \equiv \sum_{i=1}^{n} \sum_{j=1}^{n} V_{ij} \qquad \text{A.3}$$

$$D \equiv BC - A^2 > 0 \qquad \text{A.4}$$

The mathematical functions of the MV efficient frontier for the parabola (A.5) and the hyperbola (A.6) are as follows:

$$\sigma^2 = \frac{C}{D}E^2 - 2\frac{A}{D}E + \frac{B}{D} \quad \text{A.5}$$

$$E = \frac{A}{C} + \frac{1}{C}\sqrt{D(C\sigma^2 - 1)} = \frac{A}{C} + \frac{\sqrt{D}}{C}\sqrt{(C\sigma^2 - 1)} \quad \text{A.6}$$

Because the standard deviation is generally used to measure risk, we construct our model in the $[E, \sigma]$ phase space. More specifically, we build our model with hyperbola. Due to the structure of model requires a benchmark and a suboptimal efficient frontier, we need two hyperbola functions for (fund) market (A.7) and each company (A.8).

$$E = f_m(\sigma) = \frac{A_m}{C_m} + \frac{1}{C_m}\sqrt{D_m(C_m\sigma^2 - 1)} \quad \text{A.7}$$

$$E = f_c(\sigma) = \frac{A_c}{C_c} + \frac{1}{C_c}\sqrt{D_c(C_c\sigma^2 - 1)} \quad \text{A.8}$$

To simplify the solution, let, $\psi = \frac{A}{C}$, $\xi = \frac{\sqrt{D}}{C}$, and $\gamma = C$, then hyperbola functions are as follows:

$$E = f(\sigma) = \psi + \xi\sqrt{(\gamma\sigma^2 - 1)} \quad \text{A.9}$$

$$E_m = f_m(\sigma) = \psi_m + \xi_m\sqrt{(\gamma_m\sigma^2 - 1)} \quad \text{A.10}$$

$$E_c = f_c(\sigma) = \psi_c + \xi_c\sqrt{(\gamma_c\sigma^2 - 1)} \quad \text{A.11}$$

To calculate the total efficiency loss of each company, we need to integrate the difference between hyperbola functions. TEL_c denotes the area between those two hyperbolas for each company (Equations A.12–A.15).

$$TEL_c = \int_{\sigma_{c,min}}^{\sigma_{c,max}} [f_m(\sigma) - f_c(\sigma)] d\sigma \quad \text{A.12}$$

$$TEL_c = \int_{\sigma_{c,min}}^{\sigma_{c,max}} \left[\left(\psi_m + \xi_m\sqrt{(\gamma_m\sigma^2 - 1)}\right) - \left(\psi_c + \xi_c\sqrt{(\gamma_c\sigma^2 - 1)}\right)\right] d\sigma \quad \text{A.13}$$

$$TEL_c = \int_{\sigma_{c,min}}^{\sigma_{c,max}} \left[\left(\psi_m - \psi_c\right) + \left(\xi_m\sqrt{(\gamma_m\sigma^2 - 1)} - \xi_c\sqrt{(\gamma_c\sigma^2 - 1)}\right)\right] d\sigma \quad \text{A.14}$$

$$TEL_c = \int_{\sigma_{c,min}}^{\sigma_{c,max}} (\psi_m - \psi_c) d\sigma + \int_{\sigma_{c,min}}^{\sigma_{c,max}} \left(\xi_m\sqrt{(\gamma_m\sigma^2 - 1)}\right) d\sigma - \int_{\sigma_{c,min}}^{\sigma_{c,max}} \xi_e\sqrt{(\gamma_c\sigma^2 - 1)} d\sigma \quad \text{A.15}$$

To simplify the integration a partition is needed ($TEL_c = TEL_c^1 + TEL_c^2 - TEL_c^3$). The integrations of each part are as follows:

$$TEL_c^1 = \int_{\sigma_{c,min}}^{\sigma_{c,max}} (\psi_m - \psi_c) d\sigma = \psi_m\sigma - \psi_c\sigma + c \Big|_{\sigma_{c,min}}^{\sigma_{c,max}} \quad \text{A.16}$$

$$TEL_c^2 = \int_{\sigma_{c,min}}^{\sigma_{c,max}} \left(\xi_m\sqrt{(\gamma_m\sigma^2 - 1)}\right) d\sigma \quad \text{A.17}$$

$$\int f_m(\sigma) = \int\left(\xi_m\sqrt{(\gamma_m\sigma^2 - 1)}\right) d\sigma \quad \text{A.18}$$

$$\sigma = \frac{1}{\sqrt{\gamma_m}}t \Rightarrow d\sigma = \frac{1}{\sqrt{\gamma_m}}dt \quad \text{A.19}$$

$$\int f_m(t) = \xi_m \int \sqrt{\left(\gamma_m\left(\frac{1}{\sqrt{\gamma_m}}t\right)^2 - 1\right)} \frac{1}{\sqrt{\gamma_m}}dt \quad \text{A.20}$$

$$\int f_m(t) = \frac{\xi_m}{\sqrt{\gamma_m}} \int \sqrt{t^2 - 1}dt \quad \text{A.21}$$

$$\text{acosh}(\alpha) = cosh^{-1}(\alpha)$$

$$cosh(2\alpha) = 1 + 2sinh^2(\alpha)$$

$$cosh^2(\alpha) - sinh^2(\alpha) = 1$$

$$sinh(2\alpha) = 2sinh(\alpha)cosh(\alpha)$$

$$sinh(acosh(t))cosh(acosh(t)) = t\sqrt{t^2 - 1}$$

$$\int cosh(2\alpha) = \frac{1}{2}sinh(2\alpha)$$

Some useful theorems and conversions for the next solution steps:

$$\left.\begin{array}{l} t = cosh(\alpha) \Rightarrow \text{acosh}(t) = \alpha \\ dt = sinh(\alpha)d\alpha \\ \sqrt{t^2 - 1} = \sqrt{cosh^2(\alpha) - 1} = sinh(\alpha) \end{array}\right\} \quad \text{A.22}$$

$$\int f_m(\alpha) = \frac{\xi_m}{\sqrt{\gamma_m}} \int sinh(\alpha)sinh(\alpha)d\alpha \quad \text{A.23}$$

$$\int f_m(\alpha) = \frac{\xi_m}{\sqrt{\gamma_m}} \int sinh^2(\alpha)d\alpha \quad \text{A.24}$$

$$\int f_m(\alpha)=\frac{\xi_m}{\sqrt{\gamma_m}}\int\frac{cosh(2\alpha)-1}{2}d\alpha \quad \text{A.25}$$

$$\int f_m(\alpha)=\frac{\xi_m}{\sqrt{\gamma_m}}\left(\frac{sinh(2\alpha)}{4}-\frac{\alpha}{2}+c\right) \quad \text{A.26}$$

$$\int f_m(\alpha)=\frac{\xi_m}{\sqrt{\gamma_m}}\left(\frac{sinh(\alpha)cosh(\alpha)}{2}-\frac{\alpha}{2}+c\right) \quad \text{A.27}$$

$$\int f_m(t)=\frac{\xi_m}{\sqrt{\gamma_m}}\left(\frac{sinh(acosh(t))cosh(acosh(t))}{2}-\frac{acosh(t)}{2}+c\right) \quad \text{A.28}$$

$$\int f_m(t)=\frac{\xi_m}{\sqrt{\gamma_m}}\left(\frac{t\sqrt{t^2-1}}{2}-\frac{acosh(t)}{2}+c\right) \quad \text{A.29}$$

$$\int f_m(\sigma)=\frac{\xi_m}{\sqrt{\gamma_m}}\left(\frac{\sqrt{\gamma_m}\sigma\sqrt{\left(\sqrt{\gamma_m}\sigma\right)^2-1}}{2}-\frac{acosh\left(\sqrt{\gamma_m}\sigma\right)}{2}+c\right) \quad \text{A.30}$$

$$TEL_c^2=\int f_m(\sigma)=\left(\frac{\xi_m\sigma\sqrt{\gamma_m\sigma^2-1}}{2}-\frac{\xi_m acosh\left(\sqrt{\gamma_m}\sigma\right)}{2\sqrt{\gamma_m}}+c\right) \quad \text{A.31}$$

$$TEL_c^3=\int f_c(\sigma)=\left(\frac{\xi_c\sigma\sqrt{\gamma_c\sigma^2-1}}{2}-\frac{\xi_c acosh\left(\sqrt{\gamma_c}\sigma\right)}{2\sqrt{\gamma_c}}+c\right) \quad \text{A.32}$$

$$TEL_c=TEL_c^2-TEL_c^3+TEL_c^1 \quad \text{A.33}$$

$$TEL_c=\left[\frac{\xi_m\sigma\sqrt{\gamma_m\sigma^2-1}}{2}-\frac{\xi_m acosh\left(\sqrt{\gamma_m}\sigma\right)}{2\sqrt{\gamma_m}}\right]-\left[\frac{\xi_c\sigma\sqrt{\gamma_c\sigma^2-1}}{2}-\frac{\xi_c acosh\left(\sqrt{\gamma_c}\sigma\right)}{2\sqrt{\gamma_c}}\right]+[(\psi_m-\psi_c)\sigma]\Big|_{\sigma_{c,min}}^{\sigma_{c,max}} \quad \text{A.34}$$

B. The Proof of Minimum Efficiency Loss for Further Studies

To calculate the minimum and the maximum of a function, we should take the first order derivation and set it equal to zero. For efficiency losses, the first order derivation of the TEL_c function is as follows.

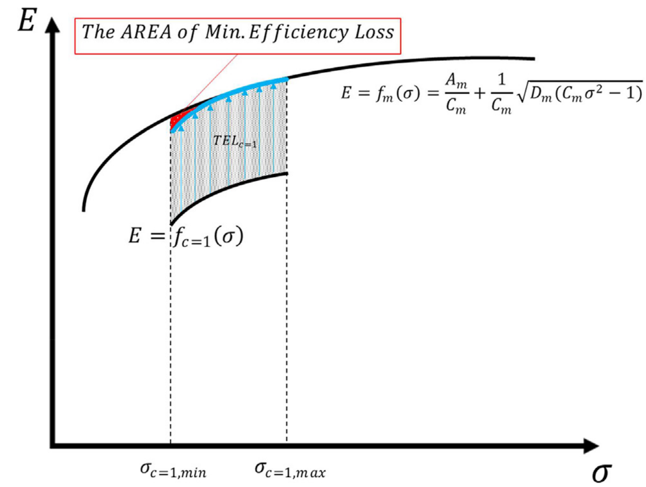

Figure B. Illustration of the total minimum efficiency loss.

Recall the A.34 as the function of total efficiency loss of a company (c).

$$TEL_c=\left[\frac{\xi_m\sigma\sqrt{\gamma_m\sigma^2-1}}{2}-\frac{\xi_m acosh\left(\sqrt{\gamma_m}\sigma\right)}{2\sqrt{\gamma_m}}\right]-\left[\frac{\xi_c\sigma\sqrt{\gamma_c\sigma^2-1}}{2}-\frac{\xi_c acosh\left(\sqrt{\gamma_c}\sigma\right)}{2\sqrt{\gamma_c}}\right]+[(\psi_m-\psi_c)\sigma]\Big|_{\sigma_{c,min}}^{\sigma_{c,max}} \quad \text{A.34}$$

To simplify the derivation of A.34, we identify the function ($f(\sigma)$) as B.1:

$$f(\sigma)=\left[\frac{\xi_m\sigma\sqrt{\gamma_m\sigma^2-1}}{2}-\frac{\xi_m acosh\left(\sqrt{\gamma_m}\sigma\right)}{2\sqrt{\gamma_m}}\right] \quad \text{B.1}$$

Define:

$$f^{Part-1}(\sigma)=\frac{\xi_m\sigma\sqrt{\gamma_m\sigma^2-1}}{2} \quad \text{B.2}$$

$$f^{Part-2}(\sigma)=\frac{\xi_m acosh\left(\sqrt{\gamma_m}\sigma\right)}{2\sqrt{\gamma_m}} \quad \text{B.3}$$

Derivation of Part-1:

$$f^{Part-1}(\sigma)=\frac{1}{2}\xi_m\sigma\sqrt{\gamma_m\sigma^2-1} \quad \text{B.4}$$

$$u(\sigma)=\xi_m\sigma,\ v(\sigma)=\sqrt{\gamma_m\sigma^2-1} \quad \text{B.5}$$

$$f'(\sigma)=\frac{1}{2}[u'(\sigma)v(\sigma)+u(\sigma)v'(\sigma)] \quad \text{B.6}$$

$$u'(\sigma)=\xi_m \quad \text{B.7}$$

$$v'(\sigma)=\gamma_m\sigma\frac{1}{\sqrt{\gamma_m\sigma^2-1}} \quad \text{B.8}$$

$$f'(\sigma)=\frac{1}{2}\left[\xi_m\sqrt{\gamma_m\sigma^2-1}+\xi_m\sigma\gamma_m\sigma\frac{1}{\sqrt{\gamma_m\sigma^2-1}}\right] \quad \text{B.9}$$

$$f'(\sigma)=\frac{1}{2}\xi_m\left[\sqrt{\gamma_m\sigma^2-1}+\gamma_m\sigma^2\frac{1}{\sqrt{\gamma_m\sigma^2-1}}\right] \quad \text{B.10}$$

$$f'(\sigma)=\frac{1}{2}\xi_m\left[\frac{\gamma_m\sigma^2-1+\gamma_m\sigma^2}{\sqrt{\gamma_m\sigma^2-1}}\right] \quad \text{B.11}$$

$$f'(\sigma)=\frac{1}{2}\xi_m\left[\frac{2\gamma_m\sigma^2-1}{\sqrt{\gamma_m\sigma^2-1}}\right] \quad \text{B.12}$$

$$f'(\sigma)=\frac{1}{2}\xi_m\left[\frac{2(\gamma_m\sigma^2-1/2)}{\sqrt{\gamma_m\sigma^2-1}}\right] \quad \text{B.13}$$

$$f'(\sigma)=\xi_m\left[\frac{\gamma_m\sigma^2-1/2}{\sqrt{\gamma_m\sigma^2-1}}\right] \quad \text{B.14}$$

$$f'(\sigma)=\xi_m\left[\frac{\gamma_m\sigma^2-1+(1/2)}{\sqrt{\gamma_m\sigma^2-1}}\right] \quad \text{B.15}$$

$$f'(\sigma)=\xi_m\left[\frac{\gamma_m\sigma^2-1}{\sqrt{\gamma_m\sigma^2-1}}+\frac{1}{2\sqrt{\gamma_m\sigma^2-1}}\right] \quad \text{B.16}$$

$$f'(\sigma)=\xi_m\left[\sqrt{\gamma_m\sigma^2-1}+\frac{1}{2\sqrt{\gamma_m\sigma^2-1}}\right] \quad \text{B.17}$$

Derivation of Part-2:

$$f^{Part-2}(\sigma)=\frac{\xi_m acosh\left(\sqrt{\gamma_m}\sigma\right)}{2\sqrt{\gamma_m}} \quad \text{B.3}$$

A useful theorem:

$$acosh(x)=\ln\left(x+\sqrt{x^2-1}\right)$$

$$x=\sqrt{\gamma_m}\sigma,\ z(\sigma)=\sqrt{\gamma_m}\sigma+\sqrt{\gamma_m\sigma^2-1} \quad \text{B.18}$$

$$acosh(x)=\ln(z(\sigma)) \quad \text{B.19}$$

$$\frac{d(\ln(z(\sigma)))}{dz}=\frac{z'(\sigma)}{z(\sigma)} \quad \text{B.20}$$

$$f(\sigma)=\frac{\xi_m}{2\sqrt{\gamma_m}}acosh\left(\sqrt{\gamma_m}\sigma\right) \quad \text{B.21}$$

$$f(\sigma)=\frac{\xi_m}{2\sqrt{\gamma_m}}\left[ln\left(\sqrt{\gamma_m}\sigma+\sqrt{\gamma_m\sigma^2-1}\right)\right] \quad \text{B.22}$$

$$z'(\sigma)=\sqrt{\gamma_m}+\gamma_m\sigma\frac{1}{\sqrt{\gamma_m\sigma^2-1}} \quad \text{B.23}$$

$$\frac{z'(\sigma)}{z(\sigma)}=\left[\sqrt{\gamma_m}+\gamma_m\sigma\frac{1}{\sqrt{\gamma_m\sigma^2-1}}\right]\times\frac{1}{\sqrt{\gamma_m}\sigma+\sqrt{\gamma_m\sigma^2-1}} \quad \text{B.24}$$

Combining the derivation of part-1 and part-2:

$$f_m'(\sigma)=\left[\xi_m\left[\sqrt{\gamma_m\sigma^2-1}+\frac{1}{2\sqrt{\gamma_m\sigma^2-1}}\right]\right]-\left[\left[\sqrt{\gamma_m}+\gamma_m\sigma\frac{1}{\sqrt{\gamma_m\sigma^2-1}}\right]\times\frac{1}{\sqrt{\gamma_m}\sigma+\sqrt{\gamma_m\sigma^2-1}}\right] \quad \text{B.25}$$

B.25 should be rearranged for company c.

$$f_c'(\sigma)=\left[\xi_c\left[\sqrt{\gamma_c\sigma^2-1}+\frac{1}{2\sqrt{\gamma_c\sigma^2-1}}\right]\right]-\left[\left[\sqrt{\gamma_c}+\gamma_c\sigma\frac{1}{\sqrt{\gamma_c\sigma^2-1}}\right]\times\frac{1}{\sqrt{\gamma_c}\sigma+\sqrt{\gamma_c\sigma^2-1}}\right] \quad \text{B.26}$$

According to derivations above, the derivation of A.34 is as follows.

$$\frac{d(TEL_c)}{d\sigma}=f_m'(\sigma)-f_c'(\sigma)+(\psi_m-\psi_c) \quad \text{B.27}$$

For calculating the minimum efficiency loss, Equation B.27 should be set to zero and solved for σ. However, this is out of the scope of this study.

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