



# Wave behaviors of Kundu–Mukherjee–Naskar model arising in optical fiber communication systems with complex structure

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## Abstract

Rogue waves are very mysterious and extra ordinary waves. They appear suddenly even in a calm sea and are hard to be predicted. Although nonlinear Schrödinger equation provides a perspective, it alone can neither detect rogue waves nor provide a complete solution to problems. Therefore, some approximations are still mandatory for both obtaining an exact solution and predicting rogue waves. Such as Kundu–Mukherjee–Naskar (KMN) model which allows obtaining lump-soliton solutions considered as rogue waves. In this study the functional variable method is utilized to obtain the analytical solutions of KMN model that corresponds to the propagation of soliton dynamics in optical fiber communication system.

**Keywords** KMN model · Functional variable method · Rogue waves

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## 1 Introduction

Wave motion is very predictable in basic level and can be explained very deterministic way. However, it becomes much more complicated with taking the nonlinear interactions into account. This type of systems called nonlinear or dynamical systems. Although, there has been some development in the mathematical field to understand the behavior of a nonlinear system we are still far from predicting the behavior of such systems deterministically. The first studies on this issue date back almost 150 years to Riemann and Stokes. And the studies are continuing with increasing importance and interest (Rezazadeh et al. 2018, 2019; Rezazadeh 2018). In the last decade this interest increased more than before (Kundu et al. 2021; Yang et al. 2021; Alam et al. 2021; Ma et al. 2021; Alam and Osman 2021; Gómez-Aguilar et al. 2021; Barman et al. 2021). Many different analytical methods are applied to understand and explain physical behavior of the nonlinear systems. (Barman et al. 2021; Osman et al. 2018; Liu et al. 2019a, b; Ding et al. 2019; Ekici et al. 2016; Eslami and Mirzazadeh 2016)

The basic idea is concentrated on two types of wave behaviors which are hyperbolic (Pettersson et al. 2019) and dispersive. Hyperbolic wave behavior can be formulated mathematically in terms of hyperbolic partial differential equations. Klein-Gordon (Kurt 2019) is a prototype for hyperbolic wave equation. Dispersive waves cannot be characterized easily. Nonhyperbolic waves generally categorized as dispersive waves. However, classification is made on the type of solution rather than on the equations. Korteweg-DeVries equation (Kurt et al. 2017) can be a good example for dispersive waves. There are many equations developed for determining the wave behavior of a dynamical system (Gao et al. 2020; Rezazadehd et al. 2018; Raza et al. 2019a, b; Tasbozan et al. 2019; Atilgan et al. 2019; Kurt 2019; Kurt et al. 2017; Seadawy et al. 2019, 2020; Ali et al. 2018a, b; Raslan et al. 2017; Sedeeg et al. 2019; Sulaiman et al. 2019a, b; Sulaiman and Bulut 2019, 2020).

For a small intersection of a spatiotemporal system the dynamics can be assumed as linear. However, they must be evaluated in terms nonlinear dynamics due to significant modulation of the wave amplitude originated from cumulative nonlinear interactions. Nonlinear Schrödinger (NLS) equation is a very common equation which is providing a canonical design of involucre dynamics of a quasi-monochromatic planar wave propagating in a weakly nonlinear dispersive medium when dissipative effects are insignificant (Liu et al. 2018; El-Dessoky and Islam 2019; Seadawy and Cheemaa 2019).

NLS is employed for many situational models such as propagation of a wave in a Kerr type (Zhang et al. 2011a, b) or non-Kerr type (Liu 2010) medium. Most of them are not fully integrable which means exact solutions cannot be obtained directly. Only approximate numerical solutions with no stable solitons can be obtained (Zhang and Simos 2016a, b). Approximations cannot predict rogue waves which can be defined as “localized and isolated surface waves, apparently appear from nowhere, make a sudden hole in the sea just before attaining surprisingly high amplitude and disappear again without a trace” KMN (Kundu et al. 2014). They proposed a model to by extension of NLS to have an integrable form which allows lump-soliton can be considered as rogue wave model;

$$iq_t = d_1 q_{xx} - d_2 q_{yy} + 2iq \left( \sqrt{d_1} j^x - \sqrt{d_2} j^y \right), \quad j^a \equiv qq_a^* - q^* q_a. \quad (1.1)$$

and then, they replaced the conventional amplitude-like nonlinear term with the a current-like nonlinear term which allows them to obtain a fully integrable form of NLS;

$$iq_t + q_{xy} + 2iq(qq_x^* - q^*q_x) = 0. \quad (1.2)$$

In this study the wave solutions of KMN model

$$iq_t + \alpha q_{xy} + i\beta q(qq_x^* - q^*q_x) = 0. \quad (1.3)$$

which describes the propagation of soliton dynamics in optical fiber communication system.

Yıldırım (Yıldırım 2019) obtained dark, bright and singular solitons by using trial equation technique for KMN model. Rizvi et al. (Rizvi et al. 2020) used csch method, extended Tanh–Coth method and extended rational sinh-cosh method to get the exact solutions of KMN model. Talarposhti et al. (Talarposhti et al. 2020) employed Exp-function method to yield the optical soliton solutions of considered KMN model.

This work is structured as follows: In Sect. 2, mathematical analysis of KMN model is given. In Sect. 3, we demonstrate the structure of the functional variable method. In Sect. 4, we apply this method to find some wave solutions of the equation written above. In Sect. 5, we give the results and discussion, Sect. 6 gives the conclusion of the whole research.

## 2 Mathematical analysis

In order to get started, the following hypothesis is selected:

$$q(x, y, t) = P(\eta) \exp [i\Phi(x, y, t)], \quad (2.1)$$

where  $P(\xi)$  represents the amplitude portion and

$$\xi = \chi_1 x + \chi_2 y - \sigma t, \quad (2.2)$$

and the phase portion of the soliton is defined as

$$\Phi(x, y, t) = -\vartheta_1 x - \vartheta_2 y + \varpi t + \theta_0. \quad (2.3)$$

Here,  $\vartheta_1$  and  $\vartheta_2$  are the frequencies of the soliton in the  $x$ - and  $y$ -directions respectively while  $\varpi$  is the wave number of the soliton and finally  $\theta_0$  is the phase constant. Also, the parameters  $\chi_1$  and  $\chi_2$  in (2.2) represent the inverse width of the soliton along  $x$ - and  $y$ -directions respectively, while  $\sigma$  represents the velocity of the soliton. Inserting (2.1) along with (2.2) and (2.3) into (1.1) and decomposing into real and imaginary parts, the following pair of equations, respectively yield

$$\alpha \chi_1 \chi_2 P'' - (\omega + \alpha \vartheta_1 \vartheta_2) P - 2\beta \vartheta_1 P^3 = 0, \quad (2.4)$$

$$\sigma = -\alpha(\vartheta_1 \chi_2 + \vartheta_2 \chi_1). \quad (2.5)$$

Equation (2.4) is transformed into the following one

$$P'' = \frac{\omega + \alpha \vartheta_1 \vartheta_2}{\alpha \chi_1 \chi_2} P + \frac{2\beta \vartheta_1}{\alpha \chi_1 \chi_2} P^3. \quad (2.6)$$

### 3 The functional variable method

This section presents the brief descriptions of the functional variable method (Zerarka and Ouamane 2010; Eslami et al. 2017; Bekir et al. 2015). While applying this method discretization or normalization is not needed, this is the main advantage of the method. Also, nonlinear partial differential equation is converted into nonlinear ordinary differential equation by the help of wave transform and chain rule. This process makes the solution easier and faster.

Suppose that a the NLEE, say in two independent variables to  $x$  and  $t$  is given by

$$G(u, u_t, u_{xx}, \dots) = 0, \tag{3.1}$$

where  $G$  is a function of  $u, u_t, u_{xx}, \dots$  and the subscripts denote the partial derivatives of  $u(x, t)$  with respect to  $x$  and  $t$ .

A transformation  $u(x, t) = U(\eta)$ ,  $\eta = x - \sigma t$  converts the NLEE (3.1) to a nonlinear ODE

$$F(U, U_\eta, U_{\eta\eta}, \dots) = 0, \tag{3.2}$$

where  $F$  is a function of  $U, U_\eta, U_{\eta\eta}, \dots$  and its derivatives point out the ordinary derivatives with respect to  $\eta$  and where  $\sigma$  is constant to be determine.

Then we make a transformation in which the unknown function  $U$  is considered as a functional variable in the form:

$$U_\eta = \Omega(U), \tag{3.3}$$

and some successive derivatives of  $U$  are

$$\begin{aligned} U_{\eta\eta} &= \frac{1}{2}(\Omega^2)', \\ U_{\eta\eta\eta} &= \frac{1}{2}(\Omega^2)'' \sqrt{\Omega^2}, \\ U_{\eta\eta\eta\eta} &= \frac{1}{2}[(\Omega^2)''' \Omega^2 + (\Omega^2)''(\Omega^2)'], \\ &\vdots \end{aligned} \tag{3.4}$$

where “'” stands for  $\frac{d}{dU}$ .

The ODE (3.2) can be reduced in terms of  $U, F3.4$  and its derivatives upon using the expressions of Eq. (3.4) into Eq. (3.2) gives

$$H(U, \Omega, \Omega', \Omega'', \dots) = 0. \tag{3.5}$$

by integrating of Eq. (3.5), Eq. (3.5) can be written with respect to  $H$ , and it is found the appropriate solutions by using Eq. (3.3) for the investigated problem.

### 4 Solutions to the Eq. (1)

In this Section we obtain wave solutions of the KMN model by using the functional variable method described in Sect. 3.

Following Eq. (3.4), it is easy to deduce from Eq. (2.6) an expression for the function  $\Omega(U)$

$$\frac{1}{2}(\Omega^2)' = \frac{\omega + \alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2}P + \frac{2\beta\vartheta_1}{\alpha\chi_1\chi_2}P^3. \quad (4.1)$$

Integrating Eq. (4.1) and setting the constant of integration to  $\Xi$  yields

$$\Omega^2 = \frac{\omega + \alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2}P^2 + \frac{\beta\vartheta_1}{\alpha\chi_1\chi_2}P^4 + \Xi, \quad (4.2)$$

or

$$\Omega = P_\eta = \pm \sqrt{\frac{\omega + \alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2}P^2 + \frac{\beta\vartheta_1}{\alpha\chi_1\chi_2}P^4 + \Xi}. \quad (4.3)$$

Using Eqs. (2.1), (2.2) and (2.3), we obtain the following wave solutions of the KMN model.

If  $\frac{\omega + \alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2} > 0$ ,  $\frac{\beta\vartheta_1}{\alpha\chi_1\chi_2} < 0$  and  $\Xi = 0$ , we obtain the following bright soliton solutions

$$q_1^\pm(x, y, t) = \pm \sqrt{-\frac{\omega + \alpha\vartheta_1\vartheta_2}{\beta\vartheta_1}} \operatorname{sech} \left( \sqrt{\frac{\omega + \alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2}} (\chi_1x + \chi_2y + (\alpha(\vartheta_1\eta_2 + \vartheta_2\eta_1))t) \right) \\ \times \exp [i(-\vartheta_1x - \vartheta_2y + \varpi t + \theta_0)]. \quad (4.4)$$

If  $\frac{\omega + \alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2} > 0$ ,  $\frac{\beta\vartheta_1}{\alpha\chi_1\chi_2} < 0$  and  $\Xi = 0$ , we obtain the following singular soliton solutions

$$q_2^\pm(x, y, t) = \pm \sqrt{\frac{\omega + \alpha\vartheta_1\vartheta_2}{\beta\vartheta_1}} \operatorname{csch} \left( \sqrt{\frac{\omega + \alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2}} (\chi_1x + \chi_2y + (\alpha(\vartheta_1\eta_2 + \vartheta_2\eta_1))t) \right) \\ \times \exp [i(-\vartheta_1x - \vartheta_2y + \varpi t + \theta_0)]. \quad (4.5)$$

If  $\frac{\omega + \alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2} < 0$ ,  $\frac{\beta\vartheta_1}{\alpha\chi_1\chi_2} > 0$  and  $\Xi = 0$ , we obtain the following periodic wave solutions

$$q_3^\pm(x, y, t) = \pm \sqrt{-\frac{\omega + \alpha\vartheta_1\vartheta_2}{\beta\vartheta_1}} \operatorname{sec} \left( \sqrt{-\frac{\omega + \alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2}} (\chi_1x + \chi_2y + (\alpha(\vartheta_1\eta_2 + \vartheta_2\eta_1))t) \right) \\ \times \exp [i(-\vartheta_1x - \vartheta_2y + \varpi t + \theta_0)], \quad (4.6)$$

$$q_4^\pm(x, y, t) = \pm \sqrt{-\frac{\omega + \alpha\vartheta_1\vartheta_2}{\beta\vartheta_1}} \operatorname{csc} \left( \sqrt{-\frac{\omega + \alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2}} (\chi_1x + \chi_2y + (\alpha(\vartheta_1\eta_2 + \vartheta_2\eta_1))t) \right) \\ \times \exp [i(-\vartheta_1x - \vartheta_2y + \varpi t + \theta_0)]. \quad (4.7)$$

If  $\frac{\omega + \alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2} < 0$ ,  $\frac{\beta\vartheta_1}{\alpha\chi_1\chi_2} > 0$  and  $\Xi = -\frac{(\omega + \alpha\vartheta_1\vartheta_2)^2}{4\alpha\beta\vartheta_1\chi_1\chi_2}$ , we obtain the following dark soliton solution

$$q_5^\pm(x, y, t) = \pm \sqrt{-\frac{\omega + \alpha\vartheta_1\vartheta_2}{2\beta\vartheta_1}} \tanh\left(\sqrt{-\frac{\omega + \alpha\vartheta_1\vartheta_2}{2\alpha\chi_1\chi_2}}(\chi_1x + \chi_2y + (\alpha(\vartheta_1\eta_2 + \vartheta_2\eta_1))t)\right) \times \exp[i(-\vartheta_1x - \vartheta_2y + \varpi t + \theta_0)]. \tag{4.8}$$

If  $\frac{\omega + \alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2} < 0$ ,  $\frac{\beta\vartheta_1}{\alpha\chi_1\chi_2} > 0$  and  $\Xi = -\frac{(\omega + \alpha\vartheta_1\vartheta_2)^2}{4\alpha\beta\vartheta_1\chi_1\chi_2}$ , we obtain the following singular dark soliton solutions

$$q_6^\pm(x, y, t) = \pm \sqrt{-\frac{\omega + \alpha\vartheta_1\vartheta_2}{2\beta\vartheta_1}} \coth\left(\sqrt{-\frac{\omega + \alpha\vartheta_1\vartheta_2}{2\alpha\chi_1\chi_2}}(\chi_1x + \chi_2y + (\alpha(\vartheta_1\eta_2 + \vartheta_2\eta_1))t)\right) \times \exp[i(-\vartheta_1x - \vartheta_2y + \varpi t + \theta_0)]. \tag{4.9}$$

If  $\frac{\omega + \alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2} > 0$ ,  $\frac{\beta\vartheta_1}{\alpha\chi_1\chi_2} > 0$  and  $\Xi = -\frac{(\omega + \alpha\vartheta_1\vartheta_2)^2}{4\alpha\beta\vartheta_1\chi_1\chi_2}$ , we obtain the following periodic wave solutions

$$q_7^\pm(x, y, t) = \pm \sqrt{\frac{\omega + \alpha\vartheta_1\vartheta_2}{2\beta\vartheta_1}} \tan\left(\sqrt{\frac{\omega + \alpha\vartheta_1\vartheta_2}{2\alpha\chi_1\chi_2}}(\chi_1x + \chi_2y + (\alpha(\vartheta_1\eta_2 + \vartheta_2\eta_1))t)\right) \times \exp[i(-\vartheta_1x - \vartheta_2y + \varpi t + \theta_0)], \tag{4.10}$$

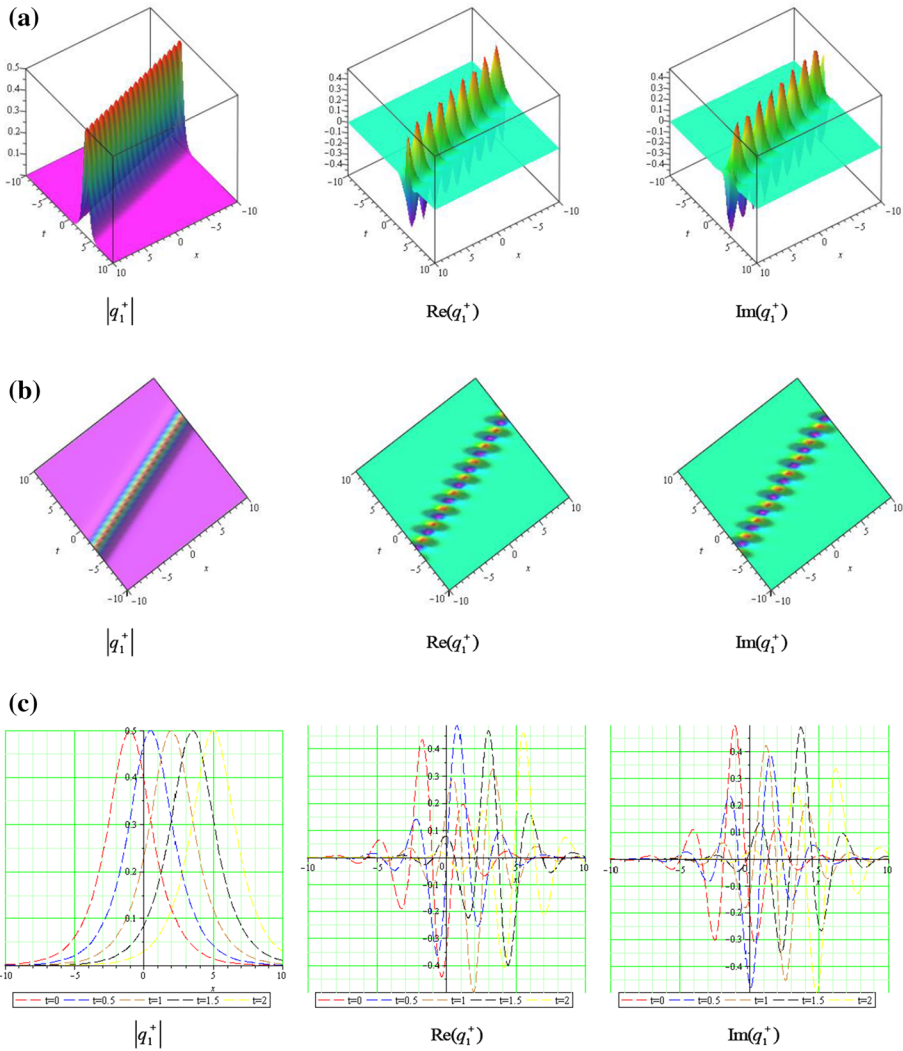
$$q_8^\pm(x, y, t) = \pm \sqrt{\frac{\omega + \alpha\vartheta_1\vartheta_2}{2\beta\vartheta_1}} \cot\left(\sqrt{\frac{\omega + \alpha\vartheta_1\vartheta_2}{2\alpha\chi_1\chi_2}}(\chi_1x + \chi_2y + (\alpha(\vartheta_1\eta_2 + \vartheta_2\eta_1))t)\right) \times \exp[i(-\vartheta_1x - \vartheta_2y + \varpi t + \theta_0)]. \tag{4.11}$$

### 5 Results and discussion

Figure 1 shows the graphs obtained from the space–time mapping of the solution  $q_1$ . It can be seen from the figure that the waves have a spatiotemporally extended homoclinic breather wave structure. In this respect, it can be concluded that this  $q_1$  solution can be useful in examining the dynamic behavior of rogue waves. It can also be seen that breather waves extend periodically along with time while extending at a certain angle with the X-axis spatially.

Figure 2 shows the graphs obtained from the space–time mapping of the solution  $q_4$ . Interestingly, although this solution seems to be the solution of heteroclinic waves at first glance, a careful look reveals the difference of the situation. This solution shows the existence of periodically extended homoclinic waves both spatially and temporally.

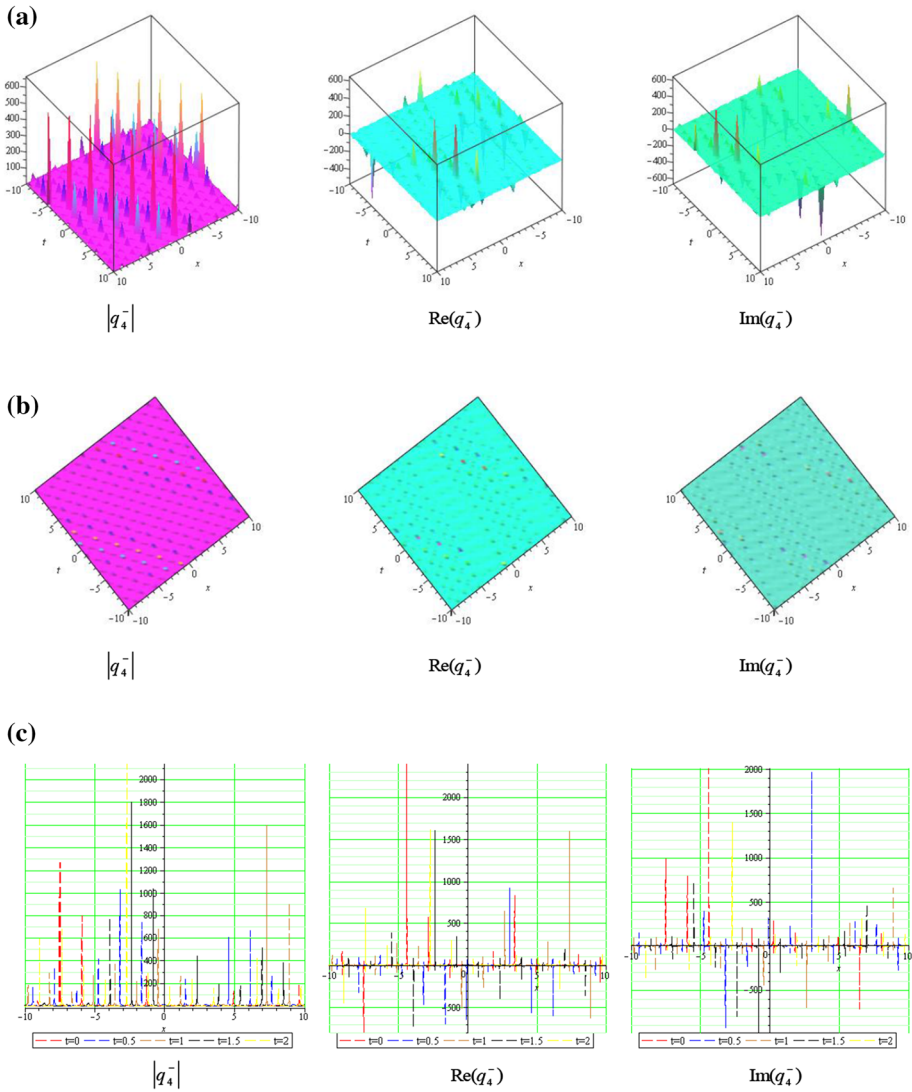
Figure 2 shows the graphs obtained from the space–time mapping of the solution  $q_5$ . From this solution, the existence of singular waves extended in time can be seen Fig. 3.



**Fig. 1** a 3D-plot for  $q_1^+$  b the contour plot for  $q_1^+$  c 2D-plot for  $q_1^+$  at  $t = 0, t = 0.5, t = 1, t = 1.5, t = 2$ . Respectively, when  $\omega = 3, \alpha = 1, \vartheta_1 = -2, \vartheta_2 = 1, \beta = 2, \chi_1 = 1, \chi_2 = 1, \eta_1 = 1, \eta_2 = 2, \varpi = 2, \theta_0 = 1$  and  $y = 0.5$

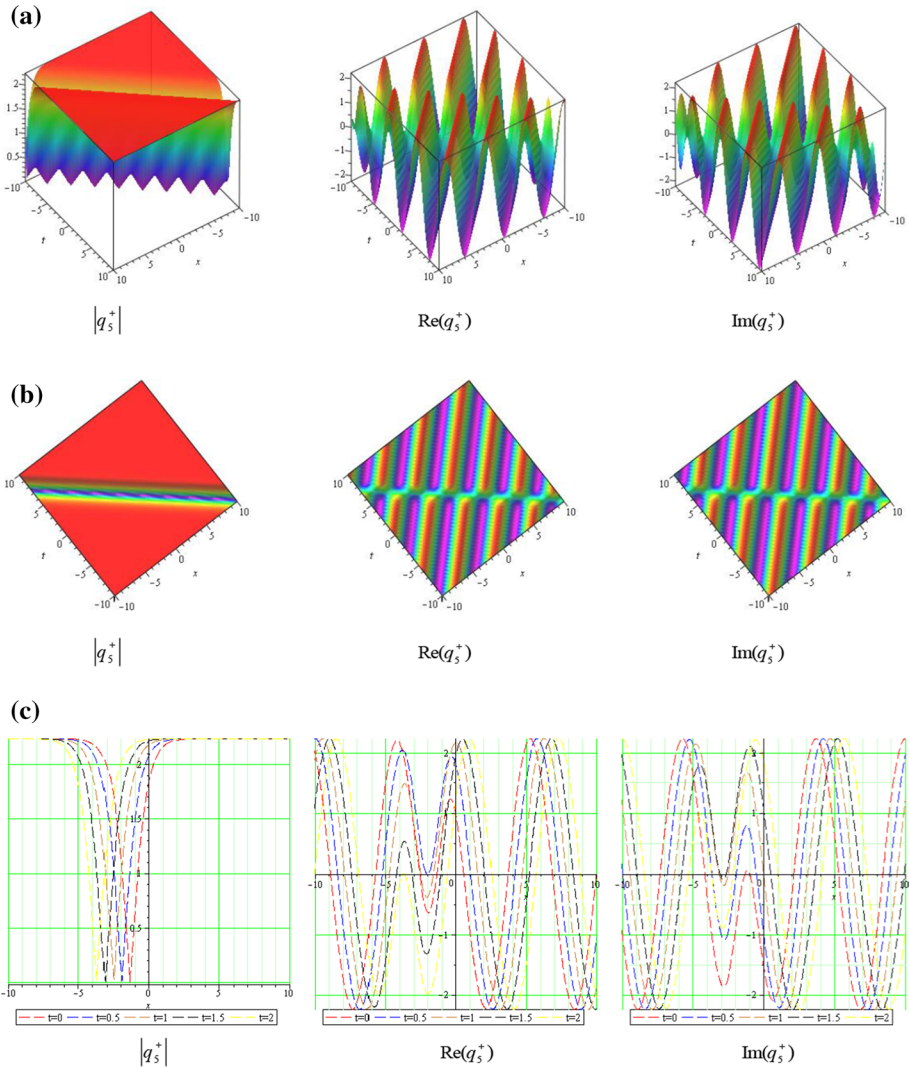
### 6 Conclusion

In this article functional variable method is applied to KMN model successfully to get the wave solutions of considered model. This model was first suggested not only to express the oceanic rogue waves but also to model the optical fiber communication. The solutions show that considered method fit well for nonlinear KMN model with complex structure. By the help of this work it is seen that functional variable method can be used as a powerful tool to obtain the exact solutions of nonlinear partial differential equations.



**Fig. 2** **a** 3D-plot for  $q_4^-$  **b** the contour plot for  $q_4^-$  **c** 2D-plot for  $q_4^-$  at  $t = 0, t = 0.5, t = 1, t = 1.5, t = 2$ . Respectively, when  $\omega = -4, \alpha = 1, \vartheta_1 = 0.5, \vartheta_2 = 1, \beta = 2, \chi_1 = 1.75, \chi_2 = 1.5, \eta_1 = 1, \eta_2 = 0.5, \varpi = 2, \theta_0 = 0$  and  $\gamma = 1.5$





**Fig. 3** **a** 3D-plot for  $q_1^+$  **b** the contour plot for  $q_1^+$  **c** 2D-plot for  $q_1^+$  at  $t = 0, t = 0.5, t = 1, t = 1.5, t = 2$ . Respectively, when  $\omega = -2, \alpha = 1, \theta_1 = 1, \theta_2 = 1, \beta = 0.1, \chi_1 = 1.5, \chi_2 = 1, \eta_1 = 0.75, \eta_2 = 1, \varpi = 1, \theta_0 = 1$  and  $y = 2$

### References

Alam, M.N., Osman, M.S.: New structures for closed-form wave solutions for the dynamical equations model related to the ion sound and Langmuir waves. *Commun. Theor. Phys.* **73**(3), 035001 (2021a)

Alam, M.N., Bonyah, E., Fayz-AI-Asad, M., Osman, M.S., Abualnaja, K.M.: Stable and functional solutions of the Klein–Fock–Gordon equation with nonlinear physical phenomena. *Phys. Scr.* **96**(5), 055207 (2021b)

Ali, K.K., Nuruddeen, R.I., Raslan, K.R.: New hyperbolic structures for the conformable time-fractional variant bussinesq equations. *Opt. Quant. Electron.* **50**(2), 61 (2018a)

- Ali, K.K., Nuruddeen, R.I., Hadhoud, A.R.: New exact solitary wave solutions for the extended (3+1)-dimensional Jimbo–Miwa equations. *Results Phys.* **9**, 12–16 (2018b)
- Atilgan, E., Senol, M., Kurt, A., Tasbozan, O.: New wave solutions of time-fractional coupled Boussinesq–Whitham–Broer–Kaup equation as a model of water waves. *China Ocean Eng.* **33**(4), 477–483 (2019)
- Barman, H.K., Roy, R., Mahmud, F., Akbar, M.A., Osman, M.S.: Harmonizing wave solutions to the Fokas–Lenells model through the generalized Kudryashov method. *Optik* **229**, 166294 (2021)
- Bekir, A., Güner, Ö., Aksoy, E., Pandir, Y.: Functional variable method for the nonlinear fractional differential equations. In: *AIP Conference Proceedings*, vol. 1648, No. 1, p. 730001. AIP Publishing LLC (2015)
- Ding, Y., Osman, M.S., Wazwaz, A.M.: Abundant complex wave solutions for the nonautonomous Fokas–Lenells equation in presence of perturbation terms. *Optik* **181**, 503–513 (2019)
- Ekici, M., Mirzazadeh, M., Eslami, M.: Solitons and other solutions to Boussinesq equation with power law nonlinearity and dual dispersion. *Nonlinear Dyn.* **84**(2), 669–676 (2016)
- El-Dessoky, M.M., Islam, S.: Resonant optical solitons of nonlinear Schrödinger equation with dual power law nonlinearity. *Phys. A: Stat. Mech. Appl.* **543**, 122445 (2019)
- Eslami, M., Mirzazadeh, M.: Optical solitons with Biswas–Milovic equation for power law and dual-power law nonlinearities. *Nonlinear Dyn.* **83**(1–2), 731–738 (2016)
- Eslami, M., Rezazadeh, H., Rezazadeh, M., Mosavi, S.S.: Exact solutions to the space-time fractional Schrödinger–Hirota equation and the space-time modified KDV–Zakharov–Kuznetsov equation. *Opt. Quantum Electron.* **49**(8), 279 (2017)
- Gao, W., Rezazadeh, H., Pinar, Z., Baskonus, H.M., Sarwar, S., Yel, G.: Novel explicit solutions for the nonlinear Zoomeron equation by using newly extended direct algebraic technique. *Opt. Quant. Electron.* **52**(1), 1–13 (2020)
- Gómez-Aguilar, J.F., Osman, M.S., Raza, N., Zubair, A., Arshed, S., Ghoneim, M.E., Emad, E.M., Abdel-Aty, A.H.: Optical solitons in birefringent fibers with quadratic-cubic nonlinearity using three integration architectures. *AIP Adv.* **11**(2), 025121 (2021)
- Kundu, A., Mukherjee, A., Naskar, T.: Modelling rogue waves through exact dynamical lump soliton controlled by ocean currents. *Proc. R. Soc. Math. Phys. Eng. Sci.* **470**(2164), 20 (2014)
- Kundu, P.R., Almusawa, H., Fahim, M.R.A., Islam, M.E., Akbar, M.A., Osman, M.S.: Linear and non-linear effects analysis on wave profiles in optics and quantum physics. *Results Phys.* **23**, 103995 (2021)
- Kurt, A.: New periodic wave solutions of a time fractional integrable shallow water equation. *Appl. Ocean Res.* **85**, 128–135 (2019)
- Kurt, A., Tasbozan, O., Baleanu, D.: New solutions for conformable fractional Nizhnik–Novikov–Veselov system via (G'/G)-expansion method and homotopy analysis methods. *Opt. Quantum Electron.* **49**(10), 333 (2017)
- Liu, J., et al.: Abundant exact solutions for the higher order non-linear Schrodinger equation with cubic-quintic non-Kerr terms. *Commun. Nonlinear Sci. Numer. Simul.* **15**(12), 3777–3781 (2010)
- Liu, L., Tian, B., Wu, X.Y., Sun, Y.: Higher-order rogue wave-like solutions for a nonautonomous nonlinear Schrödinger equation with external potentials. *Phys. A: Stat. Mech. Appl.* **492**, 524–533 (2018)
- Liu, J.G., Osman, M.S., Wazwaz, A.M.: A variety of nonautonomous complex wave solutions for the (2+ 1)-dimensional nonlinear Schrödinger equation with variable coefficients in nonlinear optical fibers. *Optik* **180**, 917–923 (2019a)
- Liu, J.G., Eslami, M., Rezazadeh, H., Mirzazadeh, M.: Rational solutions and lump solutions to a non-isospectral and generalized variable-coefficient Kadomtsev–Petviashvili equation. *Nonlinear Dyn.* **95**(2), 1027–1033 (2019b)
- Ma, W.X., Osman, M.S., Arshed, S., Raza, N., Srivastava, H.M.: Different analytical approaches for finding novel optical solitons in the single-mode fibers. *Chin. J. Phys.* (2021) (in press)
- Osman, M.S., Rezazadeh, H., Eslami, M., Neirameh, A., Mirzazadeh, M.: Analytical study of solitons to benjamin-bona-mahony-peregrine equation with power law nonlinearity by using three methods. *Univ. Polit. Buchar. Sci. Bull.-Ser. A-Appl. Math. Phys.* **80**(4), 267–278 (2018)
- Pettersson, P., Doostan, A., Nordstrom, J.: Level set methods for stochastic discontinuity detection in nonlinear problems. *J. Comput. Phys.* **392**, 511–531 (2019)
- Raslan, K.R., El-Danaf, T.S., Ali, K.K.: Exact solution of the space-time fractional coupled EW and coupled MEW equations. *Eur. Phys. J. Plus* **132**(7), 319 (2017)
- Raza, N., Aslam, M.R., Rezazadeh, H.: Analytical study of resonant optical solitons with variable coefficients in Kerr and non-Kerr law media. *Opt. Quantum Electron.* **51**(2), 59 (2019a)
- Raza, N., Afzal, U., Butt, A.R., Rezazadeh, H.: Optical solitons in nematic liquid crystals with Kerr and parabolic law nonlinearities. *Opt. Quantum Electron.* **51**(4), 107 (2019b)

- Rezazadeh, H.: New solitons solutions of the complex Ginzburg–Landau equation with Kerr law nonlinearity. *Optik* **167**, 218–227 (2018)
- Rezazadeh, H., Mirhosseini-Alizamini, S.M., Eslami, M., Rezazadeh, M., Mirzazadeh, M., Abbagari, S.: New optical solitons of nonlinear conformable fractional Schrödinger–Hirota equation. *Optik* **172**, 545–553 (2018)
- Rezazadeh, H., Korkmaz, A., Eslami, M., Mirhosseini-Alizamini, S.M.: A large family of optical solutions to Kundu–Eckhaus model by a new auxiliary equation method. *Opt. Quant. Electron.* **51**(3), 84 (2019)
- Rezazadehd, H., Mirzazadeh, M., Mirhosseini-Alizamini, S.M., Neirameh, A., Eslami, M., Zhou, Q.: Optical solitons of Lakshmanan–Porsezian–Daniel model with a couple of nonlinearities. *Optik* **164**, 414–423 (2018)
- Rizvi, S.T.R., Afzal, I., Ali, K.: Dark and singular optical solitons for Kundu–Mukherjee–Naskar model. *Mod. Phys. Lett. B* **34**(06), 2050074 (2020)
- Seadawy, A.R., Cheemaa, N.: Propagation of nonlinear complex waves for the coupled nonlinear Schrödinger Equations in two core optical fibers. *Phys. A: Stat. Mech. Appl.* **529**, 121330 (2019)
- Seadawy, A.R., Ali, K.K., Nuruddeen, R.I.: A variety of soliton solutions for the fractional Wazwaz–Benjamin–Bona–Mahony equations. *Results Phys.* **12**, 2234–2241 (2019)
- Seadawy, A.R., Nuruddeen, R.I., Aboodh, K.S., Zakariya, Y.F.: On the exponential solutions to three extracts from extended fifth-order KdV equation. *J. King Saud Univ. Sci.* **32**(1), 765–769 (2020)
- Sedeeg, A.K.H., Nuruddeen, R.I., Gómez-Aguilar, J.F.: Generalized optical soliton solutions to the (3+ 1)-dimensional resonant nonlinear Schrödinger equation with Kerr and parabolic law nonlinearities. *Opt. Quantum Electron.* **51**(6), 173 (2019)
- Sulaiman, T.A., Bulut, H.: The solitary wave solutions to the fractional Radhakrishnan–Kundu–Lakshmanan model. *Int. J. Mod. Phys. B* **33**(31), 1950370 (2019)
- Sulaiman, T.A., Bulut, H.: Optical solitons and modulation instability analysis of the (1+ 1)-dimensional coupled nonlinear Schrödinger equation. *Commun. Theor. Phys.* **72**(2), 025003 (2020)
- Sulaiman, T.A., Bulut, H., Atas, S.S.: Optical solitons to the fractional Schrödinger–Hirota equation. *Appl. Math. Nonlinear Sci.* **4**(2), 535–542 (2019a)
- Sulaiman, T.A., Nuruddeen, R.I., Mikail, B.B.: Dark and singular solitons to the two nonlinear Schrödinger equations. *Optik* **186**, 423–430 (2019b)
- Talarposhti, R.A., Jalili, P., Rezazadeh, H., Jalili, B., Ganji, D.D., Adel, W., Bekir, A.: Optical soliton solutions to the (2+1)-dimensional Kundu–Mukherjee–Naskar equation. *Int. J. Mod. Phys. B* **34**(11), 2050102 (2020)
- Tasbozan, O., Kurt, A., Tozar, A.: New optical solutions of complex Ginzburg–Landau equation arising in semiconductor lasers. *Appl. Phys. B* **125**(6), 104 (2019)
- Yang, M., Osman, M.S., Liu, J.G.: Abundant lump-type solutions for the extended (3 + 1)-dimensional Jimbo–Miwa equation. *Results Phys.* **23**, 104009 (2021)
- Yıldırım, Y.: Optical solitons to Kundu–Mukherjee–Naskar model in birefringent fibers with trial equation approach. *Optik* **183**, 1026–1031 (2019)
- Zerarka, A., Ouamane, S.: Application of the functional variable method to a class of nonlinear wave equations. *World J. Modell. Simul.* **6**(2), 150–160 (2010)
- Zhang, L., Simos, T.E.: An efficient numerical method for the solution of the Schrödinger equation. *Adv. Math. Phys.* (2016a) (in press)
- Zhang, W., Simos, T.E.: A high-order two-step phase-fitted method for the numerical solution of the Schrödinger equation. *Mediterr. J. Math.* **13**(6), 5177–5194 (2016b)
- Zhang, Z.Y., Li, Y.X., Liu, Z.H., Miao, X.J.: New exact solutions to the perturbed nonlinear Schrödinger's equation with Kerr law nonlinearity via modified trigonometric function series method. *Commun. Nonlinear Sci. Numer. Simul.* **16**(8), 3097–3106 (2011a)
- Zhang, Z.Y., Gan, X.Y., Yu, D.M.: Bifurcation behaviour of the travelling wave solutions of the perturbed nonlinear Schrödinger equation with Kerr law nonlinearity. *Z. Naturforschung A* **66**(12), 721–727 (2011b)