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Soliton solutions for time fractional ocean engineering models with Beta derivative

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ABSTRACT

In this study, the authors obtained the soliton and periodic wave solutions for time fractional symmetric regularized long wave equation (SRLW) and Ostrovsky equation (OE) both arising as a model in ocean engineering. For this aim modified extended tanh-function (mETF) is used. While using this method, chain rule is employed to turn fractional nonlinear partial differential equation into the nonlinear ordinary differential equation in integer order. Owing to the chain rule, there is no further requirement for any normalization or discretization. Beta derivative which involves fractional term is used in considered mathematical models. Obtaining the exact solutions of these equations is very important for knowing the wave behavior in ocean engineering models.

1. Introduction

Fractional calculus can be expressed as a generalized version of known calculus. In the beginning, fractionalizing the differentiation and integration seems to be a paradox. But by the time the necessity of this process is understood. Fractional calculus has neither geometrical nor physical explanation. So, a question can be asked: "What is the advantage of fractionalizing the differentiation or integration?". The answer is quite hard but can be explained as follows. By taking the global correlation into evaluation the historical dependence of the evolution of system analysis can be stated easily with fractional calculus. The theoretical mathematical models coincide with the experimental results when fractional calculus is used while establishing the model. In addition to these, fractional calculus has clearer physical importance and a simpler representation with a nonlinear model. Due to these advantages, fractional calculus became a very popular topic nowadays [16–24]. There are many different definitions of fractional calculus. The most popular ones are Grünwald-Letnikov, Riemann-Liouville, Caputo, Weyl, Hadamard, Atangana-Baleanu, Caputo- Fabrizio, conformable, etc. that are given in Oldham and Spanier [1], Miller and Ross [2], Podlubny [3] and Khalil et al. [4]. Each one has many different properties. Common property satisfied by all of them is linearity [1–3]. As a result of such diversity, the following questions arise. "What is the advantages of these derivatives over the other ones?" or "What is the advantages and disadvantages of these fractional operators?". For instance, initial/boundary value problems modelled by using the Riemann-Liouville operator must contain its conditions in the Riemann-Liouville sense. Caputo annihilated this disadvantage by his fractional operator named Caputo fractional derivative [3]. In addition to these, the derivative of a constant is not zero when the Riemann-Liouville fractional operator is used. Despite that, the derivative of constant is zero by using both Caputo and conformable derivatives. Also, Caputo, Riemann-Liouville, Grünwald-Letnikov and some other derivatives do not satisfy chain rule, the formula of the derivative of the product of two functions, formula of the derivative of the quotient of two functions, the property$D^{\beta}D^{\alpha}(f) = D^{\beta+\alpha}$ and so on [4]. After then Atangana expressed some basic criteria that fractional derivative operators must provide that need to be satisfied for the operator to be called fractional derivative [5]. Also, a new fractional operator that satisfies sixteen criteria be called fractional derivative was introduced by Atangana in his book called beta derivative with a fractional term [5].

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Definition 1. [5] Let $a \in \mathbb{R}$ and $g : [a, \infty] \to \mathbb{R}$ be a function. Then $\beta-$derivative of $g$is defined as:

$$ {}_0^A D_t^\beta g(t) = \begin{cases} \lim\limits_{\varepsilon \to \infty} \dfrac{g\left(t+\varepsilon\left(t+\frac{1}{\Gamma(\beta)}\right)^{1-\beta}\right)-g(t)}{\varepsilon} & for\, all\, t \geq 0,\ \beta = 0, \\ g(t) & for\, all\, t \geq 0,\ 0 < \beta \leq 1 \end{cases} $$

where Γ the gamma-function

$$ \Gamma(\nu) = \int_0^\infty t^{\nu-1} e^{-t} dt. $$

Atangana has proven some basic theorems by using this new fractional operator. He represented partial β derivative, $\beta-$divergence, $\beta-$gradient, $\beta-$curl, Clairaut's theorem for partial β-derivative, Green's theorem for β-derivative, Stokes's theorem for β-derivative, etc [5]. He expressed the fractional beta integral of a function as follows.

Definition 2. [5] Let $f : [a, b] \to \mathbb{R}$ be a continuous function on (a, b). The $\beta-$integral of the function $f$is given as follows.

$$ {}_0^A I_t^\beta (f(t)) = \int_0^t \left(x + \frac{1}{\Gamma(\beta)}\right)^{\beta-1} f(x) dx. $$

Recently, this new operator attracted great attention. A lot of studies are made using beta derivatives with fractional term.

In this study the analytical results for the Ostrovsky Equation (OE)

$$ uD_t^\beta D_x^2 u - D_x u D_t^\beta D_x u + u^2 D_t^\beta u = 0, $$

arising as a model of nonlinear waves in a rotating ocean and the analytical results for (1 + 1)-dimensional symmetric regularized long wave (SRLW) equation

$$ D_t^{(2\beta)} u + D_x^2 u + uD_t^\beta D_x u + D_t^\beta u D_x u + D_t^{(2\beta)} D_x^2 u = 0, $$

that the scientists benefit from while studying physical oceanography, dam breaking problems, river flooding, breakwater construction and control, coastal engineering and wave propagation in tsunami estimation.

Beta derivative is used as a tool in many studies. For instance, Zafar et al. [6] obtained the exact solutions of the DNA Peyrard–Bishop equation by three distinct methods. Gurefe [7] used the Kudryashov method to obtain the analytical solutions of some nonlinear fractional partial differential equations with beta derivative. Zahran and Khater [8] employed modified extended tanh-function method to Bogoyavlenskii equations with beta fractional derivative. Atangana and Alqahtani [9] modlled the spread of river blindness disease with beta derivative. Sub equation method is applied to some optical solitons by Martinez et al.[10]. Also, some other works can be seen in the literature [11–13].

Also, recently some studies on the wave profiles analysis of different mathematical models are made. For instance, Varsoliwala and Singh [25] expressed the mathematical modelling of atmospheric internal waves phenomenon and its solution. Kumar et. al. [26] obtained the abundant closed-form wave solutions and dynamical structures of soliton solutions to BLMP equation. Fahim et. al. [27] studied on wave profile analysis of a couple of (3+ 1)-dimensional nonlinear evolution equations. Kim et. al. [28] stated the analytical solution for one-dimensional nonlinear consolidation of saturated multi-layered soil.

To the best of our knowledge, it is firstly seen in the literature that the exact solutions of these equations have been obtained by using beta derivative.

2. Considered method

In this part a short explanation of the mETF [8] is given. Consider the following nonlinear partial differential equation:

$$ P(u, D_t^\beta u, D_x u, D_t^{(2\beta)} u, D_{xx} u, \ldots) = 0, \tag{1} $$

where $D_t^{(2\beta)} u$ means two times sequential beta derivative of function $u(x, t)$. By using the wave transformation$\chi = kx + \frac{c(\frac{1}{\Gamma(\beta)}+t)^\beta}{\beta}$, Eq. (1) turns into a nonlinear ordinary differential equation (NODE) as

$$ P(u, u', u'', u''', \ldots) = 0. \tag{2} $$

where the prime shows integer order differentiation of function u with respect to new independent variable χ. In the wave transform c denotes the harmonic wave frequency and k indicates the wave number. The new step is that the solution we are regarding in the form

$$ u(\phi) = A_0 + \sum_{j=1}^m \phi^j (\mathrm{A}_j + \mathrm{B}_j \phi^{-2j}). \tag{3} $$

where

$$ \phi' = b + \phi^2. \tag{4} $$

the parameter b can be evaluated later and $\phi = \phi(\chi)$, $\phi' = \frac{d\phi}{d\chi}$. By the balancing procedure the parameter m can be evaluated. Putting the Eqs. (3) and (4) into Eq. (2) will produce a nonlinear algebraic equation system with respect to A_j, B_j, b, c, k $(j = 1, 2, \ldots, m)$ by vanishing all the coefficients ofϕ^j. By using computer software, the values of constants A_j, B_j, b, k, c $(j = 1, 2, \ldots, m)$ can be examined. Considering these values and the following solution cases for Eq. (4)

Case 1. When $b < 0$ Hyperbolic function solution

$$ \phi(\chi) = -\sqrt{-b} \tanh\left(\sqrt{-b}\chi\right), $$
$$ \phi(\chi) = -\sqrt{-b} \coth\left(\sqrt{-b}\chi\right). $$

Case 2. When $b = 0$ Rational function solution

$$ \phi(\chi) = \frac{1}{\chi}. $$

Case 3. When $b > 0$ Hyperbolic function solution

$$ \phi(\chi) = \sqrt{b} \tan\left(\sqrt{b}\chi\right), $$
$$ \phi(\chi) = -\sqrt{b} \cot\left(\sqrt{b}\chi\right). $$

After than putting the values of $A_j, B_j, b, k, c$and the above mentioned cases in Eq. (3) respectively concludes the exact solutions for Eq. (1).

3. Applications of considered method

In this section, the exact solutions for OE equation [15] and SRLW equation [14] is obtained by using mETF where $D_t^\beta u$ means beta derivative of unknown function $u(x, t)$ with fractional term.

3.1. Solutions for SRLW equation

Consider the (1+1) dimensional SRLW equation

$$ D_t^{(2\beta)} u + D_x^2 u + uD_t^\beta D_x u + D_t^\beta u D_x u + D_t^{(2\beta)} D_x^2 u = 0. \tag{5} $$

Using the chain rule [5] and the transformation$\chi = kx + \frac{c(\frac{1}{\Gamma(\beta)}+t)^\beta}{\beta}$ for beta derivative concludes the following differential equation

$$ (k^2c^2 + ku)u'' + kc(u')^2 + k^2c^2 u^{(iv)} = 0 \tag{6} $$

where u dependent variable and χ is new independent variable. With the help of the balancing procedure in Eq. (6) $m = 2$ is obtained. Hence the auxiliary solution of the equation can be organized as

$$\psi(\chi) = a_0 + a_1\phi(\chi) + a_2\phi^2(\chi) + a_3(\phi(\chi))^{-1} + a_4(\phi(\chi))^{-2}. \tag{7}$$

Subrogating the Eq. (7) into Eq. (6) and regarding Eq. (4) lead to an equation involving the function $\phi(\chi)$ and its powers. Vanishing all the coefficients of the powers yields an equation system with respect to variables $b, k, c, a_0, a_1, a_2, a_3, a_4$. Solving this system with the aid of symbolic computer software, following solution sets yields.

Set 1:

$$a_0 = \frac{-8bc^2k^2 - c^2 - k^2}{ck},\ a_1 = 0,\ a_2 = 0,\ a_3 = 0,\ a_4 = -12b^2ck.$$

Set 2:

$$a_0 = \frac{-8bc^2k^2 - c^2 - k^2}{ck},\ a_1 = 0,\ a_2 = -12ck,\ a_3 = 0,\ a_4 = -12b^2ck.$$

Substituting the solution sets in Eq. (7), the solution cases for Eq. (4) and wave transformation the exact solutions for SRLW equation can be evaluated as follows

Solutions For Set 1.

$$u_1(x,t) = \frac{-8bc^2k^2 - c^2 - k^2}{ck} + 12bck\coth^2\left(\sqrt{-b}\left(\frac{c\left(\frac{1}{\Gamma(\beta)} + t\right)^{\beta}}{\beta} + kx\right)\right),$$

$$u_2(x,t) = \frac{-8bc^2k^2 - c^2 - k^2}{ck} + 12bck\tanh^2\left(\sqrt{-b}\left(\frac{c\left(\frac{1}{\Gamma(\beta)} + t\right)^{\beta}}{\beta} + kx\right)\right),$$

$$u_3(x,t) = \frac{-8bc^2k^2 - c^2 - k^2}{ck} - 12bck\cot^2\left(\sqrt{b}\left(\frac{c\left(\frac{1}{\Gamma(\beta)} + t\right)^{\beta}}{\beta} + kx\right)\right),$$

$$u_4(x,t) = \frac{-8bc^2k^2 - c^2 - k^2}{ck} - 12bck\tan^2\left(\sqrt{b}\left(\frac{c\left(\frac{1}{\Gamma(\beta)} + t\right)^{\beta}}{\beta} + kx\right)\right).$$

Solutions For Set 2.

$$u_5(x,t) = \frac{-8bc^2k^2 - c^2 - k^2}{ck} + 12bck\coth^2\left(\sqrt{-b}\left(\frac{c\left(\frac{1}{\Gamma(\beta)} + t\right)^{\beta}}{\beta} + kx\right)\right) + 12bck\tanh^2\left(\sqrt{-b}\left(\frac{c\left(\frac{1}{\Gamma(\beta)} + t\right)^{\beta}}{\beta} + kx\right)\right),$$

$$u_6(x,t) = \frac{-8bc^2k^2 - c^2 - k^2}{ck} - 12bck\cot^2\left(\sqrt{b}\left(\frac{c\left(\frac{1}{\Gamma(\beta)} + t\right)^{\beta}}{\beta} + kx\right)\right) - 12bck\tan^2\left(\sqrt{b}\left(\frac{c\left(\frac{1}{\Gamma(\beta)} + t\right)^{\beta}}{\beta} + kx\right)\right),$$

Let us give the graphical representations for the abovementioned equations.

Figs. 1 and 2 indicate the soliton solution and Figs. 3 and 4 represent the periodic wave solution of the SRLW equation.

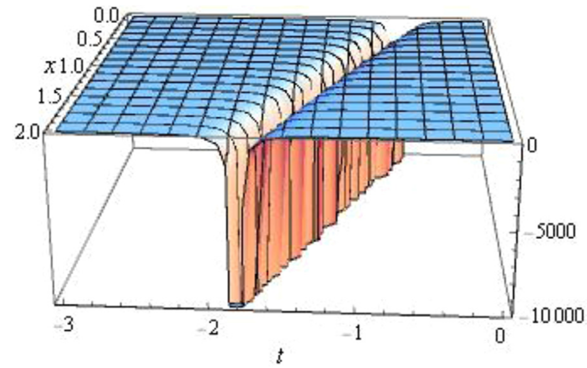

Fig. 1. Graphical representation of the $u_1(x,t)$ for $\beta = 0.8$, $c = 0.5$, $k = 0.8$, $b = -1$.

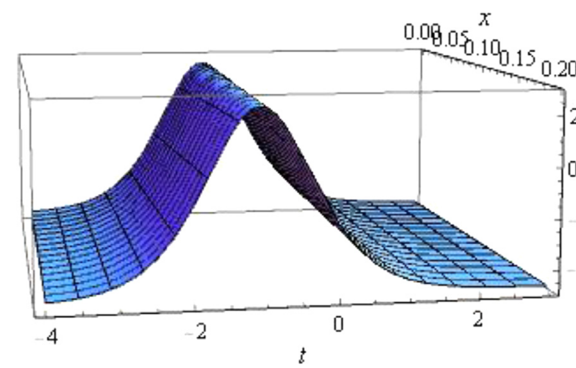

Fig. 2. Graphical simulation of the soliton solution of $u_2(x,t)$ for $\beta = 0.5$, $c = 0.5$, $k = 0.8$, $b = -1.5$.

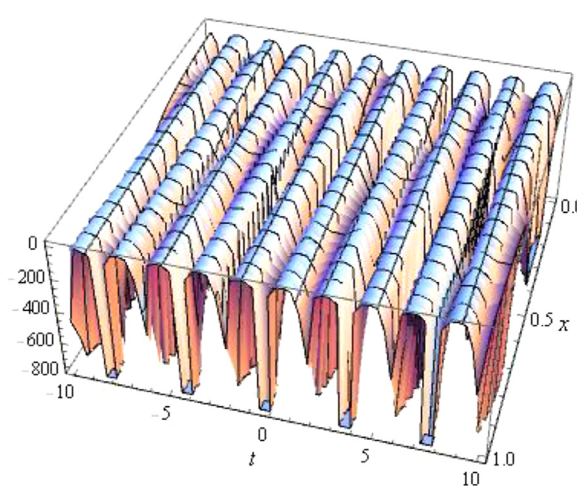

Fig. 3. Graphical representation of the solution $u_3(x,t)$ for $\beta = 0.3$, $c = 0.5$, $k = 0.8$, $b = 4$.

3.2. Solutions for OE Equation

Regard the OE as

$$uD_t^{\beta}D_x^2u - D_xuD_t^{\beta}D_xu + u^2D_t^{\beta}u = 0 \tag{8}$$

Using the chain rule [5] and the transformation $\chi = k\left(x - \frac{c\left(\frac{1}{\Gamma(\beta)}+t\right)^{\beta}}{\beta}\right)$, for beta derivative concludes the following differential equation

$$k^2(uu''' - u''u') + u^2u' = 0 \tag{9}$$

where u dependent variable and χ is new independent variable. With the help of the balancing procedure in Eq. (8) $m = 2$ is ob-

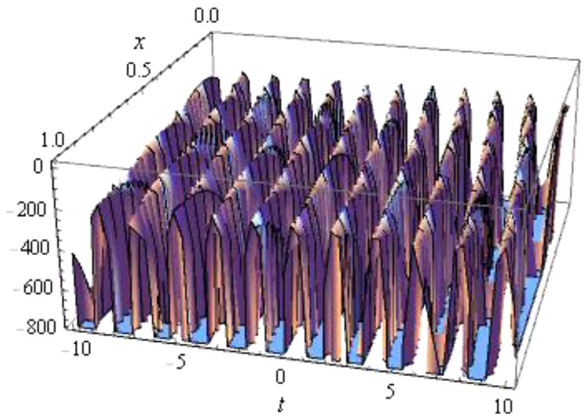

Fig. 4. 3D graphics of the periodic wave solution of $u_3(x,t)$ for $\beta=0.5$, $c=2$, $k=5$, $b=2$.

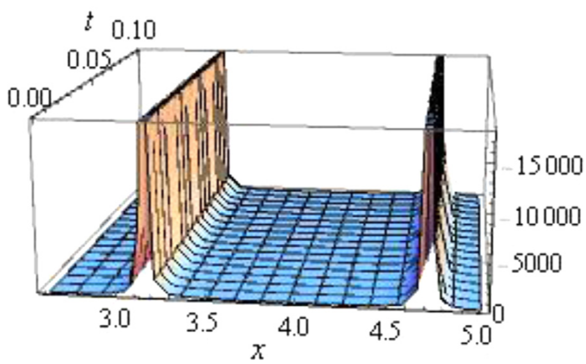

Fig. 5. 3D graphical simulation for double soliton solution of $u_1(x,t)$ for $\beta=0.9$, $c=0.001$, $k=1$, $b=1$.

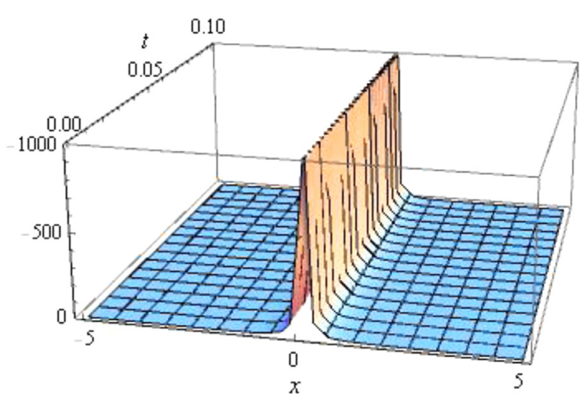

Fig. 6. Graphical representation of the cuspon solution of $u_1(x,t)$ for $\beta=0.9$, $c=0.001$, $k=1$, $b=1$.

tained. Hence the auxiliary solution of the equation can be organized as

$$\psi(\chi)=a_0+a_1\phi(\chi)+a_2\phi^2(\chi)+a_3(\phi(\chi))^{-1}+a_4(\phi(\chi))^{-2}. \tag{10}$$

Subrogating Eq. (10) into Eq. (9) and regarding Eq. (4) leads to an equation involving the function $\phi(\chi)$ and its powers. Vanishing all the coefficients of the powers yields an equation system with respect to variables $b, k, c, a_0, a_1, a_2, a_3, a_4$. Solving this system with the aid of symbolic computer software, following solution set yields.

Set 1:

$$\mathrm{a}_0=-12bk^2,\ \mathrm{a}_1=0,\ \mathrm{a}_2=-6k^2,\ \mathrm{a}_3=0,\ \mathrm{a}_4=-6b^2k^2.$$

Substituting the solution set in Eq. (10), the solution cases for Eq. (4) and wave transformation the exact solutions for OE equation can be evaluated as follows:

Solutions For Set 1.

$$\begin{aligned}u_1(x,t)=&\ 6bk^2\coth^2\left(\sqrt{-bk}\left(x-\frac{c\left(\frac{1}{\Gamma(\beta)}+t\right)^{\beta}}{\beta}\right)\right)\\&+6bk^2\tanh^2\left(\sqrt{-bk}\left(x-\frac{c\left(\frac{1}{\Gamma(\beta)}+t\right)^{\beta}}{\beta}\right)\right)-12bk^2,\end{aligned}$$

$$\begin{aligned}u_2(x,t)=&-6bk^2\cot^2\left(\sqrt{bk}\left(x-\frac{c\left(\frac{1}{\Gamma(\beta)}+t\right)^{\beta}}{\beta}\right)\right)\\&-6bk^2\tan^2\left(\sqrt{bk}\left(x-\frac{c\left(\frac{1}{\Gamma(\beta)}+t\right)^{\beta}}{\beta}\right)\right)-12bk^2.\end{aligned}$$

The graphical representations for obtained results can be given as follows.

In Fig. 5 double soliton solution is obtained. When the Fig. 6 is considered it is seen that a cuspon solution is obtained.

4. Conclusion

In this article, the periodic wave solutions and soliton solutions for OE and SRLW equations are obtained. Both of these equations are arising as a mathematical model in ocean engineering. Our work reported here is a first step towards understanding the structural and physical behaviour of ocean models. We hope that our work will be very useful in better understanding the wave occurring at coastal and harbor regions of the oceans. We believe our manuscript is very timely and will interest the broad range of scientists working on ocean engineering models. A soliton or solitary wave is a self-reinforcing wave packet that continues its shape while it propagates at a constant velocity. Solitons are caused by a cancellation of nonlinear and dispersive effects in the medium. Solitons are the solutions of an extensive class of weakly nonlinear dispersive partial differential equations describing physical and engineering systems. A periodic travelling wave is a periodic function of one-dimensional space that moves with constant speed. Consequently, it is a special type of spatiotemporal oscillation that is a periodic function of both space and time. Periodic travelling waves play a fundamental role in many mathematical equations, including self-oscillatory systems, excitable systems and reaction-diffusion–advection systems.

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Declaration of Competing Interest

Authors declare that they have no conflicts of interest to report regarding the present study.

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