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Research Article

Mathematics student teachers' task design processes: The case of History, Theory, Technology, and Modeling

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This study adopts a holistic single-case design to explain the task design processes of mathematics student teachers (MSTs) regarding History, Theory, Technology, and Modeling (HTTM). A criterion sampling method was used to select nine MSTs who had successfully completed *algorithms and programming* course. Video analyses, written answer sheets, scratch papers, and GeoGebra files were used to obtain data, including the views of MSTs on HTTM task design. Data analysis was performed using a content analysis method based on the theoretical framework of HTTM learning. The results revealed that HTTM design processes included task, focus/origin, problem, design (prototype), results, and approved reports. Furthermore, the mental steps that connected these basic components were found to be investigating, exploring, designing, evaluating, revising, and reporting. One of the key challenges experienced by the MSTs was found to be spending a great amount of time especially while determining a focus. The study has been finalised with a set of suggestions for future designs.

Keywords: HTTM learning process; Mathematical modeling; Technology-aided mathematics education; Task design; Mathematics student teacher

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1. Introduction

With the rise of the constructivist paradigm shaped by cognitive and social understanding in education in the late 20th century, mathematical modeling has become one of the most significant concepts in the literature. In addition to facilitating the evaluation of students (Borromeo Ferri, 2013; Peter Koop, 2009), this paradigm provides learning environments that are rich in cognitive and metacognitive processes (Borromeo Ferri, 2006; Lesh & Doerr, 2003; Stillman et al., 2007), where students can develop the necessary 21st-century skills and knowledge by using it in learning activities or problems. Mathematical modeling has emerged as one of the most influential tools in mathematics education today. Mathematical modeling is essentially the process of mathematically expressing real-world situations and events through mathematical models (Berry & Houston, 1995; Blum & Niss, 1991). The mathematical model that emerges in the mathematical modeling process refers to mathematical representations in which the relationship between two or

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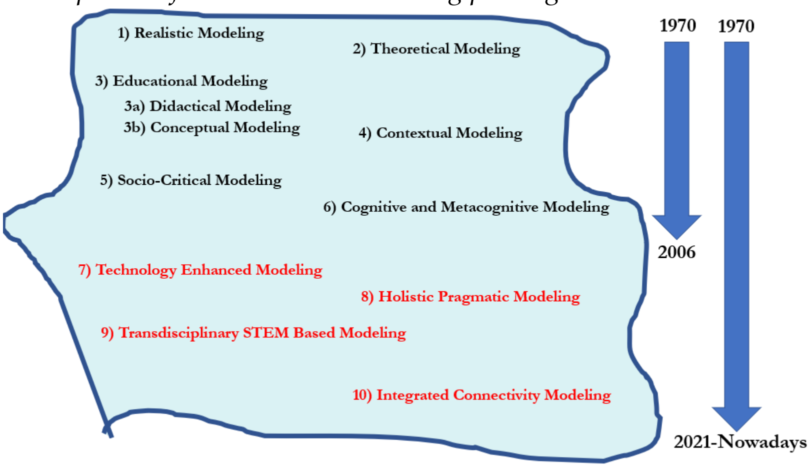
more variables related to a real-life situation is explained in different ways such as graphs, equations, inequalities, and system of equations (Berry & Houston, 1995; Kapur, 1982).

Numerous researchers have studied mathematical modeling in education since 1970 from various perspectives. Kaiser and Sriraman (2006), and Blomhøj (2009) divided these different perspectives into six groups as realistic/applied modeling, theoretical/epistemological modeling, educational modeling (didactic and conceptual), contextual modeling, socio-critical modeling, and cognitive modeling. Despite their differences in basic philosophies, goals, subject areas, and perspectives on education, these six approaches play a vital role in supporting each other from several perspectives. *Realistic modeling* deals with associating mathematical information with different real-life contexts by utilizing problem situations in engineering and other fields. Rather than primarily focusing on real-life context, *theoretical modeling* tries to reveal the mathematical concepts and theories in the solutions of modeling problems as well as the relationships between these. *Socio-critical modeling* focuses on socio-cultural, ethno-mathematics and contemporary dimensions of mathematics. *Educational modeling* considers conceptual learning as the most basic goal and gives particular importance to the objectives and outcomes of the curriculum for this purpose. In educational modeling approach, if necessary, limitations can be made in mathematical modeling problems according to the levels of students. *Contextual modeling* contains non-artificial and meaningful real-life situations. *Cognitive and metacognitive modeling* focuses on students' mental actions in the modeling process.

Based on the fact that the classification summarized above does not take into account mathematical modeling studies carried out since 2006, it can be claimed that current classifications are needed today. Kaiser and Sriraman (2006) had already drawn our attention to the scarcity of cognitive modeling studies, and it is clear that in the past 15 years, both cognition-metacognition studies and technology integration in mathematical modeling have increased significantly. Mathematical modeling is also considered the basic teaching method in integrated STEM (Çorlu, 2021). As a learning approach, STEM integrates the disciplines of science, technology, engineering, and mathematics (Çorlu, 2021). Looking at the most recent studies, it can be suggested that the connectional integrity paradigm (Kılıç, 2021), which is predicated on the chaos theory and based on the neuroscientific studies of the last 10 years, will be distinguished in mathematical modeling approaches in education. Kılıç (2021) noted that the most advanced information processing system is life itself rather than the human brain and that all the biological and physical things exist in an extraordinary interconnectedness. The more a person can reach this interconnectedness within the complexity, the more qualified brain he will have. Mathematical modeling will come to the fore in human's reaching this complex connectedness. In this sense, Figure 1 can be interpreted as a new classification for modeling perspectives.

Figure 1
Development of mathematical modeling paradigms in math education



Note. (1 and 2 → scientific paradigm, 3,4,5 and 6→ education based paradigm, 7,8,9 and 10→ integrative paradigm)

The six different modeling perspectives discussed by Kaiser and Sriraman (2006) basically aim to provide learning and to raise individuals who will be more successful in real life. However, different points primarily focused by each of them determine their priorities and cause them to leave some important points aside. While realistic modeling emphasizes daily life too much, it does not care about the theory behind daily life as much as theoretical modeling. Actually, both are important. Contextual modeling requires working like an engineer and handles daily life situations and does not want to simplify them. In educational modeling, on the other hand, simplification of the real-life context according to the student level is a possible strategy. The underlying reason is that in order for the student to learn a concept, a learning environment suitable for his/her mental level is necessary. Socio-critical modeling aims to study current complex problems such as COVID and ignores other problems (such as historical problems). Cognitive and metacognitive modeling focuses more on the depth and richness of mental actions; however, it does not focus on determining the emergence of disciplinary and interdisciplinary concepts and possible misconceptions in the process (*educational modeling cares.*). In technology-enhanced modeling, technology is the focus and other perspectives are more in the background. As can be seen, each perspective looks at events from its own perspective (Blomhøj, 2009). For this reason, learning environments that will be designed with **a holistic pragmatic understanding** with the embracement of the strenghts of these different approaches will provide an environment where important points that are sometimes put aside are highlighted. Designed with this perspective, HTTM incorporates different perspectives in modeling and presents a learning process that takes into account interdisciplinary thinking strategies such as computational thinking and STEM, which are essential in the 21st century. For example, HTTM cares about socio-critical modeling in designing current modeling problems. In this way, students also reveal the link between the historical problem and the current problem (Hıdıroğlu & Özkan Hıdıroğlu, 2016). This brings a different and rich perspective to the dual modeling process (Saeki & Matsuzaki, 2013), which is prominent in cognitive and metacognitive modeling. As another example, HTTM incorporates both realistic and theoretical modeling, emphasizing the evolution of theories and real-life problems in history of science. HTTM cares about the newspaper article such as contextual modeling and includes in the modeling problem into the HTTM learning process in its natural state in the history of science. The HTTM learning approach carries disciplines such as history, art, architecture, biology, chemistry, geography, physics, technology, engineering and mathematics to the process and offers a current and different bridge that will provide interdisciplinary learning environments such as STEM. HTTM feeds on historical problems and these problems such as contextual ones create suitable environments for interdisciplinary learning environments. Çorlu et al. (2014), regard mathematical modeling as the core skill of the mathematics in the Integrated STEM approach. The HTTM learning process creates environments that will serve for Integrated STEM framework by Çorlu et al. (2014), nine different STEM integration models by Bybee (2013), and the last three of STEM integration levels (multidisciplinary, interdisciplinary and transdisciplinary) by Vasquez et al. (2013). In this sense, HTTM (History/ Theory/ Technology/ Modeling) learning process, developed with a holistic pragmatic understanding supported by postmodernism, aims to create richer learning environments by using other modeling approaches together more effectively.

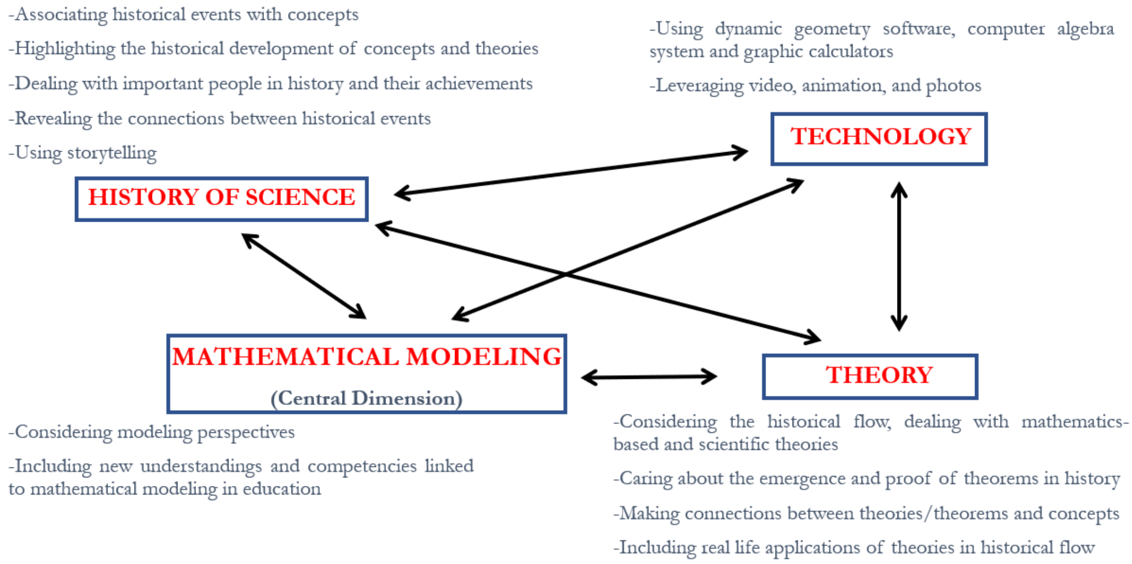
2. Holistic Pragmatic Modeling: HTTM Learning Process

2.1. Dimensions of the HTTM learning process

In the most general sense, HTTM learning model aims to develop mathematical modeling skills in the environments enriched with theories in mathematics, physics, chemistry and biology, history of science, and technology in the learning process (Hıdıroğlu & Özkan Hıdıroğlu, 2016). Since HTTM learning model is formed by the synthesis of the modeling approaches, its most basic dimension is modeling. Modeling in HTTM represents the processes of creating mathematical models, which are the basic parts of mathematics, physics, chemistry and biology (see Figure 2).

Figure 2

Dimensions of HTTM learning process (Hıdıroğlu, 2021)



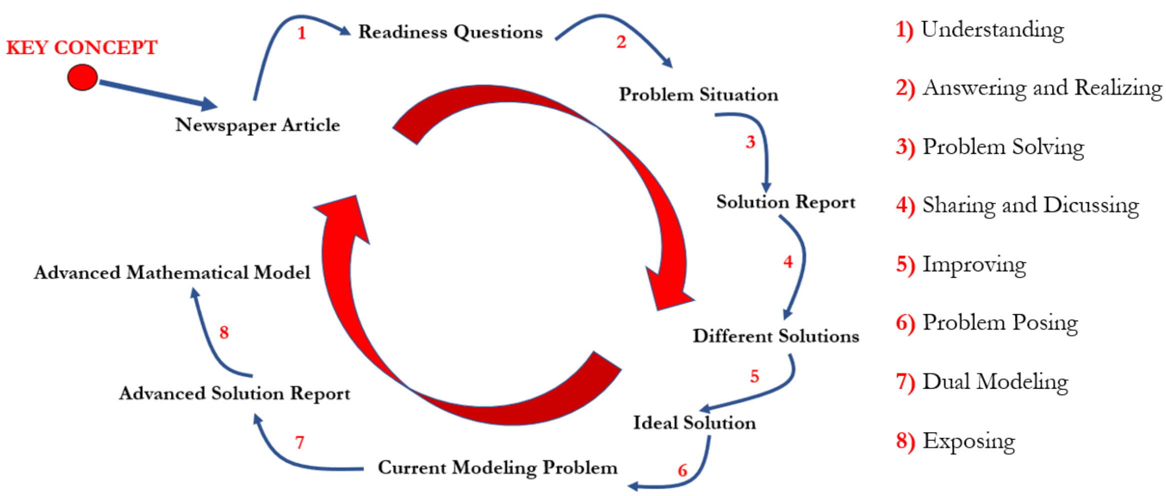
According to HTTM learning approach, *the history of science* gives important clues about how the conceptual development should be in the learning process. Developments at all levels in history, such as the formation of the concepts in science, the development of the theories, and the studies in the field of engineering, are of great importance in constructing the learning processes. *Theories*, on the other hand, are one of the most essential building blocks in the history of science. Since the theories progress in parallel with the development of basic concepts, learning environments that will be designed considering the historical development of the theories serve as an important map for the flow of conceptual development in the learning process. According to HTTM, revealing the relationships between historical progress and theories and concepts will provide a rich mental process in both the development of concepts and the discovery of theories in the learning process. In HTTM learning process, as Swetz (1994) emphasized, including the lives and works of the figures in mathematics/science field, identifying historical problems and revealing their importance, performing activities based on historical problems or discoveries, and using historical films or videos in classroom instruction are critical strategies. In HTTM learning process, the history of science reveals the development process of scientific knowledge and concepts. How the mathematical problems in the history of science are solved, where their solutions are used and how they can support future studies, are of great importance in revealing the development processes of the concepts. According to HTTM, rather than the difficulties in learning processes without technology, the difficulties faced in technology-enriched environments are noteworthy; because learning is affected by existing technology and there is need for individuals who can use this technology effectively in real life and reach innovations (Hıdıroğlu & Can, 2020). Ellis et al. (2020) proposed a classification into the roles of technology with four dimensions [(a) engineering product, (b) instructional technology, (c) computational thinking, and (d) tools and practices used by science, mathematics engineering practitioners)]. In HTTM learning process, in the engineering product role of technology, advanced mathematical models that can be used as a prototype are essential while computer software, video, animation and photographs are essential in the instructional technology role. On the other hand, in its role of computational thinking, high-level mental processes enriched with technology emerging throughout the process are remarkable. With the enrichment of HTTM learning process with technology, considerable opportunities such as eliminating operational difficulties, concretizing abstract concepts and relationships between concepts, visualizing mathematical formulas and relationships, presenting detailed solutions, and making more general assumptions and generalizations are aimed to reach.

2.2. HTTM Learning Process

HTTM learning process includes an application process that connects nine basic components (newspaper article, readiness questions, problem situation, solution report, different solutions, ideal solution, current modeling problem, advanced solution report, and advanced mathematical models) with nine basic steps (understanding, answering and realizing, problem solving, sharing and discussing, improving, problem posing, dual modeling and exposing) (Hıdıroğlu & Özkan Hıdıroğlu, 2016, see Figure 3). HTTM learning tasks can be applied individually or in groups of 3-5 people, depending on the teacher's preference, the ease and effectiveness of application, and the student level. Group work is specifically required because students are involved in more advanced solution processes (Hıdıroğlu, 2012, 2015); on the other hand, the individual modeling process is also essential in revealing the cognitive level, identifying and evaluating weaknesses (Borromeo Ferri, 2006; Hıdıroğlu & Özkan Hıdıroğlu, 2016).

According to Hıdıroğlu and Özkan Hıdıroğlu (2016), students are given a newspaper article at the beginning of HTTM learning process and they are expected to understand the plot (there can be storytellings) based on the history of science. Next, through readiness questions, it is ensured that the students have thoroughly understood and questioned the text, and so they are expected to make predictions about the basic problem situation. Video, animation, photograph, or 3D models may be used along with the newspaper article (as in the HTTM tasks called Archimedes, the Lighthouse of Alexandria, Galileo, and the Tower of Pisa Experiment).

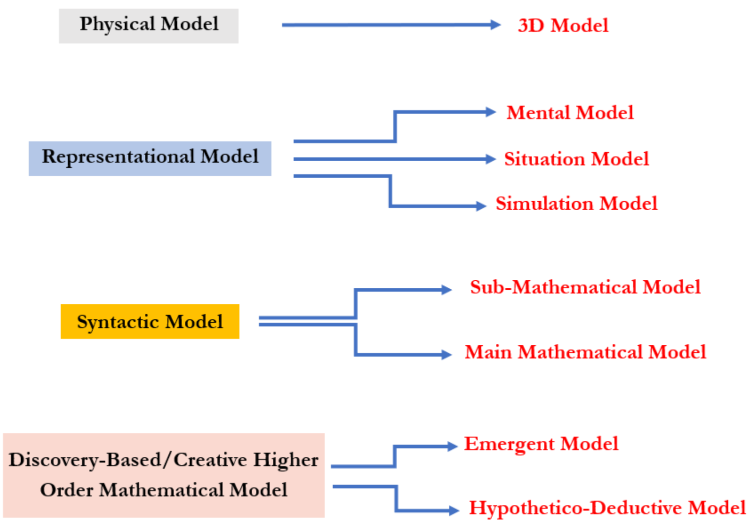
Figure 3
The learning process of HTTM model



In the third step, students solve the given problem in technology-supported mathematical modeling process. This is explained with the process model of Hıdıroğlu (2015) which consists of nine basic steps, nine basic components and 55 sub-steps (see Figure 3). In the third step, dynamic mathematics and geometry software are involved in the solution process and enrich the mental actions in mathematical modeling. In the fourth step, different solutions are presented and discussed in the classroom setting. In the fifth step, students are asked to refine their solutions by considering different ideas emerging in the discussion. In the sixth step, a current mathematical modeling problem is designed based on the HTTM task. The seventh step of HTTM learning process, which is a similar process, is explained with dual modeling of Matsuzaki and Saeki (2013) and Saeki and Matsuzaki (2013). Dual modeling involves the use of thoughts or mathematical models in previous mathematical modeling problems in the new problem and associating these problems. In the seventh step, the solution based on the history of science and the solution of the current problem are intertwined and dual modeling emerges. In the eighth step, advanced mathematical models (emergent model and hypothetico-deductive model) are created in a different and more advanced form from several aspects. Different models such as physical (3D puzzle), mental, simulation, syntactic, emergent models that emerge in HTTM learning process, play a key role in simplifying a theory, explaining it in detail, revealing a concept, associating concepts, revealing deficiencies or misconceptions in conceptual development, and describing real life and interdisciplinary relationships. There are different models that can emerge at different mental levels in the learning process. According to HTTM, the structure of the mathematical models produced in the learning process and the interaction of these models with each other provide mental support to the learner in conceptual learning and encourage creativity, and so prepare the environment for the creation of advanced mathematical models (see Figure 4).

Figure 4

The different models revealed in HTTM learning process (Hıdıroğlu & Özkan Hıdıroğlu, 2016)



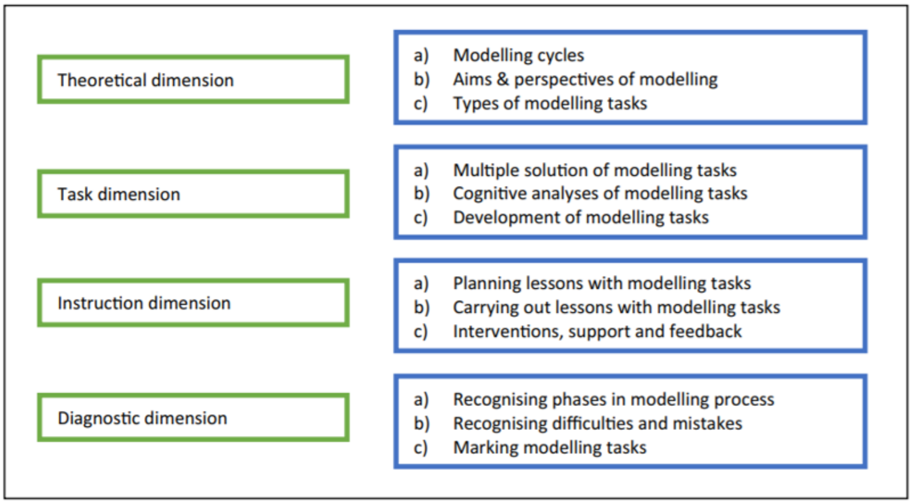
3. Mathematical Modeling Task Design

Basically, mathematical modeling includes defining a problem in a real-world context, developing a mathematical representation of the problem, determining a mathematical solution, interpreting the solution in the original context, and evaluating the validity of the solution (Geiger et al., 2022). The perpetually improvable open-ended nature of real-world problems both provides richness in the process and explains the cyclical nature of modeling (Blomhøj & Jensen, 2003). None of the process models that explain mathematical modeling require a path to follow necessarily; because both individual mental differences and real-life context as well as the complex relationship between these two are deep (Geiger et al., 2022; Hıdıroğlu & Bukova Güzel, 2017). Transitions between the steps can occur non-sequentially and in a non-linear way throughout the cycle. Therefore, the design of mathematical modeling tasks is crucial in creating rich mental environments.

Task design is one of the main research topics in mathematics education (Watson & Ohtani, 2015). It is highly essential for task designers to realize that the details of task design can affect students and their performances (Sierpinska, 2004). According to Venturini and Sinclair (2017), a task should provide students with hints to improve their mathematical reasoning skills and therefore should consist of well-prepared questions and instructions. In addition, it is also suggested to provide opportunities to use multiple strategies in mathematical activities, instead of a single pathway to a solution (Yerushalmy et al., 2017). Watson and Ohtani (2015) emphasized that the content of the task is of vital importance in the task design, that the tasks should be at the core of the classroom environment, and they enable students to understand the nature of mathematics in the context by improving their real-life experiences. It is difficult to come up with a strict classification of tasks, since the complexity of a task depends strongly on the student's knowledge or competence level (Venturini & Sinclair, 2017). Therefore, teachers need to adjust the tasks devised or used according to the levels of their classes in order for them to effectively implement the tasks and achieve the intended goals (Komatsu & Jones, 2019). Designing "good" tasks necessitates an interface between learner and task, between practice and theory, and between actual and intended practice (Liljedahl et al., 2007).

Questions regarding how to enhance mathematical modeling competencies best, still remain unresolved (Geiger et al., 2022). Borromeo Ferri (2018) and Borromeo Ferri and Blum (2010) introduced dimensions that were developed theoretically from a competency-based perspective, presented in Figure 5. While one of the four basic dimensions is the task dimension, task design has a complex nature that includes the dominance of these four basic dimensions. Although this structure provides us the scope of modeling competencies, the relationship between these structures is not that simple.

Figure 5
Model for competencies needed in teaching mathematical modeling (Borromeo Ferri, 2018)



Maaß (2010) remarked that the quality of a task is the key to developing modeling competence. According to Maaß, the first step in enhancing the quality is to define the focus of the modeling task, the nature of its relationship with reality, the type of model used, the type of representation, the clarity of a task, its cognitive demand, and its mathematical content. This is related to the quality of the process of task design. Reviewing the studies in the literature, it is seen that while there often exist several explanations about the selection of suitable modeling tasks for certain student groups, the number of studies on how to design and implement these tasks is limited (Geiger et al., 2022). The design and implementation framework for mathematical modeling tasks (DIFMT) presented by Geiger et al. (2022) consists of two components including tasks design and pedagogical architecture. One of the few frameworks to address this gap is the design principles provided by Galbraith (2006) for real-world tasks. It should be noted that Galbraith's principles provide necessary conditions, although not sufficient, for an effective task (Geiger et al., 2022). While task design, from the perspective of Galbraith (2006; see Table 1), is primarily concerned with the development of a modeling task, Pedagogical Architecture shaped by the perspective of Czocher (2017) is about classroom implementation part of it. Although these principles are of great importance in the design process, a process model that explains the mathematical modeling task design process has not been encountered in mathematical modeling in the literature.

Table 1
Task design principles (Galbraith, 2006)

Nature of Problem
Problems should be open-ended and contain both mathematical and non-mathematical information. The degree of open-endedness depends on students' previous experience with modeling. Less experienced students may need additional scaffolding questions or information. More experienced students should be expected to engage in less limited problems.
Relevance and Motivation
Problems should be linked to students' real-life experiences. This is related with the factors such as students' age, year, personal circumstances, living conditions, etc. Problems may need to be contextualized for specific student groups.
Accessibility
There is an opportunity to define and specify mathematically managable questions from a general problem statement. Is there a mathematical approach accessible to students? Problems should be traceable from the student group's point of view.
Feasibility of Approach
It is possible to devise a solution process for students which includes using appropriate mathematics, making necessary assumptions, and combining the necessary data. Teachers must overcome the problem.
Feasibility of Outcome
The solution of the mathematics of the basic problem is possible for students together with interpretation. Expectations regarding the type of response, such as arithmetic and generalized solutions, depend on the characteristics and year level of the particular student group engaged.
Didactical Flexibility
The problem can be structured into sequential questions that maintain the integrity of the real situation. (These can be occasionally given as hints or to provide organized assistance by scaffolding a line of investigation.) (This includes thinking about how it might be implemented after working on the problem. For example, students are guided through a sequence of questions to structure the problem and perhaps they are supported through the description of the subsections of the problem.

While there are few studies on the difficulties encountered in designing a mathematical modeling task in the literature, it can be said that the studies explaining the mathematical modeling task design process are insufficient. In addition, Venturini and Sinclair (2017), Deniz (2014), Jankvist and Niss (2019) and Sağıroğlu (2018) emphasized that teachers are not very willing and have difficulties in designing modeling activities. The most important reason for this situation is seen as their lack of experience in mathematical modeling. In this sense, it will be important that the working group consists of experienced and skilled people in the relevant field in order to obtain a rich data set in studies that will explain the mental actions in the design process of mathematical modeling task. When the literature is examined, while there is limited study on the design process of mathematical modeling activities, this study is the first study on the HTTM task design process.

4. Importance of the Study

The HTTM learning approach has a well-grounded structure as it is put forward by considering all the perspectives that exist in mathematical modeling in education. In this study, the design process of the activities used in the HTTM learning process will be revealed and it will be explained which mental actions are revealed in this process. In this way, the possible effectiveness of HTTM in mathematics teacher competencies is exemplified and a strategy (HTTM-enriched learning environments) that can be followed for the empowerment of mathematics teachers is presented to researchers. Since the HTTM learning process is fed by learning strategies that are effective in quality learning and developing 21st century skills in mathematics education, different explanations can be made for the importance of using HTTM in education. This study provides an important overview of interdisciplinary learning approaches such as STEM in mathematics education and studies on the history of mathematics and technology integration in the literature.

The quality of the students, who are the individuals of the future, depends on the equipped teachers who train them (Schommer, 1990). With the design of rich mental environments, multiple associations [integrated connectivity according to Kılıç (2021) from a more recent perspective] come to the fore and effective learning environments can be created in a short time (Schoenfeld, 1992). The mental richness in the HTTM task design process contains good examples of how mathematics learning environments can be improved. During the HTTM task design process, effective learning environments emerge for the simultaneous development of technology, history of science, mathematical modeling and design-based skills of mathematics student teachers and mathematics teachers. Mathematics teachers and mathematics student teachers who have developed these skills will be equipped to apply HTTM activities more effectively in their lessons. Revealing the HTTM design process is also a guide for explaining the sub-skills necessary for the development of 21st century basic skills. These sub-skills will be important in the development of rubrics to be designed to evaluate the design processes of STEM and HTTM tasks, and in defining current teacher competencies. On the other hand, considering the components and steps in the mathematical modeling process while solving mathematical modeling problems is a mental act based on metacognition or self-regulation (Maaβ, 2010). Similarly, knowing the design processes of such activities in order to create rich learning environments such as the HTTM learning process will be important for their empowerment in the context of metacognition or self-regulation. Considering the existing modeling perspectives and innovative ideas in education (STEM, computational thinking, scientific inquiry, engineering design process), the HTTM learning process creates environments that ensure the development of the basic competencies included in the integrated STEM teaching framework as stated by Çorlu (2021). In this way, the HTTM learning process can be viewed as a key learning perspective that strengthens the bond between STEM and mathematical modeling.

An important goal of mathematics education is to enable students to learn how to effectively use their mathematical and technological knowledge and skills while solving real world problems (Clayton, 1999). In this sense, HTTM is an essential tool in achieving this goal, as it reveals the technology-enriched mathematical modeling cycle. NCTM (2000) mentions three possible roles of technology in mathematics education. In the first role, technology unlocks skills such as estimation, reasoning and mathematical modeling in solving complex real-world problems. In the second role, technology integrated mathematical tasks supports conceptual learning. In the third role, intellectual interaction evolves with technology integration, and new and high-level mental connections are revealed with conceptual linking.

With the support of computer algebra systems and dynamic geometry software in mathematics education, multiple representations are revealed, relations between geometric and algebraic representations are compared, concepts are associated, and mental transitions can be made between different disciplines (Hıdıroğlu & Can, 2020). Teachers' successful integration of technology into their classrooms is at the center of strong debate in the field of teacher education (Lee & Lee, 2014). According to Lakshminarayanan and McBride (2015), in technology integration in education, how technology can enrich education, help students develop 21st century skills, and support high order thinking, creativity, student participation and teamwork are important research topics. In line with these considerations, it is thought that HTTM will provide a rich perspective on technology integration into education and is a learning approach worth working on.

Honey et al. (2014) mentioned three different roles in technology integration. In the first role, technology is an engineering product. In the HTTM learning process, the solver thinks like an engineer, designs a current mathematical modeling problem by feeding on his solution in the historical problem, and obtains high-level mathematical models (emergent and hypothticodeductive models). In the second role, technology can be defined as instructional technology used to improve teaching and learning. The HTTM learning process is powered by GeoGebra, video, animation and photographs and supports a learning process that will serve for the second role. In the third role, technology can be defined as tools used by science, mathematics and engineering professionals. In the HTTM learning process, the solver looks for different solutions to real world problems and actively incorporates technology into the process. Sivaraj et al. (2019) and Ellis et al. (2020) talk about the role of computational thinking as distinct from these three roles of technology. Csizmadia et al. (2015) explain computational thinking, which is accepted as one of the important skills of the 21st century, with the sub-skills of abstraction, evaluation, algorithmic thinking, decomposition and generalization. Angeli et al. (2016) described the components of computational thinking as abstraction, generalization, decomposition, algorithmic thinking and debugging. The HTTM learning process creates mental environments that will reveal and develop these skills in both plugged and unplugged sub-processes. White (2014) stated that one of the barriers to technology integration is that educators think technology only as computers. HTTM creates learning environments to support the four different roles of technology.

Current mathematics teachers and mathematics student teachers can provide students with new learning opportunities where they can uncover and develop different mathematical skills through effective technology integration (Hıdıroğlu, 2015; Hollebrands, 2007; Larson & Miller, 2011; Sinclair, 2004). Therefore, it is considered important to teach mathematics student teachers how they can use technology in their teaching and how they can integrate it into their lessons. Teacher training institutions play an important role in enabling teacher candidates to integrate technology into their courses (Hofer & Grandgenett, 2012). Supporting all these considerations, the HTTM learning process can be considered as an important learning strategy in teacher empowerment and teacher training in mathematics education (perhaps in science education i.e. Hıdıroğlu & Can, 2020).

The history of science and mathematics is an important resource for students to follow the emergence and development of concepts and to realize the solutions of historical problems as if they were living in that period (Berlinghoff & Gouvea, 2004; Reimer & Reimer, 1992; Swetz, 1994). According to illumination approach of Jankvist (2009) and strategy of addition of Fried (2001), problems selected from the history of science and their solutions, contributions of scientists to science and effective videos, photographs/pictures about them should be included in the curriculum. The HTTM learning process starts with a historical problem and requires establishing connections between the historical problem and the current problem in the continuation of the process. With the integration of the history of science in mathematics education, by focusing on the problems and solutions of the old scholars, creative and different solutions are discovered with the limited conditions in history, and more effective learning environments are created by considering the development processes of the mathematical or scientific concepts (Siu, 2003; Swetz, 1994; Veljan, 2000). If a person learning the science also learns the history of that science, then s/he can acquire deep and comprehensive knowledge of the concepts (Göker, 1997). In this way, one can understand the importance of discoveries and inventions in history and know how concepts and theorems have developed. At the same time, an interdisciplinary understanding is gained by revealing the relationship of mathematics with other sciences with the help of theories (Hıdıroğlu & Özkan Hıdıroğlu, 2016). Newspaper article and problem situation in HTTM tasks are designed to serve this purpose. In this respect, the use of learning approaches that allow the interaction between mathematics and history of science in the mathematics learning process, such as the HTTM learning process, is supported by many researchers (Bidwell, 1993; Ernest, 1998; Gulikers & Blom, 2001; Leng, 2006; Liu, 2003; McBride & Rollins, 1977).

Teachers note that they can improve their mathematical/didactic design skills and content knowledge while developing or revising mathematical tasks (Pepin, 2015). Learners can foster their cognitive skills and participate actively in mathematical conversations if they engage in open-ended and non-routine activities at an appropriate level (Simon & Tzur, 2004). While Jankvist and

Niss (2019) stressed the benefit of modeling activities to diagnose the difficulties related to the development of students' modeling competence, Hernandez-Martinez and Vos (2018) highlighted the importance of the originality of the modeling task. Tan and Ang (2016) underlined the tendency of teachers to direct their efforts towards predetermined solutions. While Schukajlow and Krug (2014) and Schukajlow et al. (2015) mentioned teachers' recommendations for openness to more than one possible solution, de Oliveira and Barbosa (2010) underlined teachers' desire to intervene in the process in ways that reduce their cognitive difficulties in the implementation process. Gardner (2007) and Anderson and Krathwohl (2001), by referring to the step of exploring, mentioned that the design of learning environments that will reveal or enhance such skills is quite difficult. HTTM learning process expects students to present a current mathematical modeling problem and solution looking at the history of science-based problem and its solution, and this emerging multi-structure dual modeling process allows students to design advanced mathematical models (Hıdıroğlu, 2018).

Dynamic and interactive learning environments allows for the attainment of the goals that are hard to reach in mathematics learning (e.g. reasoning, argumentation, conceptual understanding, making assumptions and inferences, mathematical thinking and open-ended problem solving) (Venturini & Sinclair, 2017; Yerushalmy, Nagari-Haddif, G., & Olsher, 2020). The use of digital technologies in mathematical task design has been recommended and studied by many mathematics education researchers around the world (Çevikbaş & Kaiser, 2021). The pedagogical potential of The Digital International Media Literacy Education [DIMLE] may differ for different learners (Leung, 2011). Some activities may lose their potential when digital tools are used, as they may degrade mathematics to experimental practice, limit the exploring step and hinder meaningful learning processes (Venturini & Sinclair, 2017). In summary, the pedagogical importance and effectiveness of DIMLEs depend on how they are designed and used in mathematics classrooms (Leung, 2017). Although technology, especially DIMLE, supports the creation of a meaningful learning environment that allows problem solving and supports creativity, it is not a panacea (Çevikbaş & Kaiser, 2021).

Designers unearth their metacognitive skills by addressing issues that can be mathematically useful in the application stage of the mathematical modeling task they will design (Niss & Blum, 2020). Hıdıroğlu and Bukova Güzel (2015; 2016) and Niss (2010) explained such forward-looking actions in mathematical modeling with the metacognitive skill of prediction, and according to Niss (2010), in order for a real-life situation to be mathematically applicable, its anticipated features should be focused beforehand. In this sense, the designer should envisage suitable mathematical representations that will emerge in the later stages of the solution (Hıdıroğlu & Özkan Hıdıroğlu, 2016). Such representations should be understandable to the solver and the solver should have sufficient experience. It is also essential to predict what kind of mathematical solution to the problem will be provided by the mathematical models that may emerge (Hıdıroğlu & Bukova Güzel, 2017). Czocher (2017), Jankvist and Niss (2019), Stillman and Brown (2014) also mentioned the key role of prediction in mathematization; however, the modeller must also predict potential mathematical representations, relationships, procedures, and problem-solving strategies (Geiger et al., 2018). As designers must predict how students will react, what scaffolding strategies need to be prepared, and where challenges will arise, we argue that prediction is key to the design of modeling tasks like HTTM.

Although there has been some research on task design (e.g., Czocher, 2017; Galbraith, 2006; Maaß, 2010), the present study is original in that it deals with a new learning process (HTTM learning process) in the literature, proposes a process model about the engineering design concept and the design of mathematical modeling activities in addition to the basic principles, and reveals the mental difficulties and important elements at the time of design. This study was carried out with a small sample and it is a limitation. However, it is believed that it will inspire researchers and teachers in terms of the design and implementation of HTTM learning process, which allows for the development of fundamental skills in the 21st century. In general, the study provides insights into the basic components and steps in the design process of mathematical modeling activities and provides teachers with guidance to enhance teaching competency in modeling.

Although there are some studies on the mathematical modeling task design in the literature, studies on HTTM tasks are limited. The underlying reason for this situation can be the fact that the HTTM learning approach is new and this approach is difficult to work on because it covers different areas (such as history of science, mathematical modeling, technology enriched mathematics education, relationship of theory and real world, problem posing). Therefore, there is a need for comprehensive research on the HTTM learning process.

The research has some limitations as in any qualitative research. These limitations are stated in the relevant sections throughout the article. Some of the limitations of the study are as follows: The study was conducted with nine pre-service mathematics teachers; data analysis was carried out with two experts; and the data were collected through students' designs for only one learning task. Some of the points that make the study important are that the authors and analysts have conducted many scientific studies in this field before and have experience in such learning environments, nine mathematics student teachers have experienced such environments before, and they have high order skills in this field. Since the aim of the study is to reveal the rich mental actions needed while designing HTTM tasks, a sample that can design qualified HTTM tasks was selected in the study. The fact that the mathematics student teachers took courses on current approaches in mathematics education in their undergraduate education, that they were more willing to learn and apply new approaches in education, and that they were better in 21st century essential teacher skills were effective in their selection for the study. The sample also received an eight-week training in the "Algorithm and Programming" course, which included technology-assisted mathematical modeling and HTTM learning process. In this way, a rich data set was reached in the study. It is believed that the HTTM task design process revealed in this study will be an important resource in the development of rubrics to be used to evaluate individuals in such learning environments, in explaining required teacher competencies for future, and in research on mathematical modeling and interdisciplinary task design.

In this regard, the main research question discussed in this study is as follows: What is the HTTM (History/ Theory/ Technology/ Modeling) task design process (including main components and steps) of mathematics student teachers?

5. Methodology

5.1. Research Design

The study was conducted with a holistic single case design, a qualitative research method. A holistic single case study design is a method in which there is a single unit of analysis, contradictory and idiosyncratic situations are studied, and a theory is tested (Yin, 1984). The unit of analysis of this study is the mental actions performed by mathematics student teachers while designing an HTTM task. The case in this study is the HTTM task design process. The study also has the characteristics of exploratory and explanatory case studies developed by Yin (2003). In this sense, the study explains in detail the components (what) considered in the design process of the HTTM task and how the process works (how) between these components. The elements taken into account in the HTTM task design process are the key components in the process model. In this respect, the study is an example of Yin's exploratory holistic single case study. On the other hand, the main actions taken into account in the HTTM task design process are the basic steps in the process model. In this respect, the study is an example of Yin's descriptive holistic single case study. The main strategy followed in the study to reveal the HTTM task design process was clinical interviews and thinkalouds.

5.2. Participants

The participants of the study were comprised of nine middle school mathematics student teachers studying at a public university. Criterion sampling method, one of the purposive sampling methods, was used in the selection of the participants. The criteria to determine the participants included being a second-year mathematics student teacher completing "Algorithm and Programming" course successfully, which includes HTTM learning process, and willingness to participate in the study. The main reason for working with mathematics student teachers in the study is that they are more familiar with new learning approaches such as HTTM learning process due to undergraduate level courses and they have more opportunities to experience this process than mathematics teachers. In the study, it was desired to work with students who have high ability to use technology in mathematical modeling, who know the HTTM learning approach, who have experienced the HTTM learning process and who have had rich thinking processes in the applications within the scope of the "Algorithm and Programming" course. In addition, before the application, the participants used software such as GeoGebra, Matlab, Cabri, TinkerPlots in lessons such as Calculus, Analytical Geometry, Geometry, Foundations of Mathematics I/II, Using Computers in Mathematics Education lessons. In this way, it is aimed to provide a deeper and richer explanation of the HTTM task design process by obtaining a rich data set. Each participant was given a pseudonym (see Table 2).

Table 2
Some information about the participants

Pseudonym	*Design Process (min)*	*The HTTM Task Designed*
Mert	118	Tower Bridge
Ayça	135	Chinese Civilization and the Mountain Problem
Meryem	119	Ancient Greek Mathematics and the Dinosaur Problem
Özge	110	Phoenician Maritime
Yağmur	145	Ancient Egypt and the Nile River
Kumru	112	Hezârfen Ahmed Çelebi and Flight
Yiğit	122	Mathematics for Daily Needs “The Lotus Problem”
Çağıl	137	Da Vinci Bridge
Belma	139	Mimar Sinan and Selimiye Mosque
Average Duration	*≈126*	

5.3. Instruments

A case study is a qualitative research method in which the researcher comprehensively examines one or more cases and defines the situations and the situation-related themes that are confined over time by using various data collection tools (observations, interviews, audio-visuals, documents, reports) including multiple sources (Creswell, 2007). In this direction, the data of the study were obtained from the video transcripts of approximately 19 hours of clinical interviews with nine mathematics student teachers regarding the HTTM task design processes, GeoGebra files as the documents collected throughout the study about the task, scratch papers, written reports, and the observation notes taken during the research. In holistic single case studies, Piaget's clinical interview enables the individual to express herself freely and reveals the implicit and deep thought processes (which is the analysis unit of the study) (Opper, 1977).

5.4. Data Collection

The study was integrated into Algorithm and Programming course taught by the researcher to the middle school mathematics student teachers, and within the scope of this course, a theoretical and practical instruction was carried out on HTTM learning process over a period of eight week (see Table 3). After the instruction they had received, nine mathematics student teachers who were successful in the course and willing to attend in the study were asked to design an HTTM task individually. The design processes were recorded, and the documents they produced in the process and the researcher observation notes were used in the data analysis.

Table 3
HTTM task design process of the participants

Week	*Content*	*Learning Material to be Used*
1	Explaining the relationship between model-modeling and mathematical model-mathematical modeling with examples. Creating a discussion environment on the importance of mathematical modeling in mathematics teaching.	Real life photos
2	Discussion and demonstration of different mathematical modeling problems in the literature and their example solutions Explaining different perspectives on mathematical modeling in mathematics education with examples.	Important mathematical modeling problems in the literature and their possible example solutions
3	Identifying general features of mathematical modeling problems through discussions and examining their place among other problems.	Scientific studies that classify the problems in the literature
4	Exemplifying the technology-supported mathematical modeling process with different solutions and discussing about the learning environment it creates.	Swing Problem, Height-Foot Length Problem, Stadium Problem, Meadow Problem, Stairs Problem (GeoGebra-aided environment)
5	Creating a learning process supported with a pre-task before introducing HTTM and creating a discussion environment for the process.	Lighthouse of Alexandria and Archimedes HTTM task (GeoGebra-aided environment)
6	Discussing the emergence of HTTM learning approach, its learning objectives, principles, dimensions and learning process, and explaining these in detail with examples through the pre-task	The first scientific study on HTTM (GeoGebra-aided environment)
7	Application the learning process of the HTTM task named Eratosthenes and the Calculation of the Earth’s Circumference	Eratosthenes and the Calculation of the Earth's Circumference HTTM task (GeoGebra-aided environment)
8	Application of the learning process of the HTTM task called the Maya civilization and Mayan (Kukulkan) pyramid	The Maya civilization and Mayan (Kukulkan) pyramid HTTM task (GeoGebra-aided environment)
9	Performing the process of designing an HTTM task	The process will be video-recorded.
10	Performing the process of designing an HTTM task	The process will be video recorded.
11	Performing the process of designing an HTTM task	The process will be video-recorded.

Within the scope of the Algorithm and Programming course, a learning environment was created on the concept of mathematical modeling and its use in mathematics education. Afterwards, different mathematical modeling problems were given to the mathematics student teachers. Ideas about why these problems were mathematical modeling problems were discussed. Scientific studies on technology supported mathematical modeling and mathematical modeling problems used in these studies were discussed in the classroom environment. The basic principles and parts of HTTM learning approach were explained. Theoretical informations and examples were provided for mathematics student teachers about the use of various mathematics software (such as GeoGebra, Cabri 3D, Code.org, thinkerplots), how to develop HTTM tasks using these softwares and how to integrate these tasks into mathematics education. Within the scope of the course, face-to-face lectures were conducted for eight weeks. In addition, videos coveringthe talks of important scientists in the field explaining the importance of the basic components of HTTM were used as e-course materials. The mathematics student teachers were asked to write a discussion article on these talks. With the help of "Lighthouse of Alexandria and Archimedes HTTM task (Hıdıroğlu & Özkan Hıdıroğlu, 2016)", "Eratosthenes and the Calculation of the Earth's Circumference HTTM task (Hıdıroğlu, 2019a)" and "The Maya Civilization and Mayan (Kukulkan) pyramid HTTM task (Hıdıroğlu, 2019b)", the mathematics student teachers were provided the opportunity to experience the HTTM learning process. At the end of the instruction, the mathematics student teachers were asked to design an HTTM task that would reveal a comprehensive HTTM learning process based on the mathematics curriculum by using the knowledge they learned individually. The data of the study were obtained from the design processes of the HTTM task, which were revealed at the end of the instruction, performed by nine mathematics student teachers selected within the scope of Algorithm and Programming course.

5.5. Data Analysis

In the data analysis process of the study, the data analysis process applied in grounded theory (open, axial and selective coding) and the coding based on theoretical framework approach introduced by Strauss and Corbin (1990) were applied. The design process model obtained in the study was created entirely based on data. However, while explaining the structures in the process, studies in the literature were considered for theoretical soundness. In this way, it is aimed to make the theoretical framework strong (Creswell, 2007). In order to ensure the validity and reliability of the study, the design process models in the literature (*design process models including 1. Double Diamond, 2. Deep Dive, 3. Stanford, 4. IDEO, 5. Donut, 6. Stage Gate, 7. IBM Model, 8. 5D, 9. Google Design Sprint*) were especially examined in detail, and the components and the steps in the models were determined and their features were revealed. These components and steps were considered in the process of creating codes and categories during the analysis. This stage was of critical importance especially in terms of identifying open codes in the first stage of the data analysis and providing an idea about possible axial and selective codes in the next stage. The design process was not based on only one of the models; rather, the parts of the different models in the process were taken into account and associated with the data, and in this way, it was aimed to report a more valid and reliable design process in the literature. The main reason for not sticking to a certain design process is not to lose the realities in the data that may arise from the original structure of the HTTM learning process. In addition, HTTM learning process introduced by Hıdıroğlu and Özkan Hıdıroğlu (2016) was the basic theoretical framework considered in the data analysis process. In this way, it is determined how and where the basic parts of HTTM emerged, and the properties of the basic steps of the HTTM task design process and its components were explained in more detail in the HTTM task design process. In the study, it was requested not to ignore the strong collaboration existing in the data between the task design process and the HTTM learning process. In the data analysis process conducted with two researchers, the interrater reliability was determined to be 91%. Analysts came together and shared with each other what they thought about which code fell into which basic step. For different analysis results, two researchers came together to discuss the reasons for the disagreements and reached a consensus. Thus, the codes and related themes took their final form. Basic components obtained in data analysis are especially important for key indicators explaining basic steps of HTTM task design process. Basic components are the case themes and basic steps are the process themes. Indicators related to the situation and process themes (basic components/basic steps) that emerged in the study are presented in Table 4.

Table 4
Themes and indicators obtained in data analysis

Themes	Indicators	
A) Task		Design the Right Thing
Investigating	- to reveal what is given, what is desired, opportunities and limitations - to broaden the viewpoint in the context of the goal (divergent thinking) - to search for an effective origin/focus in the direction of the task - think superficially in a large intellectual space	
B) Focus/Origin		
Exploring	- to restrict the viewpoint in the context of the goal (convergent thinking) - to refine informations and thoughts in line with the focus/origin. - to think in detail in a small inteclectual space - to obtain the most promising main mathematical problem to use in the task	
C) Problem		Design the Thing Right
Designing	- to enrich the mathematical problem by revealing its specific points - to act on different possible solutions of the mathematical problem - to structure in detail all the parts and functioning of the task in a wider intellectual field by benefiting from the problem (divergent thinking) - to make detailed justifications for the structure of the task - to aim to achieve the first design/prototype that includes all basic parts of the task	
D) Design/Prototype		
Evaluating	- to reveal the different results obtained from the design - to explain the effectiveness, limitations, shortcomings and advantages of the design - to benefit from peer-assessments, self-assessments and expert opinions - to highlight details in the basic parts (the steps/ the components) of the design (convergent thinking)	
E) Results		
Reporting	- to highlight the important ideas that should be written in the report containing the design - to supplement existing explanations with detailed mathematical expressions - to sort what should be written in the report - to make the design more understandable and remarkable (convergent thinking)	
Approved Report		
Revising	- to question the causes of undesirable situations (which want to be corrected). - to overview thoughts (convergent or divergent thinking) - to identify the source of the troubles/errors in the design - to create and to implement high level strategies and assumptions to eliminate troubles/errors	

In the data analysis, the themes were formed with selective codes and indicators appeared as axial codes. The HTTM learning process and different design processes were especially helpful in the creation of open codes. When deemed necessary, the basic concepts of these different approaches were used in the naming of the axial code and selective codes in the data analysis. For example; one of the selective codes (indicators) that explains the axial code, designing, is to make detailed justifications for the structure of the task. One of the helpful open codes in the emergence of this selective code (*to make detailed justifications for the structure of the task*) is justifying the use of GeoGebra for multiple representations in mathematical analysis in the technology-enriched mathematical modeling process in the HTTM learning process. The bolded parts in the findings are evidences exemplifying indicators.

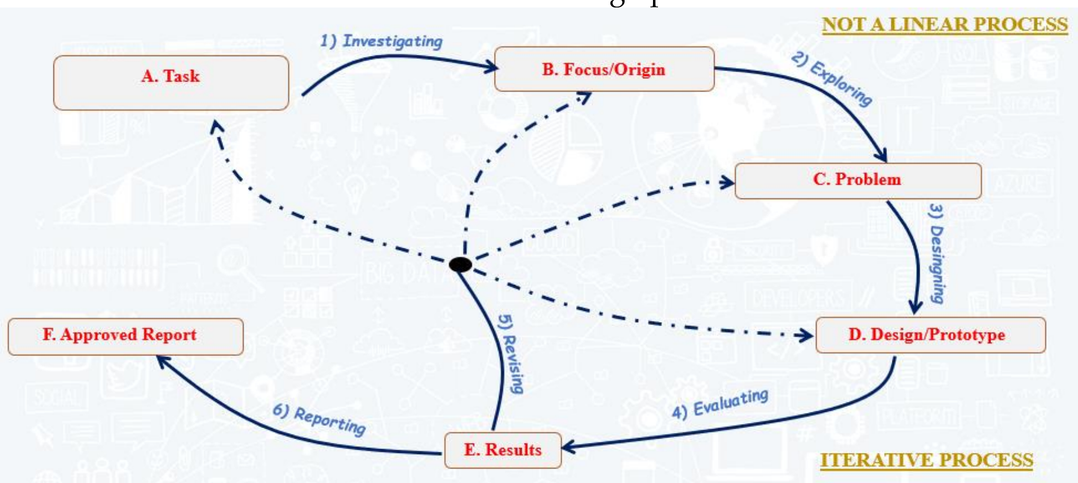
The overall steps followed in the data analysis process of the study are as follows: (1) coding the data, (2) identifying initial categories, (3) making the codes suitable for the categories, (4) finalisation of the categories, (5) defining the general characteristics of the categories, (6) making distinctions between the categories, (7) organizing the findings in line with the purpose of the study as well as the codes and categories obtained.

6. Findings

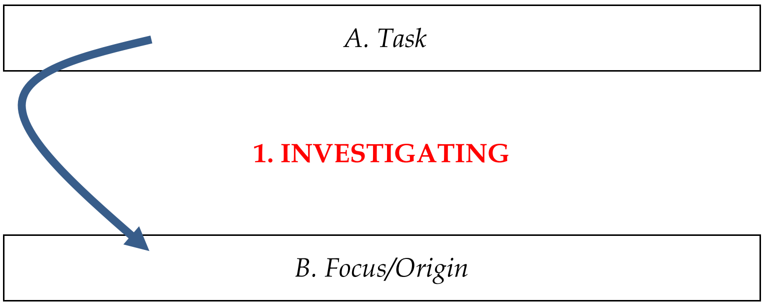
As a result of the data analysis, the categories that emerged in the HTTM task design processes of the mathematics student teachers consisted of process categories and case categories. The process categories in the HTTM design process were determined as follows: 1) investigating, 2) exploring, 3) designing, 4) evaluating, 5) revising, and 6) reporting while case categories included a) task, b) focus/origin, c) problem, d) design/prototype, e) results, f) approved report (see Figure 5).

While determining the components in the process, the main goals of the students at the time of designing in the process were considered. Then, the common patterns performed in the transition process between these components were identified, the processes were completed, and the features of the processes were revealed. The data indicated that the mathematics student teachers involved many complex and nonlinear mental transitions while designing the HTTM tasks. For instance, a designer transitioned frequently from the designing or evaluating step to investigating or exploring steps for certain reasons in the process. This happens because of the mental difficulties encountered in the process or because of the fact that each new information triggers and enhances the mental structure and the designer manages the process consciously (metacognition).

Figure 5
Mathematics student teachers' HTTM task design process



According to the findings, the main goal in the transition from A to C in the process model, was to choose the right problem to solve (*Design the Right Problem*). The main goal in the transition from C to E is to solve the problem correctly (*Design the Problem Right/Correctly*). The main goal in the transition from E to F is to solve the right problem correctly. The components in each step are of vital importance in the non-linear transitions between the steps. For example, in HTTM tasks, components such as newspaper article, readiness questions, problem situation and problem solution appeared as the subcomponents in the design process. Besides, the problem situation or the newspaper article, which are two components of HTTM, became the starting point/origin for some students in the design process and assumed the role of a main component. Factors such as the purpose of the task, classroom management, time management, classroom organization, physical environment, teacher intervention, dealing with multiple focuses/origins, materials/tools to be used, student roles, student prior knowledge, student difficulties and misconceptions, assessment and evaluation, flexibility, drawing attention, learning methods and techniques, use of different representations, interdisciplinary structure, role of technology, possible keys to the solution, necessary mathematical concepts, curriculum, and student level were encountered as the sub-components considered in the design of HTTM tasks. In the design process, the way the mathematics student teachers approached to the process was consequential. It was observed that the first two steps took a shorter time if the focus was more on the designing step rather than the investigating and exploring steps in the process. On the other hand, in cases that really require a new exploration, the first two steps were observed to take longer.



In the step of investigating, which is the first stage of the HTTM design process, the mathematics student teachers generally acted in line with the task given to them and performed what they were asked to do and what was expected of them and managed the things available and unavailable. Since they were asked to create a HTTM learning activity in the task, they thought about what they should do for this. They collected data on a wider range (divergent thinking) in line with the task. Within this wide range, different points such as mathematics curriculum, mathematical concepts, HTTM, history of mathematics, open-ended problems, mathematical modeling, and technology integration into the mathematics learning process were considered. At this step, when there is no specific focus yet, an effort is exerted to move from a wide spectrum to a narrower and deeper point and to centre on a specific focus. In the study, the mathematics student teachers were in search of a starting point so that they could narrow their thinking areas in the task design. The step of investigating necessitates looking at an overview of the general situation and coming up with some insights into determining the focus. In order to find the right problem, it is necessary to narrow down the focus. In the first step, the main purpose is to determine a wide range of ideas and options in the context of the problem and to come up with an effective focus, and thus to direct the attention to a special area of the relevant focus (2nd step-exploring-convergent thinking). In the step of investigating, determining the main area of the design, reviewing digital or printed resources, taking experiences into account, and generating effective ideas by considering various perspectives were proven to be effective factors.

During the step of investigating in the design process, Mert believed that he should better understand HTTM learning process within the scope of the task and conducted investigation on this. Then, he emphasized that he should read about the distinctive features of HTTM learning process and prepare a design related to its steps. Similarly, Yiğit investigated about HTTM by stating that it is crucial to know the steps of HTTM learning process well during this investigation process.

> Mert: I need to know more about HTTM, and I need to remember some of its features. When we look at HTTM learning process, there are eight steps. …Therefore, I reviewed the article and the lighthouse task you provided beforehand and its solution in detail. Then I read your other works. …
>
> Yiğit: I think it is necessary to know these steps well for an effective process. After all, I will design my task considering this process.

Çağıl started the step of investigating with the newspaper article, which is one of the basic components of HTTM. In this sense, she investigated about an event, person, or situation in the history of science. She stated that she faced some difficulties while deciding on her focus because what she was planning to focus for the newspaper article had been used before and it would not be suitable for the middle school level. At this step, although Çağıl stated that she didn't focus on trigonometry as it is not covered at middle school level, she could not realize that she could arrange it to make it suitable for middle school students' pre-learning of geometry.

> Çağıl: I need to *create a newspaper article* here. Thus, I *searched about this in the history of science*. …I can look at *the leading scientists of these periods*. …Actually, I thought of Biruni's Calculation of the Earth's Circumference. But since this task would *not be suitable for middle school students*, as they need to know about the values of tan, sec., I gave up.

Yaren considered various situations in the history of science as the subject of the newspaper article. For example, she thought that by dealing with Cartesian coordinate plane, she could focus on the way the sailors found their ways at that time; however, she noted that she would have problems in two dimensional and three-dimensional associations. In addition, she emphasized that she did not use the Lotus Problem and a ratio-proportion-based problem about the Parthenon temple because she could not create a context.

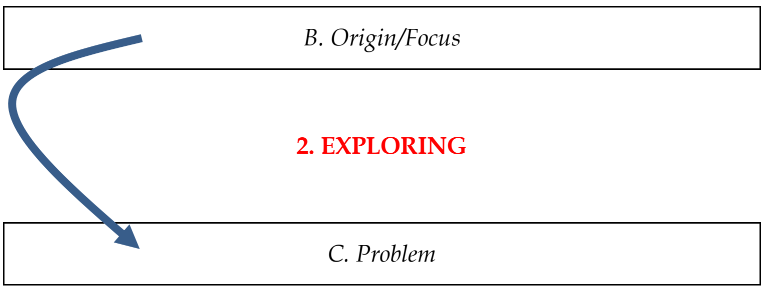
> Yaren: *Cartesian coordinate plane*. You know, I read about the sailors' use of the coordinate plane as latitude-longitude in the history of mathematics book. (She attempted to work on it.) …I don't want to do this because I think I will have trouble in a two-dimensional universe. (The student then thought of asking the students about the Lotus Problem, the proportional relations in the Parthenon temple.). I don't know how to progress in the Parthenon problem either, *it's nice but I don't know how to create a complete context*.

In the step of investigating, Özge thought that she could look at civilizations other than the usual well-known topics in order to determine a specific focus. Then she decided that she could benefit from the videos she had watched before about the history of science, so she made a search to find out what could be done about this.

> Özge: There are many civilizations in history. *I can start from civilizations other than well-known topics*. I have to do some research. I think I can find some interesting things this way. *…For example, I once watched a movie. There were scenes about the construction of the Egyptian pyramids. It would be great if I could find its videos*. I can also increase the motivation of students. I can also benefit from movies (Searching and studying historical movies.).

Kumru tried to come up with a focus by considering many problem situations and contexts. For example, she considered the relationship between the Fibonacci series and COVID, but gave up this context since she couldn't exactly foresee how she could make associations with the history of science and how to reach the mathematical solution. As a result of her investigations, she found videos and interesting stories about the Galata Tower and determined her starting point believing that she could create a mathematical fiction with this.

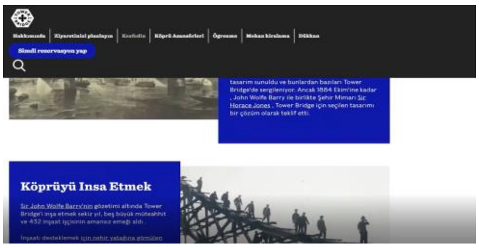
> Kumru: I wonder if there is a relationship between the Fibonacci series and COVID, and it is a current problem. (She investigated about it, but she thought it might not be exactly suitable for HTTM.). But how can I include the history of science into the process (Investigating.) … But I couldn't find a mathematical relationship. … (She studied textbooks. She watched videos on history of science.) *…There are amazing videos and legends about the Galata Tower*. I can make use of this situation.



In the second step of the HTTM task design process namely exploring, the designer narrows his focus and refines the information collected to approach a more specific problem intended. As the starting point of the design process has already been determined on the way to this step, it is aimed to reach a deeper focus in a more limited area in the exploring step. In this sense, it can be said that while divergent thinking was observed in the step of investigating, convergent thinking was observed in the step of exploring. This step was characterised by some considerations about the task parts with a deep focus. This is the nature of the exploring step to achieve an effective mathematical problem. This step includes analysing, evaluating and reviewing data or ideas in order to determine the final problem or idea to work on. In the exploring step, the frame and structure of the problem are tried to be shaped with the information obtained, and it is aimed to select the mathematical problem in general terms by putting forward different ideas about what can be done about the focus. The designer aims to choose the most promising problem idea for further development with his/her experiences and internet resources.

Mert determined the Tower Bridge Model Eliciting Activities [MEA] task as the focus in line with the information he gathered in the step of investigating. Afterwards, by narrowing his focus within the scope of this content, he started the step of exploring the content and thinking about the final problem he would use. In this step, Mert examined the architectural features of the Tower Bridge in detail and gathered information on how he could handle the mathematical problem and structure the newspaper article. Mert used internet resources related to the context for this. Since the information obtained here is used to determine the final problem, this step is expressed as the exploration process based on the focus.

> Mert: There are some *mathematical modeling problems* here (looked at some resources from the Internet). When I was looking at these problems, I saw model-building activities in an article, and it caught my attention. I think this *Tower Bridge MEA task would be good for middle school students* (He started to do a historical search on this structure.). …The site gives information about its construction and the bridges of that time. There is a lot of information here (He read in detail and took notes.)



Meryem, believing that it was suitable for out-of-school learning environments, determined the history museums as the focus, narrowed her focus to identify the problem in the step of exploring, and thought about the content that she could use in the relevant newspaper article about museums and in the problem situation.

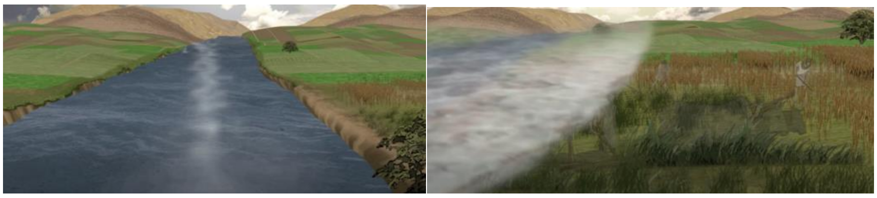
> Meryem: I will *make use of history museums* here. In this way, I can create out-of-school learning environments. We will have a chance to practice this task by taking the students there, and I think it would be good. The important thing here is to choose the right problem (She collected information from the websites of history museums in different cities.). I will decide on the newspaper article according to that (problem).

Özge dealt with mathematics in maritime history as her starting point, narrowed her focus to gather information for the newspaper article within this context and searched for a suitable mathematical problem. She stated that she could consider the distance between ships, and in this way she could design a problem suitable for middle school level.

> Özge: I'm going to do something about *maritime mathematics* here. I remember the task of Alexandria lighthouse performed before; it was really nice (He searched for videos he could use for a while but couldn't find it). …I can support the newspaper article with images even if I can't find a video (Tried to select map images from previous sources.). For example, I can do something related to the distance between ships for middle school level. This is also an important situation in the history because warships took this into account in order to catch up with each other.

Yaren thought that there were a lot of fascinating animations about the Nile River and that she could create a problem situation regarding the calculation of the fertile land formed with tide in the video.

> Yaren: There are a lot of videos about Egypt. I have to choose one of them (she looked at different videos.). …*The videos about the Nile River* will be good for me. *TRT School's animation about Egypt* is suitable. It also describes the Nile River with visuals. [She chose the video titled Mathematics Stories Part 1 (The Rising Sun in Egypt).]. …The video mentions about tides and fertile lands. This can be an appropriate problem in calculating plots.



Before passing on to the step of exploring, Kumru was highly hesitant while deciding on her focus and spent too much time in the step of investigating. Later, she narrowed her focus after she obtained some information related to the legends of Galata tower and Hezarfen Ahmed Çelebi. She then looked for mathematical problems in this context in the step of exploring. She gave up the idea considering that it was too difficult for students to find the formula for flying, which was discussed at that time. Taking into account the information that she found in Hezarfen's flying experience, she thought that she could do something about the route of flying and that this would be appropriate for the level of the students, and she put forward ideas to clarify the problem situation.

> Kumru: I think the topic of *Hezarfen Ahmed Çelebi in the legends of Galata Tower* would be very nice (In this part, she focused on Hezarfen.) …There is a formula he uses to fly here. Can I use this? (He asks himself.) *Can the students find the formula* (Thinks and gives up.)? I don't know how I can do this as it is difficult. (He is doing some searching to select a problem about Hazerfen.). Hezerfen descended from Galata tower to Doğancılar Square (Continued to collect information.) *It seems easier for middle school level to do something related to the distance on the route taken.*

Kumru later included Google Earth program into the process for technology integration and to improve children's map knowledge. With the purpose of clarifying the problem situation and determining the limitations and possible solutions, she thought about what information students could obtain from the map with the use of Google Earth.

> Kumru: I can use *Google Earth* to include technology in the process and to develop children's map knowledge and skills. When we *write the location we want here and hover over* it, it gives the altitude above sea level. Students can use it. They can access the necessary information related to the distance. Actually, I came up with the problem situation by asking about the distance.

C. Problem

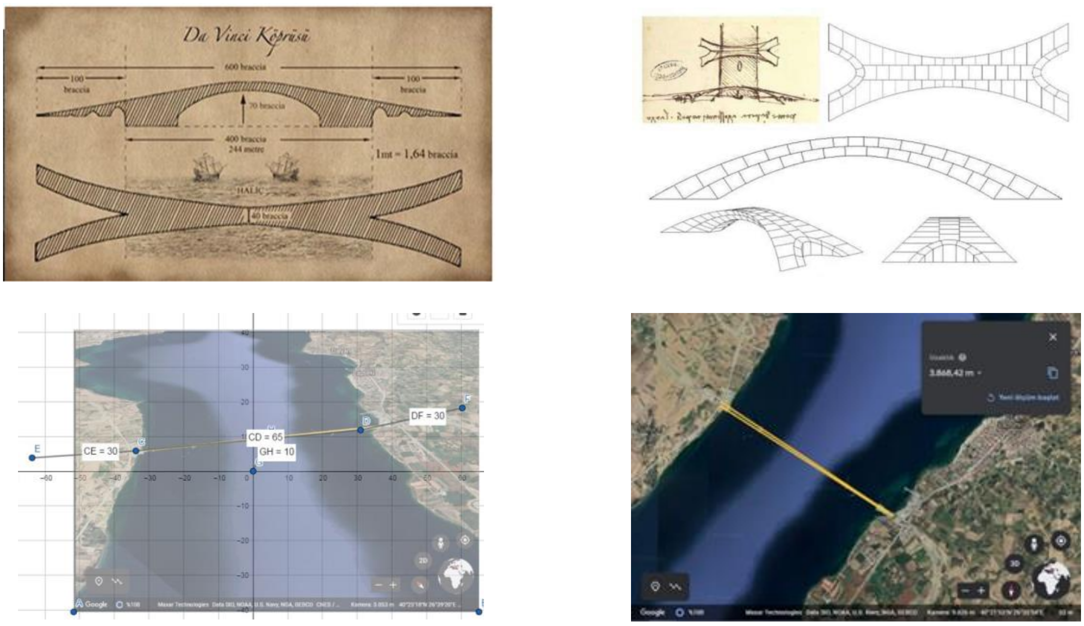
3. DESIGNING

D. Design/Prototype

The designer, who clarifies the problem situation in the step of designing, first focuses on developing possible solutions (at least one) that explain this problem in detail and are suitable for the student's level. Here, the scope of the problem (for example, how it can be used for different levels) is determined by considering possible different solutions besides one solution. While designing, the designer shapes the newspaper article, readiness questions, possible problems in current problem design and advanced level mathematical models that may arise in the solution of this current problem, which are components of the HTTM activities that come after the problem situation and solution. The first draft HTTM task, which is formed by the combination of the parts revealed at the end of the design process, will be a prototype of the design (the first design). The step of designing includes rapid prototyping through iterative processes and illustrating the process and steps in detail. Argumentation actions in designing occur with more comprehensive and deep justifications.

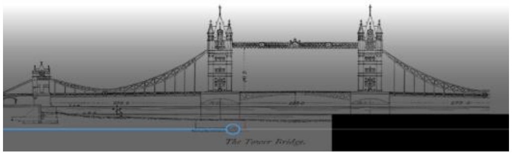
At this step, Çağıl stated that giving the pictures about Da Vinci Bridge in the newspaper article would provide the students with the opportunity to create more comprehensive mental representations of the real-life situation (mental model) and to reach different solutions. Çağıl noted that she intentionally did not give some features of the context to the students and that she expected them to search about it and find themselves. In addition, she stated that she aimed to arouse curiosity by providing short videos about the Da Vinci Bridge with the newspaper article.

> Çağıl: *Giving the pictures of the bridge in the newspaper article* will enable the children to imagine the modeling in their minds more easily and try *different solutions.* …I deliberately left out some aspects of the context of the problem. I wanted them to search and find out what is asked there, namely the features of the bridge on which they would practice. …. In addition to the newspaper article, I will provide *short videos about Da Vinci Bridge,* and in this way, by watching these videos in the process, students will have more curiosity.



Although Mert read several different sources during the designing step, he emphasized that he didn’t want the newspaper article to be too long, and so did not include everything into the task so that the students would not get bored. Mert also stated that he tried to make the newspaper article mathematics-based and asked the students to make a search by not providing every detail related to the content. Besides, Mert emphasized that, based on his own learning experiences, he could have the students make a three-dimensional model of the bridge in the process, so that he could involve students in the process and get more extensive solutions.

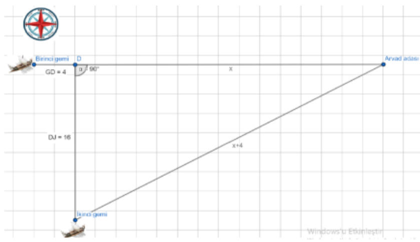
> Mert: I read and got information from about seven different internet sources; but I didn't want to include them all in my newspaper article. Because I thought *students would get bored if my newspaper article was too long*. That's why, I thought I should *centre the newspaper article around the math in the bridge*. …In the newspaper article, it was also important to get students to make a search by not providing them with every single detail.



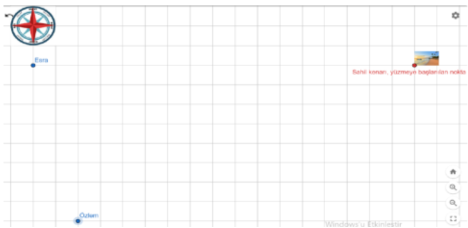
> Mert: *Since this is an important bridge, there were also its 3D models*. You had made a 3D model of it in the task of the Alexandria lighthouse. I thought that I could also have the students make this model in this task. In this way, I thought that I could both *involve the students in the process and get better solutions*.

In her design, Özge thought about possible different solutions to the problem situation in which she related the distances of two ships in different locations to Arvad Island in the Phoenicians maritime historical context. Also, she thought about possible current problem situations that students could design in dual modeling in HTTM learning process and included separate solutions to each of these in the process.

The Map in the Problem Situation in the History of Science



GeoGebra File including a Possible Solution



Current Context Obtained in Dual Modeling Process-1

Current Context Obtained in Dual Modeling Process-2

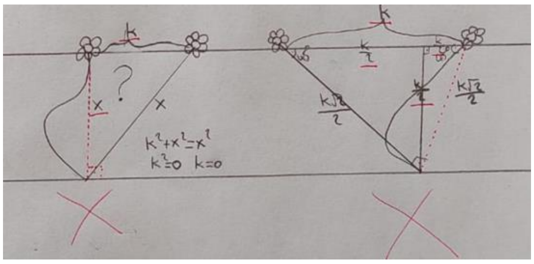
> Özge: *Here, students can replace students with ships, and schools with Arvad island*. Because students are asked for a situation that can be encountered in daily life and they spend most of their time at

school. …Students can *design a mathematical modeling problem that includes sea*, since the given problem includes sea and ship.

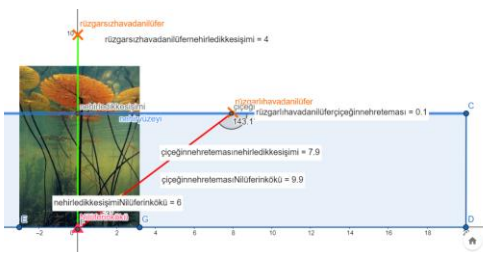
Kumru stated that the students should consider using the height of the Galata Tower, which is an important strategic factor in the solution, and that she expected them to approximately estimate its height from the photographs given to them. In this sense, Kumru mentioned an example estimation strategy while describing possible solutions.

Kumru: Students should consider using the height of the Galata Tower in the solution. I intentionally did not give direct information about this. *They should find the height of the Galata Tower from the photos I gave in the newspaper article. …For example, they can make use of the height of the car, as in the figure, just as I did.*

Yiğit presented the possible solution of the Lotus problem, which was included in the problem situation of the HTTM task, on GeoGebra and elaborated the situations that he could not explain on GeoGebra by supporting it with a written answer sheet.



An Extract of the Written Answer Sheet with a Possible Solution



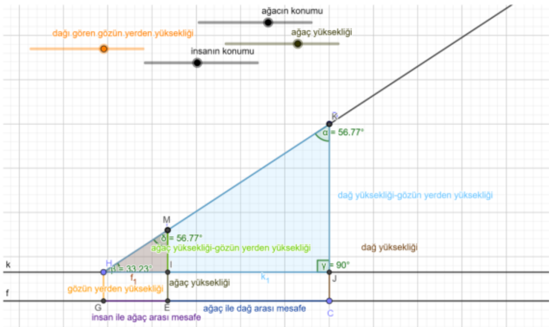
GeoGebra File with a Possible Solution

In the step of designing, based on the problem situation, Ayça mentioned different solution strategies that students could follow in the solution. Ayça also emphasized that in the dual modeling phase of HTTM learning process, learning environments outside the school can be used to create environments where students can calculate the heights of the tall buildings around them. In addition, Ayça viewed calculating the distance of a ship from the shore as a possible problem situation.

Ayça: After the solutions to the problem are finalized, the students are asked to design a current modeling problem. Here students can try *different strategies*. Students can create problems *to calculate the height of famous towers and tall buildings they see in daily life*. In order to establish similarity in these problems, they can use other objects instead of trees. They can create a problem situation on the *ground clearance of a cable car*. …One possible problem that could reveal advanced mathematical models would be the problem of *calculating the distance of a ship or an island from the shore by someone looking out to sea from the shore.* The similarity to be established here is different from that in our problem situation, although it is similar in some respects.

Ayça's Problem Situation. The studies carried out in Chinese civilization were also effective in the daily life of the people and the emperor. One day, two officials of the same status applied for an important job in the empire. The Chinese emperor summoned them before him so that he could choose between these two officials. The Chinese emperor was wandering around the garden when the officers came to the empire. Pointing his finger at a mountain they considered sacred, he said that he would give the job to those who found the height of this mountain. They had to find the length of this mountain with the resources available from the spot there were standing. If you were one of these officials, how would you calculate the height of a mountain with the methods used back then? Support and explain your solutions with mathematical models.

A GeoGebra Section from Ayça's Sample Solution



D. Design/Prototype

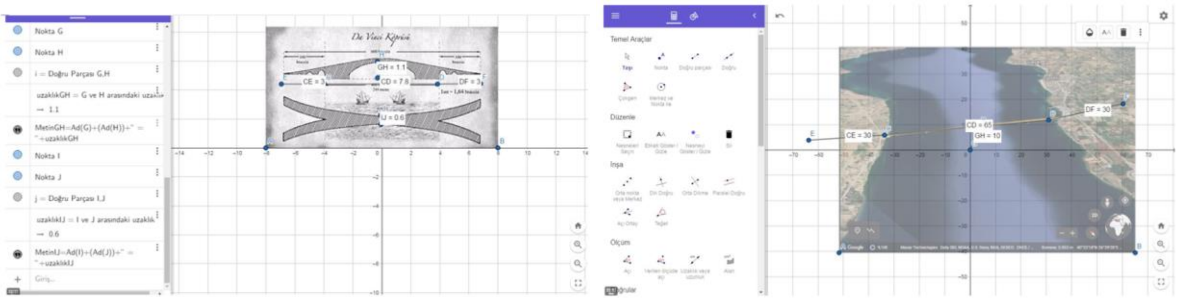
4. EVALUATING

E. Results

In the step of evaluating, the designers who come up with the design/prototype aim to reach some judgments about the effectiveness of the design by considering the outcomes they obtain from the design. For instance, they question the accuracy of the priori and other possible solutions that they have expected to emerge in HTTM task design learning process, with different strategies. At this step, convergent thinking is observed again. The designer considers all aspects of the design, offers opinions on its pros, cons and limitations, and makes some suggestions for its improvement. At this stage, self-evaluation is employed, and peer and expert opinions were also obtained.

After creating her design, Çağıl made some comments on how this task can be used at different levels. She emphasized that Da Vinci's different works could also be included in the task context. She also reported that high school students can understand this content better, yet she noted that it can be more challenging for middle school students, and therefore she didn’t regard her activity as flexible.

> Çağıl: It would be appropriate to use the problem starting from the 7th grade in the program. Because 7th grade covers the concept of ratio-proportion extensively and it is also related to algebraic expressions and equations. It is a problem that can be easily used in 8th grade in terms of congruence and similarity by using scales. Scale-similarity-ratio proportion relationship is used very easily. However, I don't think *it is flexible enough to be used before 7th grade.* The problem can *be extended on the same topic to include different works of Da Vinci. I think high school students can solve it more easily*. I think it will be challenging for the middle school student level and close to the area of convergent development. *…I don't think I have created a flexible problem, which is one of my shortcomings.*



Mert made self-evaluations after his design. He stated that he structured his newspaper article well and that his task served for two different learning outcomes. He viewed the solution of the problem situation as a positive feature in the designing and the deficiencies in structuring the dual modeling process as negative features. For him, the deficiency in dual modeling process was that he did not focus on this part sufficiently. In addition, although he had enough time, he didn't make any effort to revise this section. This is considered to be caused by Mert's insufficient self-efficacy level.

> Mert: I think I structured the newspaper article well. …The fact that it addressed two different learning outcomes considering its problem situation was a strength. I also liked the solution situation, but *I have a weakness in students' designing mathematical modeling problems from daily life*. I could not focus enough on that sense *(At this stage, he did not take any mental action to make up for this weakness in the step of revising since he accepted the situation as it was).*

Kumru reported that although she had a good grasp of some basic ideas during the design process, she did some research to come up with original ideas, and this was a waste of time for her. In this regard, it can be said that Kumru wasted plenty of time, especially in the divergent thinking part of the design process. Yiğit stressed that even though he had determined the problem situation, he experienced difficulties in deciding what information to give and to what extent in the newspaper article and problem situation. He attributed this difficulty to his lack of teaching experience at the 8th grade level.

> Kumru: During the process, I had come up with several ideas in my mind related to the task, but I wanted to find different things and made more searching. This was somehow *a waste of time for me.*

> Yusuf: After I had found the problem situation, while I was *preparing the newspaper article and my problem situation*, I was uncertain about what information to give and to what extent. I suppose one of the important reasons for this is that *I do not have a lesson or task experience with 8th grade students.*

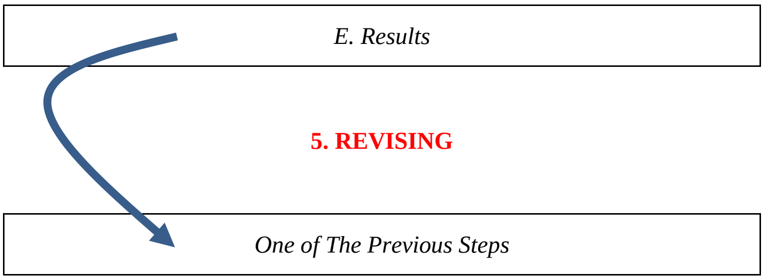
While evaluating her design, Ayça stated that the level of the task affected the number of hints given for the problem. For this reason, Ayça explained that she did not want to include advanced level mathematical studies in Chinese civilization. Ayça marked that she did not include the information that was not suitable for the student's level in her design, considering that it would increase their mental burden. She also regarded the fact that the newspaper article was not directly related to the problem situation as a shortcoming and explained the underlying reason for this as the fact that she was not able to find a more relevant story.

> Ayça: Here, *the fact that the task was designed for middle school level affected whether the number of hints to be given would be low or high*. …I didn't want to go into advanced mathematical studies conducted in Chinese civilization. Because I thought *information that is not suitable for the level of the students may cause gaps in their minds while they are reading the article*. …There could have been a story closer to my problem situation, but there was no such a story, and the resources were insufficient.

While evaluating the design process, Yiğit reported that he experienced difficulties in decision making while narrowing his focus down. He stated that he wasted a lot of time in the divergent thinking part of his design like Kumru. Yiğit attributed this situation to the difficulty in accessing resources on the history of science and his inadequacy while using GeoGebra software.

> Yusuf: *The most challenging part* for me was *the decision-making phase* before starting the task. I was very undecided about what kind of a HTTM task to design. The most important factors affecting me

in this regard were the difficulties I had in accessing the resources and my limited skill in using the GeoGebra software.



In the step of revising, if there are unexpected and undesirable situations in line with the evaluation results obtained, or if there is a flaw in the content, the previous steps are checked with the decision of the designer and so the design is revised. This step necessitates the elimination of the deficiencies existing in the design, and necessary changes are made accordingly. Revising continues unless the designer is satisfied with the design and accepts the final design.

Yiğit intended to include a video in the newspaper article; however, he could not find a video suitable for the context during the step of revising despite his efforts. Yiğit also investigated the applicability of his task to the classes below the eighth grade yet could not reach any conclusion.

> Yiğit: I *intended to include a video* in the newspaper article, but I could not find a video suitable for my problem situation and my newspaper article (He searched but could not find a video). …*I could not find examples of how this task could be adapted to the grades below 8th grade (He searched different sources and examined the curriculum in detail).*

After the evaluations about the GeoGebra solution of the problem situation, Mert, thinking that he could improve his solution, tried to revise the solution and attempted to make adjustments on it. Considering her design, Meryem concluded that she had asked too many questions, and decided to remove some of the questions she chose and reduce the number of questions.

> Mert: In fact, *I can improve this mathematical model*. Let me try one more time (Tried to make improvements on the solution again.)

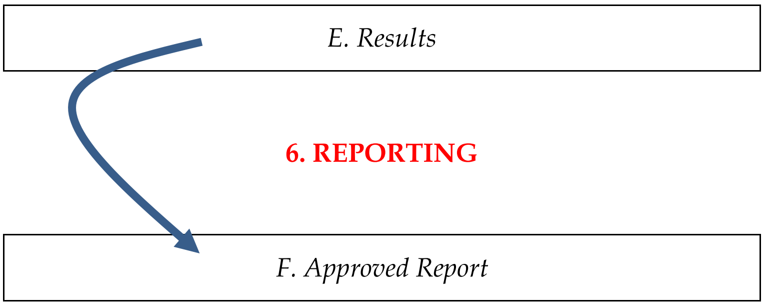
> Meryem: Actually, students can calculate the height of the school, but I asked too many questions. It will be better if I take that out.

In the evaluations regarding her design, Yağmur thought that including an extra unit conversion in the task would reveal a more effective learning process and made the necessary revisions in the design by taking into account the possible difficulties of this change in the context and its suitability for the level of the student.

> Yağmur: Actually, it would be better if I did something like this here. Some studies reveal that making conversions in the units such as km, hm, min, m can be better for students so that students can better understand the concept of area and can better integrate these into daily life. That's why *it would be better here if I ask it in acres [a land measure of about 920 square metres], instead of m^2*. But it can be difficult to refer to y and h expressions as acres for students. Because in fact, students directly create the area relation here. *Regarding this conversion subject, I think that students can make conversion calculations in GeoGebra by connecting these h and y to a slider.* Students already know how many square meters an acre is. Because in the 5th grade, land measurement units are covered. [He made the relevant revision during the design process.].

Kumru attempted to include a video in her design and found a suitable video for this. Looking at the content of the video, she examined how the video could serve the problem situation and later decided not to include it since the old and new heights of the Galata Tower were different and it would be difficult for students to reach their old height. Another idea she held was that she could make an airplane out of origami, yet she left this idea as she did not know how to construct its mathematical model.

Kumru: I want to i*nclude a video* here as well. This video is pretty good. But I have to watch the video, it shouldn't give the answer to the problem I asked. (She watched the video and added it as part of the task.) … In the past, the height of the Galata Tower was different. The students will find it according to the current height. (She thought about this contradiction.). They may not reach the conditions at that time. I should keep this in this way. …*I can actually have them make an airplane out of origami, but how can I build its mathematical model?* (After some thought, she gave up including it.)



When the designers felt satisfied with the outcomes of their designs, they moved on to the step of reporting. In the step of reporting, important issues that should be written in the report covering the design are highlighted, the existing explanations are supported with detailed mathematical expressions and the things that should be written in the report are listed. The reporting is carried out in order to present what is at hand better and elaborating the reasons more clearly rather than making structural changes as in the step of revising. What is achieved in the reporting step can be viewed as the final touch on the design process.

Expressing that she should present her design to the students in a simple and meaningful order as much as possible, Ayça revised it and made minor edits and changes in the order. Kumru, on the other hand, zoomed some of the photos, thinking that they were blurry and small. As can be seen, different from the revising step, in the reporting, there are simple and visual edits, not deep and structural ones.

Ayça: While finalizing the newspaper article, I should provide the students with the information I learned as a result of the investigations into Chinese civilization and mathematics, *in a simple and meaningful order as much as possible* (Re-read and made arrangements.).

Kumru: Here, the photos are *not clear*, they *seem small. Let me zoom the photos a little more*. Students can figure out proportions more precisely when I print them out.

Mert stated that he had to put the files in order before submitting his design and that there was several spelling mistakes, and then he arranged them. Although Çağıl thought that he had written the report in detail, he nevertheless quickly made a final revision. He submitted his report without much editing.

Mert: There are a few minor spelling mistakes. I need to fix them. (He made minor edits to finalize his report.) …I'll *put the files I need to submit in an order*. …it's almost over. I worked so hard; I don't want to forget to send the files.

Çağıl: In fact, I wrote the report in detail, but let me have a look at it again before submitting it. I may not have explained everything in the video, but I reflected everything in detail in the report. *I tried to present the explanations in a clear way from the simple one to the complex.*

7. Discussion, Conclusion and Suggestions

In this study, it was aimed to explain how the HTTM (History / Theory / Technology / Modeling) task design processes (basic components and steps) took place. In this study, mathematics student teachers' HTTM task design processes were explained through six basic components (task, focus/origin, problem, design/prototype, results, approved report) and six basic steps (investigating, exploring, designing, evaluating, revising, reporting). In addition, the mental actions performed to achieve the sub-goals gave the opportunity to reveal the characteristics of the basic steps that connect the components to each other. The basic steps were elaborated in terms of their characteristics and with the help of the studies in the literature. HTTM task design process is not in a linear but iterative-cyclical nature, and the designer goes back to the previous steps with instant turns (metacognitive/self-regulated action) if necessary. In HTTM design process, the main concern was not about time but creating a quality HTTM task. The designers generally focused on the design of the main problem and the development of related solution/s, and at certain stages employed divergent-convergent thinking similar to the double diamond model [Design Council, 2007] and IDEO (2015) design process. In HTTM task design process, divergent thinking came to the fore in the steps of *investigating and designing* while convergent thinking was observed in the steps of *exploring and evaluating*. Similarly, Hassi and Laakso (2011) defined design-based thinking as a combination of convergent and divergent approaches. Divergent and convergent thinking correspond to the concepts of induction and deduction respectively. The high-level interaction of divergent and convergent thinking leads to environments that will develop abductive thinking skills. Therefore, it is highly important.

While in the transition from task to problem (*investigating* and *exploring*) in HTTM design process, the main purpose is to choose the right task to achieve quality (*design the right problem*), in the transition from the problem to the results (*designing* and *evaluating*), the main purpose is to design it correctly (*design it right*). In HTTM task design process, similar to Borromeo Ferri (2018) and Galbraith (2006), *supportive factors* such as student level, current curriculum, physical environment, time management, teacher-student role, one/multi/inter/transdisciplinary perspective and content, flexibility, 21st century skills, application process, different solutions and the way they are presented, and context were effective in enriching the steps and non-linear transitions. The designers had a hard time identifying a starting point/focus (*investigating*) for their designs and wasted a great deal of time. The step of investigating includes discovery. FROG (2014) revealed that the most time-consuming part in the task design process is discovery (Bobbe et al., 2016). Likewise, the double diamond model (Design Council, 2007) points to the difficulty for the designer to determine a focus without being distracted within the multi-complex structure resulting from divergent thinking. While determining the focus and the main problem of HTTM task during the steps of investigating and exploring, the mathematics student teachers took into account the relevant mathematical concepts, the number and quality of conceptual relations, prominent figures in the history of science, civilizations, notable problems, possible solutions to problems, the number of ways to the solution, the relationship between the problem and current problems, improper and incomplete ideas, solutions and proofs, the content of the newspaper article and the level of the student. The step of exploring includes defining.

In the step of *investigating*, the designers searched for an event/phenomenon in the history of science for their HTTM task. The designers made predictions related to the next stages to be experienced by the students and themselves with the historical context they had chosen. Based on their predictions, realising that the context was above the class level, or they could not solve it themselves caused the designers to change their focus. In line with the Feasibility of Approach, one of the design principles developed by Geiger et al. (2022), the designers paid attention to the problems they could solve. In HTTM design process, it can be said that metacognitive prediction skills emphasized by Hıdıroğlu (2021) and Niss and Blum (2020) and outcome expectation skills in forethought phase in self-regulation proposed by Zimmerman (2002) were active throughout the process. The structure of mathematical models that might arise in the solution in the steps of *investigating and exploring*, and the prediction of what kind of mathematical solution it would provide for the problem that came to life in *the history of science* (Czocher, 2017; Hıdıroğlu & Bukova Güzel, 2016; Jankvist & Niss, 2019; Stillman et al., 2007; Stillman & Brown, 2014) became influential in their belief that they would design the right task in HTTM tasks. In HTTM design process, similar to Swetz (1994), leading scientists and fundamental historical problems became an important focus for the designers in the step of investigating. Illumination and history-based strategies by Jankvist (2009) and addition and accommodation strategies presented by Fried (2001) were observed in HTTM design process,. According to the distinction made by Jankvist's (2009) between goal and tool in the integration of the history of science, the history of science in the design process of HTTM tasks has mainly served as a tool that reveals a rich mental process.

In the step of *designing*, possible solutions (*at least one*) of the main problem that are supposed to be suitable for the student's level are presented. The scope of the problem, that is, to what extent it can be extended (*for example, how it can be used for different levels*), is determined. In the designing step, the newspaper article, readiness questions, possible current life problems that students can design, and advanced mathematical models that may arise in the solution of this current problem, which are the basic parts of HTTM tasks, were identified. The designing step involves rapid prototyping through iterative processes, illustrating the process and steps in detail. In terms of integrating technology into HTTM design process, in the first steps (investigating and exploring), video, animation, pictures and photographs played an active role in determining the starting point (focus) and the main problem. Orr et al. (1997) and Stemler (1997) emphasized that pictures, photographs, animations and videos in mathematics education help to associate multiple representations and have deep discussions, to understand the relationships between variables, and to develop appropriate strategies. Many researchers in mathematical modeling (Blomhøj, 2019; Hıdıroğlu, 2012; 2015; 2021; Hıdıroğlu & Özkan Hıdıroğlu, 2016; Lingefjärd, 2012; Ortega et al., 2019; Stillman, 2019) noted that videos and photographs enrich the mental process. In the design, extensive considerations were noticed in the integration of dynamic mathematics and geometry software into HTTM learning process. The designing process includes developing. Similar to the thought of Sierpinska (2004), the mathematics student teachers thought that the details of their design could affect students' performance, and in the designing, they focused on detailed structuring of all parts and functions of the HTTM task which has a wider range than the problem, the main component of the process (divergent thinking). In the designing, as Venturini and Sinclair (2017) stated, intellectual clues for the learning process were determined and well-prepared guiding questions were created. In parallel with Yerushalmy et al. (2017), multiple strategies that can be revealed in the activities were put forward at the designing step. In the modeling task designing process, considering the theoretical frameworks related to both modeling types and the modeling process is of great importance in explaining the rich mental process that occurs in the process (Borromeo Ferri, 2018). In this context, the designers structured their tasks by taking into account the HTTM learning process and the technology-supported mathematical modeling process.

In the step of *evaluating*, the designers questioned the effectiveness of their designs in the learning process and what they could do with it and came up with some evaluation results. The accuracy of the possible solutions expected to emerge in the learning process of HTTM design were checked through different strategies. In line with the feasibility of outcome, a design principle introduced by Geiger, Galbraith et al. (2022), the designers, in the step of evaluating, thought about real-life outcomes they could reach with the help of mathematical models that explain the history-based context and made explanations about the nature of the process. Managing the progress in the critical aspects of a task and predicting unexpected responses from students, as suggested by Geiger (2019), was observed in the step of evaluating during the design process. According to IDEO (2012), the design is tested in the evaluation, and intellectual outcomes regarding the quality and effectiveness of the design are obtained in line with what can be done with the design. In the study, the designers did not have the opportunity to practically test/try their designs in the *evaluating* step. However, they exhibited self-evaluation actions and revised their designs in line with the evaluation results they obtained, if necessary, by considering various points based on internet and experiences.

In the design process, although HTTM was designed with a holistic-pragmatic approach, the basic problem situation also emerged as multiple sub-problems with an atomistic understanding. This indicated that atomistic understanding also found its place in the design process. Kaiser and Brand (2015) argued that while the holistic approach is more effective in developing, interpreting and validating capacities, atomistic approach is better in working mathematically and supporting the development of mathematical analysis. As suggested by Geiger et al. (2022), in the step of *evaluating*, the designers arranged the open-ended structure of the problem and made some adjustments in the problem in accordance with the student's level. The designers also included sub-problems with an atomistic understanding, with the concern that students might need scaffolding in the HTTM learning process. This is in line with the didactic flexibility, a task design principle of Geiger et al. (2022). Therefore, it may be a good strategy to address the main problem as well as the supportive sub-problems in HTTM design process, but further research is needed on this subject.

In the steps of evaluating, revising, and reporting, designers display mental acts (metacognitive/self-regulated actions) through their own thinking. The designers, who evaluated their own designs in the evaluating step and obtained results related to it, were nurtured by the self-reflection phase of Zimmerman (2002). Causal attributions to designers' designs and performances affect this process. To give an example, if self-satisfaction is achieved, the process is reported, if not, the step of revising is activated. In the step of *revising*, the designers adjusted their designs, if necessary, in line with the regulatory implications. If there are unexpected/undesirable situations in the evaluation results reached in the evaluating step or if an error/problem is believed to exist in the content, necessary changes were made in the *revising* step by going back to the previous step with the designer's decision, deficiencies in the design were eliminated, and in this way the design was tried to be improved. Revising continued until the designer was satisfied with the design or approved its final version. The revision step here is similar to the redesigning steps in engineering design process model proposed by The Massachusetts Department of Education and the National Centre for Engineering and Technology Education [NCETE] (Hynes et al., 2011).

In the step of *reporting*, minor adjustments on the surface were performed rather than structural changes, and the presentation was arranged to better reflect the existing structure. In the *reporting*, there were more superficial non-structural arrangements regarding the presentation of the design that was already approved structurally. FROG (2014) referred to this kind of process as delivering (Bobbe et al., 2016). In the *reporting*, the draft was finalized to be delivered. As an example, you intend to give a birthday gift for somebody and you have bought the gift; however, the way you deliver it (like gift-wrapping) is also crucial. The step of reporting includes delivering. As in Hıdıroğlu (2015), the designers clarified the important issues that should be written in the report before submitting their reports, supported the thoughts with detailed and correct mathematical expressions, and listed what should be written in the report. National Centre for Engineering and Technology Education [NCETE] engineering design process model refers to this step as *finalize design*. Similar to Geiger et al. (2022), in the reporting step, while creating their reports, the designers conveyed their thoughts through a concise, consistent and systematic report and paid attention to the use of appropriate mathematical language in the report.

When the literature is examined, while there are limited studies on the design process of mathematical modeling activities, this study is the first example in the literature in terms of addressing the HTTM task design processes and revealing a process model. It can be said that this process model can be an important guide in determining the difficulties experienced by mathematics student teachers and mathematics teachers while developing mathematical modeling tasks and overcoming these difficulties. The fact that the study was carried out with experienced and skilled individuals in the relevant field gave the opportunity to obtain a rich process model. This process model obtained explains the mental actions in the mathematical modeling task design process and can give clues about 21st century teacher competencies. Although there are few studies in the lierature on task design in mathematical modeling, this study is similar to them in that it considers both the HTTM learning process and engineering design approaches. The multifaceted nature of HTTM may make this process model attractive for those conducting studies in other fields (engineering, history of science, interdisciplinary study). Considering that the basic skills in the parts of integrated STEM include computational thinking, mathematical modeling, engineering designing and scientific inquiry, it can be said that this design process model can be a notable resource in integrated STEM studies. In future studies, checklists or rubrics can be designed by considering this theoretical framework. In this way, the evaluation of such learning processes can be healthier and easier.

Considering that the main purpose of integrating mathematical modeling into the education process is to make the learning process more effective and to educate individuals who are suitable for the human profile needed today, to equip them with the necessary knowledge and skills to be successful in real life, The HTTM learning model, which is presented as a new understanding for the conceptual learning process and integrates the history of science, theory, technology and modeling into the same process, is a new and noteworthy perspective in enriching the learning process and integrating today's education understanding into lessons. While this learning process supports 21st century skills, it is pivotal as it fosters other modeling perspectives and future modeling approaches. HTTM learning process is one of the modeling approaches that needs to be developed most. In this sense, there is a need for more experimental studies explaining this newly emerging HTTM learning process and more resources for teachers to create the HTTM learning process. Further studies can be conducted especially on the role of HTTM learning process in integrating mathematical modeling into transdisciplinary STEM, its relationship with computational thinking, and the integration of science history and technology. In addition, difficulties faced by mathematics teachers and students during the application process of HTTM activities should be unearthed.

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