Karadeniz Fen Bilimleri Dergisi, 12(2), 560-570, 2022. DOI: 10.31466/kfbd.1036644

KFBD

Karadeniz Fen Bilimleri Dergisi
The Black Sea Journal of Sciences
ISSN (Online): 2564-7377

Araştırma Makalesi / Research Article

On Absolute Cesáro Series Space And Certain Matrix Transformations

Fadime GÖKÇE[1*]

Abstract

On recent paper, the paranormed space $|C_{\lambda,\mu}|(p)$ which is defined as the domain of Cesáro matrix in the Maddox's space $l(p)$ has been introduced and studied in (Gökçe and Sarıgöl, 2019). In this study, some characterizations of matrix operators from the Absolute Cesáro series space $|C_{\lambda,\mu}|(p)$ to the classical sequence spaces c, c_0, l_∞ are given. Also, it is shown that the matrix operators between the absolute Cesáro series space and the spaces c, c_0, l_∞ are bounded operators. Finally, certain results are obtained as a special case.

Keywords: Absolute summability, matrix transformations, Cesáro matrix, bounded linear operators.

Mutlak Cesáro Seri Uzayı Ve Bazı Matris Dönüşümleri

Öz

Son zamanlarda, Cesáro matrisinin, $l(p)$ Maddox uzayı içinde toplama alanı olarak tanımlanan $|C_{\lambda,\mu}|(p)$ paranormlu uzayı tanıtılmış ve çalışılmıştır, (Gökçe ve Sarıgöl, 2019). Bu çalışmada, $|C_{\lambda,\mu}|(p)$ mutlak Cesáro seri uzayından c, c_0, l_∞ klasik dizi uzaylarına tanımlanan matris operatörlerin karakterizasyonları verilmiştir. Ayrıca mutlak Cesáro seri uzayları ve c, c_0, l_∞ uzayları arasındaki matris operatörlerin sınırlı lineer operatör olduğu gösterilmiştir. Son olarak, özel seçimlerle bazı sonuçlar elde edilmiştir.

Anahtar Kelimeler: Mutlak Toplanabilme, matris dönüşümler, Cesáro matrisi, sınırlı lineer operatörler.

[1]Pamukkale University, Department of Statistics, Denizli, Turkey, fgokce@pau.edu.tr

[1]https://orcid.org/0000-0003-1819-3317

[*]**Sorumlu Yazar/ Corresponding Author**
Geliş/Received: 14.12.2021 **Kabul/Accepted:** 01.11.2022

1. Introduction

The summability theory is one of the most important fields of mathematics, which has various applications in analysis, applied mathematics, engineering sciences, specially quantum mechanics, probability theory, Fourier analysis, approximation theory and fixed point theory, etc. It deals with the generalization of the concept of convergence of sequences and series, and aims to assign a limit for non-convergent sequences and series using an operator described by an infinite matrix. The reason why matrices are used for a general linear operator is that a linear operator from a sequence space to another can generally be given with an infinite matrix. This reveals the importance of sequence spaces and matrix operators in summability theory. In recent times, the literature has grown up concerned with characterizing all matrix operators which transform one given sequence space into another. For example, the absolute Cesáro series space $|C_{\lambda,\mu}|(p)$ has been introduced and some matrix characterizations of the matrix classes related to the space have been examined in (Gökçe and Sarıgöl, 2019), (see also (Gökçe, 2021; Gökçe and Sarıgöl, 2020; Gökçe and Sarıgöl, 2018; Gökçe and Sarıgöl, 2019a; Güleç, 2020; Sarıgöl, 2016; Zengin and İlkhan, 2019)). In this paper, we investigate the matrix class $\left(|C_{\lambda,\mu}|(p),\Gamma\right)$ where $\Gamma=\{c,c_0,l_\infty\}$, and then we present some results.

2. Materials and Methods

Let ω stands for the set of all complex (or real) valued sequences. Any vector subspace of ω is called as a sequence space. We represent the set of all convergent, null and bounded sequences and the set of all convergent and bounded series spaces by $c, c_0, l_\infty, c_s, b_s$, respectively. Let X, Y be arbitrary sequence spaces and $U=(u_{nv})$ be any infinite matrix of complex numbers. By $U(x)=\left(U_n(x)\right)$, we denote the U-transform of the sequence $x=(x_v)$ if the series

$$U_n(x)=\sum_{v=0}^{\infty}u_{nv}x_v$$

is convergent for any integer n. If $U(x)\in Y$, whenever $x\in X$, then it is said that U defines a matrix transformation from X into Y, and the class of all infinite matrices U such that $U: X\to Y$ is represented by (X,Y). Besides, the concept of matrix domain of an infinite matrix U in a sequence space X is defined by the set

$$X_U=\{x\in\omega: U(x)\in X\}$$

which is also a sequence space.

The Maddox's space defined by

$$l(p) = \left\{x = (x_n) : \sum_{n=0}^{\infty} |x_n|^{p_n} < \infty\right\}$$

has an important role in summability theory. Note that $l(p)$ is an FK space which is a complete metrizable locally convex space with continuous coordinates $r_n: X \to \mathbb{C}$ defined by $r_n(x) = x_n$ for all $n \in \mathbb{N}$, according to its paranorm given by

$$g(x) = \left(\sum_{k=0}^{\infty} |x_k|^{p_k}\right)^{1\backslash M}$$

where $M = \max\{1; \sup_k p_k\}$. Also, this space has AK property that is every sequence $x\epsilon l(p)$ has a unique representation $x = \sum_{k=0}^{\infty} x_k e^{(k)}$ where $e^{(k)}$ is the sequence whose only non-zero term is 1 in the kth place for each $k\epsilon\mathbb{N}$ (Maddox, 1969; Maddox, 1968; Maddox,1967).

Let $\sum a_v$ be an infinite series with the sequence of its partial sum $s = (s_n)$, $\theta = (\theta_n)$ be any sequence of positive real numbers and $p = (p_n)$ be any bounded sequence of positive real numbers. The series $\sum a_v$ is said to be summable $|U, \theta_n|(p)$ if

$$\sum_{n=1}^{\infty} \theta_n^{p_n - 1} |U_n(s) - U_{n-1}(s)|^{p_n} < \infty,$$

(Gökçe and Sarıgöl, 2018).

Let $\sigma_n^{\lambda,\mu}$ be the n-th Cesáro mean (C, λ, μ) of order (λ, μ) with $\lambda + \mu \neq -1, -2, \ldots$ of the sequence (s_n), i.e.,

$$\sigma_n^{\lambda,\mu} = \frac{1}{A_n^{\lambda+\mu}} \sum_{v=0}^{n} A_{n-v}^{\lambda-1} A_v^{\mu} s_v,$$

where

$$A_n^{\lambda} = \frac{(\lambda+1)(\lambda+2)\ldots(\lambda+n)}{n!},$$

$$A_0^{\lambda} = 1, A_{-n}^{\lambda} = 0, n > 0.$$

If we consider the concept of absolute summability and Cesáro matrix with $\theta_n = n$ for all $n \in \mathbb{N}$, then, we get immediately the absolute Cesáro summability method. Let us give more clear definition of the method: if

$$\sum_{n=1}^{\infty} n^{p_n - 1} \left|\sigma_n^{\lambda,\mu} - \sigma_{n-1}^{\lambda,\mu}\right|^{p_n} < \infty,$$

then, the series $\sum a_v$ is said to be summable $|C, \lambda, \mu|(p)$.

The space consisting all series summable by the absolute Cesáro summability method can be expressed as

$$|C_{\lambda,\mu}|(p)=\left\{x:\sum_{n=1}^{\infty}n^{p_n-1}\left|\sum_{v=0}^{n}\left(\frac{A_{n-v}^{\lambda-1}}{A_n^{\lambda+\mu}}-\frac{A_{n-v-1}^{\lambda-1}}{A_{n-1}^{\lambda+\mu}}\right)A_v^{\mu}s_v\right|^{p_n}<\infty\right\}.$$

Note that according to notation of domain, the absolute Cesáro space may be redefined as $|C_{\lambda,\mu}|(p)=(l(p))_{T^{\lambda,\mu}(p)}$ where the matrix $T^{\lambda,\mu}(p)$ is given by

$$t_{nv}^{\lambda,\mu}(p)=\begin{cases}1, & n,v=0\\ \dfrac{vA_{n-v}^{\lambda-1}A_v^{\mu}}{n^{1/p_n}A_n^{\lambda+\mu}}, & 1\le v\le n\\ 0, & v>n.\end{cases}$$

Moreover, it is known that every triangle matrix has a unique inverse which is also a triangle, so $T^{\lambda,\mu}(p)$ has the inverse matrix $S^{\lambda,\mu}(p)$ such that

$$s_{nv}^{\lambda,\mu}(p)=\begin{cases}1, & n,v=0\\ v^{1/p_v}\dfrac{A_{n-v}^{-\lambda-1}A_v^{\lambda+\mu}}{nA_n^{\mu}}, & 1\le v\le n\\ 0, & v>n\end{cases}$$

where $\lambda+\mu,\mu\neq-1,-2,\ldots$.

Throughout the whole paper, we suppose that $p=(p_n)$ is any bounded sequence of positive real numbers with $0<\inf p_n<\infty$ and p_n^* is the conjugate of p_n such that $1/p_n+1/p_n^*=1$ for $p_n>0$, $1/p_n^*=0$ for $p_n=1$.

Before the main theorems, we remind certain lemmas which have important role in their proofs :

Lemma 2.1. (Grosse – Erdmann, 1993) Let $p=(p_v)$ be any bounded sequence of strictly positive numbers.

(i) If $p_v\le 1$, for all v, then,

$$U\in(l(p),c)\Leftrightarrow(a)\ \lim_{n\to\infty}u_{nv}\text{ exists for each }v,\qquad(b)\ \sup_{n,v}|u_{nv}|^{p_v}<\infty$$

$$U\in(l(p),c_0)\Leftrightarrow(c)\ \lim_{n\to\infty}u_{nv}=0\text{ for each }v,\qquad(b)\text{ holds}$$

and

$$U\in(l(p),l_\infty)\Leftrightarrow(b)\text{ holds.}$$

(ii) If $p_v>1$ for all v, then,

$$U\in(l(p),c)\Leftrightarrow(a')\ \lim_{n\to\infty}u_{nv}\text{ exists for each }v,\quad(b')\text{ there is a number M}>1\text{ such that}$$

$$\sup_n\sum_{v=0}^{\infty}|u_{nv}M^{-1}|^{p_v^*}<\infty,$$

$$U\in(l(p),c_0)\Leftrightarrow(c')\ \lim_{n\to\infty}u_{nv}=0\text{ for each }v,\qquad(b')\text{ holds}$$

and

$$U \in (l(p), l_\infty) \Leftrightarrow (b') \text{ holds.}$$

Lemma 2.2. (Malkowsky and Rakocevic, 2007) Let X be an FK space with AK property, T be triangle, S be its inverse and Y be an arbitrary subset of ω. Then, we have $U \in (X_T, Y)$ if and only if $\widetilde{U} \in (X, Y)$ and $V^{(n)} \in (X, c)$ for all n, where

$$\tilde{u}_{nv} = \sum_{j=v}^{\infty} u_{nj} s_{jv}, n, v = 0,1, \ldots$$

$$v_{mv}^{(n)} = \begin{cases} \sum_{j=v}^{m} u_{nj} s_{jv}, 0 \leq v \leq m \\ 0, \quad v > m. \end{cases}$$

Lemma 2.3. (Malkowsky and Rakocevic, 2000) Let T be a triangle. Then, for $X, Y \subset \omega, U \in (X, Y_T)$ if and only if $B = TU \in (X, Y)$.

Lemma 2.4. (Wilansky, 1984) Matrix transformations between FK-spaces are continuous.

Theorem 2.5 (Gökçe and Sarıgöl, 2018) Let $\lambda + \mu$, $\mu \neq -1, -2, \ldots$ and (p_v) be a bounded sequence of non-negative numbers. The space $|C_{\lambda,\mu}|(p)$ is a linear space with the coordinate-wise addition and scalar multiplication, and also the space is an FK-spaces with respect to the paranorm

$$g(x) = \left(\sum_{v=0}^{\infty} \left| T_v^{\lambda,\mu}(p)(x) \right|^{p_v} \right)^{1\backslash M}$$

where $M = \max\{1; \sup_v p_v\}$.

On the other hand, $|C_{\lambda,\mu}|(p)$ is linearly isomorphic to the Maddox's space $l(p)$.

In the continuation of the study, for simplicity, we take

$$\Omega_{nv}^{\lambda,\mu} = \frac{A_v^{\lambda+\mu} A_{n-v}^{-\lambda-1}}{n A_n^{\mu}}.$$

3. Findings and Discussion

Theorem 3.1. Let $\lambda + \mu$, $\mu \neq -1, -2, \ldots$, $U = (u_{nv})$ be an infinite matrix of complex numbers and $p = (p_v)$ be any bounded sequence of positive real numbers with $p_v \leq 1$ for all v. Then

(a) $U \in \left(|C_{\lambda,\mu}|(p), c \right)$ if and only

$$\sum_{k=v}^{\infty} v^{1/p_v}\Omega_{kv}^{\lambda,\mu}u_{nk} \text{ converges for all } v, \tag{1}$$

$$\sup_{m,v}\left|v^{1/p_v}\sum_{k=v}^{m} u_{nk}\Omega_{kv}^{\lambda,\mu}\right|^{p_v} < \infty \tag{2}$$

$$\lim_{n\to\infty} u_{n0} \text{ and } \lim_{n\to\infty} v^{1/p_v}\sum_{j=v}^{\infty} u_{nj}\Omega_{jv}^{\lambda,\mu}, \text{for each } v \geq 1, \text{exists} \tag{3}$$

$$\sup_{n,v}\left\{|u_{n0}|^{p_0} + \left|v^{1/p_v}\sum_{j=v}^{\infty} u_{nj}\Omega_{jv}^{\lambda,\mu}\right|^{p_v}\right\} < \infty. \tag{4}$$

(b) $U \in \left(\left|C_{\lambda,\mu}\right|(p), c_0\right)$ if and only if the conditions (1), (2), (4) hold and

$$\lim_{n\to\infty} u_{n0} = 0 \text{ and } \lim_{n\to\infty} v^{1/p_v}\sum_{j=v}^{\infty} u_{nj}\Omega_{jv}^{\lambda,\mu} = 0 \quad \text{for all } v \geq 1. \tag{5}$$

(c) $U \in \left(\left|C_{\lambda,\mu}\right|(p), l_\infty\right)$ if and only if the conditions (1), (2), (4) hold.

Proof Assume that $p_v \leq 1$ for all v. Since $\left|C_{\lambda,\mu}\right|(p) = \left(l(p)\right)_{T^{\lambda,\mu}(p)}$, it follows from Lemma 2.2 that $U \in \left(\left|C_{\lambda,\mu}\right|(p), c\right)$ if and only if $\widetilde{U} \in (l(p), c)$ and $V^{(n)} \in (l(p), c)$ where

$$\tilde{u}_{nv} = \begin{cases} u_{n0} & v = 0, n \geq 0 \\ v^{1/p_v}\sum_{j=v}^{\infty} u_{nj}\Omega_{jv}^{\lambda,\mu}, & v \geq 1, n \geq 0 \end{cases}$$

and

$$v_{mv}^{(n)} = \begin{cases} u_{n0}, & v = 0 \\ v^{1/p_v}\sum_{j=v}^{m} u_{nj}\Omega_{jv}^{\lambda,\mu}, & 1 \leq v \leq m \\ 0, & v > m. \end{cases}$$

It is obvious that by Lemma 2.1, $\widetilde{U} \in (l(p), c)$ if and only if the conditions (3), (4) hold. Also, $V^{(n)} \in (l(p), c)$ if and only if the conditions (1) and (2) hold which concludes the first part of the proof.

The remaining part of the proof can be proved in similar way. So, it has been left to reader.

Theorem 3.2. Let $\lambda + \mu, \mu \neq -1, -2, \ldots.,$ $U = (u_{nv})$ be an infinite matrix of complex numbers and $p = (p_v)$ be any bounded sequence of positive real numbers with $p_v > 1$ for all v. Then

(a) $U \in \left(\left|C_{\lambda,\mu}\right|(p), c\right)$ if and only if there exists an integer $M > 1$ such that

$$\sup_m \sum_{v=1}^{m} \left| M^{-1} v^{1/p_v} \sum_{k=v}^{m} u_{nk} \Omega_{kv}^{\lambda,\mu} \right|^{p_v^*} < \infty \tag{6}$$

$$\sup_n \left\{ |M^{-1} u_{n0}|^{p_0^*} + \sum_{v=1}^{\infty} \left| M^{-1} v^{1/p_v} \sum_{j=v}^{\infty} u_{nj} \Omega_{jv}^{\lambda,\mu} \right|^{p_v^*} \right\} < \infty \tag{7}$$

and the conditions (1), (3) hold.

(b) $U \in \left(\left|C_{\lambda,\mu}\right|(p), c_0\right)$ if and only if the conditions (1), (5), (6) and (7) hold.

(c) $U \in \left(\left|C_{\lambda,\mu}\right|(p), l_\infty\right)$ if and only if the conditions (1), (6), (7) hold.

Proof Let $p_v > 1$ for all v. Since $\left|C_{\lambda,\mu}\right|(p) = \left(l(p)\right)_{T^{\lambda,\mu}(p)}$, it follows from Lemma 2.2 that $U \in \left(\left|C_{\lambda,\mu}\right|(p), c_0\right)$ if and only if $\widetilde{U} \in (l(p), c_0)$ and $V^{(n)} \in (l(p), c)$ where the matrices $\widetilde{U}$ and $V^{(n)}$ are defined as in the above theorem. It follows from Lemma 2.1, $\widetilde{U} \in (l(p), c_0)$ if and only if the conditions (5), (7) hold. Also, $V^{(n)} \in (l(p), c)$ if and only if the conditions (1) and (6) hold. So, it completes the proof of (b).

The other part of the theorem can be proved in similar way.

Theorem 3.3. Let $U = (u_{nv})$ be an infnite matrix of complex numbers, $\theta = (\theta_n)$ be any sequence of positive real numbers, $p = (p_v)$ be any bounded sequence of positive numbers and $\Gamma = \{c, c_0, l_\infty\}$. If $U \in \left(\left|C_{\lambda,\mu}\right|(p), \Gamma\right)$, then U defines a bounded linear operator.

Proof Since the spaces c, c_0, l_∞ are BK- spaces, normed FK-spaces, using Lemma 2.4 and Theorem 2.5, the proof of Theorem can be immediately obtained.

4. Conclusions

In this section, we present some results obtained with special selections of our main theorems:

Take the matrix $L = (l_{nk})$ as

$$l_{nj} = \begin{cases} 1, 0 \le j \le n \\ 0, \quad j > n, \end{cases}$$

Since $b_s = (l_\infty)_L$ and $c_s = (c)_L$ the matrix classes $\left(\left|C_{\lambda,\mu}\right|(p), b_s\right)$ and $\left(\left|C_{\lambda,\mu}\right|(p), c_s\right)$ can be characterized as follows with Lemma 2.3 :

Corollary 4.1. Put $u(n,v) = \sum_{j=0}^{n} u_{jv}$ instead of u_{nv} in the Theorem 3.1 and Theorem 3.2. Then,

(a) if $p_v \leq 1$ for all v,

$$\left(\left|C_{\lambda,\mu}\right|(p), b_s\right) \Leftrightarrow (1),(2),(4) \text{ hold,}$$

$$\left(\left|C_{\lambda,\mu}\right|(p), c_s\right) \Leftrightarrow (1),(2),(3),(4) \text{ hold,}$$

(b) if $p_v > 1$ for all v,

$$\left(\left|C_{\lambda,\mu}\right|(p), b_s\right) \Leftrightarrow (1),(6),(7) \text{ hold,}$$

$$\left(\left|C_{\lambda,\mu}\right|(p), c_s\right) \Leftrightarrow (1),(3),(6),(7) \text{ hold.}$$

If we take $p_v = 1$ and $p_v = p$ for all v in Theorem 3.1 and Theorem 3.2, respectively, then the following results given in (Güleç, 2020) are easily obtained :

Corollary 4.2. Let $\lambda + \mu$, μ be non-negative integers, $U = (u_{nv})$ be an infinite matrix of complex numbers. Then

(a) U $\epsilon\left(\left|C_{\lambda,\mu}\right|, c\right)$ if and only if

$$\sum_{k=v}^{\infty} \frac{A_{k-v}^{-\lambda-1}}{kA_k^{\mu}} u_{nk} \text{ converges for all } v \geq 1, \tag{8}$$

$$\sup_{m,v\geq 1} \left| vA_v^{\lambda+\mu} \sum_{k=v}^{m} u_{nk} \frac{A_{k-v}^{-\lambda-1}}{kA_k^{\mu}} \right| < \infty \tag{9}$$

$$\sup_{n,v\geq 1} \left| vA_v^{\lambda+\mu} \sum_{j=v}^{\infty} u_{nj} \frac{A_{j-v}^{-\lambda-1}}{jA_j^{\mu}} \right| < \infty \tag{10}$$

$$\lim_{n\to\infty} \sum_{j=v}^{\infty} u_{nj} \frac{vA_v^{\lambda+\mu} A_{j-v}^{-\lambda-1}}{jA_j^{\mu}} \text{ exists for each } v \geq 1.$$

(b) U $\epsilon\left(\left|C_{\lambda,\mu}\right|, c_0\right)$ if and only if the conditions (8), (9), (10) hold and

$$\lim_{n\to\infty} \sum_{j=v}^{\infty} u_{nj} \frac{vA_v^{\lambda+\mu} A_{j-v}^{-\lambda-1}}{jA_j^{\mu}} = 0 \quad \text{for all } v \geq 1.$$

(c) U $\epsilon\left(\left|C_{\lambda,\mu}\right|, l_{\infty}\right)$ if and only if the conditions (8), (9), (10) hold.

Corollary 4.3. Let $\lambda + \mu$, $\mu \neq -1, -2, \ldots.$, $U = (u_{nv})$ be an infnite matrix of complex numbers and $p > 1$. Then,

(a) U $\epsilon\left(\left|C_{\lambda,\mu}\right|_p, c\right)$ if and only if

$$\sum_{k=v}^{\infty} \frac{A_{k-v}^{-\lambda-1}}{kA_k^{\mu}} u_{nk} \text{ converges for all n}, v \geq 1, \tag{11}$$

$$\sup_m \sum_{v=1}^{m} \left| v^{1/p} A_v^{\lambda+\mu} \sum_{k=v}^{m} u_{nk} \frac{A_{k-v}^{-\lambda-1}}{kA_k^{\mu}} \right|^{p^*} < \infty, \text{n} \geq 1, \tag{12}$$

$$\lim_{n\to\infty} v^{1/p} A_v^{\lambda+\mu} \sum_{j=v}^{\infty} u_{nj} \frac{A_{j-v}^{-\lambda-1}}{jA_j^{\mu}} \text{ exists for each n}, v \geq 1, \tag{13}$$

$$\sup_n \sum_{v=1}^{\infty} \left| v^{1/p} A_v^{\lambda+\mu} \frac{A_{j-v}^{-\lambda-1}}{jA_j^{\mu}} u_{nj} \right|^{p^*} < \infty \tag{14}$$

hold.

(b) U $\epsilon \left(|C_{\lambda,\mu}|_p, c_0 \right)$ if and only if the conditions (11), (12), (14) hold and

$$\lim_{n\to\infty} v^{1/p} \sum_{j=v}^{\infty} u_{nj} \frac{vA_v^{\lambda+\mu} A_{j-v}^{-\lambda-1}}{jA_j^{\mu}} = 0 \quad \text{for all n}, v \geq 1.$$

(c) U $\epsilon \left(|C_{\lambda,\mu}|_p, l_\infty \right)$ if and only if the conditions (11), (12), (14) hold.

Note that if we take $\lambda > -1, \mu = 0$ and $p_n = p$, for all n in the definition of the space $|C_{\lambda,\mu}|(p)$, it is reduced to the space

$$|C_\lambda|_p = \left\{ a = (a_v): \sum_{n=1}^{\infty} \left| \frac{1}{n^{1/p} A_n^{\lambda}} \sum_{v=0}^{n} A_{n-v}^{\lambda-1} v a_v \right|^p < \infty \right\}$$

studied in (Sarıgöl, 2016) and so the following new results are immediately obtained from our main theorems:

Corollary 4.4. Let λ be non-negative integers, $U = (u_{nv})$ be an infnite matrix of complex numbers. Then

(a) U $\epsilon(|C_\lambda|, c)$ if and only if

$$\sum_{k=v}^{\infty} \frac{A_{k-v}^{-\lambda-1}}{k} u_{nk} \text{ converges for all } v, \tag{15}$$

$$\sup_{m,v} \left| vA_v^{\lambda} \sum_{k=v}^{m} u_{nk} \frac{A_{k-v}^{-\lambda-1}}{k} \right| < \infty \tag{16}$$

$$\sup_{n,v} \left| vA_v^{\lambda+\mu} \sum_{j=v}^{\infty} u_{nj} \frac{A_{j-v}^{-\lambda-1}}{j} \right| < \infty \tag{17}$$

$$\lim_{n\to\infty}\sum_{j=v}^{\infty} u_{nj}\frac{A_{j-v}^{-\lambda-1}}{j} \text{ exists for each } v.$$

(b) U $\epsilon(|C_\lambda|, c_0)$ if and only if the conditions (15), (16), (17) hold and

$$\lim_{n\to\infty}\sum_{j=v}^{\infty} u_{nj}\frac{A_{j-v}^{-\lambda-1}}{j} = 0 \quad \text{for all } v.$$

(c) U $\epsilon(|C_\lambda|, l_\infty)$ if and only if the conditions (15), (16), (17) hold.

Corollary 4.5. Let $\lambda \neq -1, -2, \ldots,$ $U = (u_{nv})$ be an infnite matrix of complex numbers. Then,

(a) U $\epsilon\left(|C_\lambda|_p, c\right)$ if and only if

$$\sum_{k=v}^{\infty}\frac{A_{k-v}^{-\lambda-1}}{k}u_{nk} \text{ converges for all } v, \tag{18}$$

$$\sup_m \sum_{v=1}^{\infty}\left|v^{1/p}A_v^{\lambda}\sum_{k=v}^{m}u_{nk}\frac{A_{k-v}^{-\lambda-1}}{k}\right|^{p^*} < \infty, \tag{19}$$

$$\sup_n \sum_{v=1}^{\infty}\left|v^{1/p}u_{nj}A_v^{\lambda}\frac{A_{j-v}^{-\lambda-1}}{j}\right|^{p_v^*} < \infty \tag{20}$$

$$\lim_{n\to\infty}\sum_{j=v}^{\infty} u_{nj}\frac{A_{j-v}^{-\lambda-1}}{j} \text{ exists for each } v,$$

hold.

(b) U $\epsilon\left(|C_\lambda|_p, c_0\right)$ if and only if the conditions (18), (19), (20) hold and

$$\lim_{n\to\infty}\sum_{j=v}^{\infty} u_{nj}\frac{A_{j-v}^{-\lambda-1}}{j} = 0 \quad \text{for each } v\,.$$

(c) U $\epsilon\left(|C_\lambda|_p, l_\infty\right)$ if and only if the conditions (18), (19), (20) hold.

Authors' Contributions

All authors contributed equally to the study.

Statement of Conflicts of Interest

There is no conflict of interest between the authors.

Statement of Research and Publication Ethics

The author declares that this study complies with Research and Publication Ethics.

References

Gökçe, F. (2021). Compact and Matrix Operators on the Space $|\bar{N}_P^{\theta}|_k$. *Fundamental Journal of Mathematics and Applications* 4(2), 124-133.

Gökçe, F., Sarıgöl, M.A. (2020). Series spaces derived from absolute Fibonacci summability and matrix transformations. *Bollettino dell'Unione Matematica Italiana*, 13, 29-38.

Gökçe, F., Sarıgöl, M.A. (2019). Generalization of the absolute Ces`aro space and some matrix transformations. *Numerical Functional Analysis and Optimization,* 40, 1039-1052.

Gökçe, F., Sarıgöl, M.A. (2019a). Extension of Maddox's space l(μ) with Nörlund means. *Asian-European Journal of Mathematics* 12.06 (2019), 2040005.

Gökçe, F., Sarıgöl, M.A. (2018). A new series space $|\bar{N}_P^{\theta}|(\mu)$ and matrix transformations with applications. *Kuwait Journal of Science*, 45(4), 1-8.

Grosse – Erdmann, K.G. (1993). Matrix transformations between the sequence spaces of Maddox*, Journal of Mathematical Analysis and Applications*, 180(1), 223-238.

Güleç, G. C. H. (2020). Applications of measure of noncompactness in the series spaces of generalized absolute Cesaro means. *Karadeniz Fen Bilimleri Dergisi, 10*(1), 60-73.

Maddox, I.J. (1969). Some properties of paranormed sequence spaces, *Journal of the London Mathematical Society,* 1, 316-322.

Maddox, I.J. (1968). Paranormed sequence spaces generated by infinite matrices, *Mathematical Proceedings of the Cambridge Philosophical Society*, 64, 335-340.

Maddox, I.J. (1967). Spaces of strongly summable sequences, *The Quarterly Journal of Mathematics* 18, 345 -355

Malkowsky, E., Rakocevic, V. (2007). On matrix domains of triangles. *Applied Mathematics and Computation* ,189(2), 1146- 1163.

Malkowsky, E., Rakocevic, V. (2000). An introduction into the theory of sequence space and measures of noncompactness, *Zbornik Radova.(Beograd),* 9(17), 143-234.

Sarıgöl, M.A. (2016). Spaces of series summable by absolute Cesáro and matrix operators, *Communications in Mathematics and Applications*, **7**, 11-22.

Wilansky, A., (1984). *Summability through Functional Analysis*. Mathematical Studies 85, North–Holland, Amsterdam.

Zengin Alp, P., Ilkhan, M. (2019). On the difference sequence space $l_p(\hat{T}^q)$. *Math. Sci. Appl. E- Notes*, *7*(2), 161-173.