A Production Function Estimation to Input-Output Relations with Support Vector Regression

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A Production Function Estimation to Input-Output Relations with Support Vector Regression

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Abstract – Support vector machines are used for classification with the input vectors using a decision surface into a high dimensional feature space. In this paper, the mostly known Cobb-Douglas production function is examined the input-output relations in a textile industry. A support vector regression (SVR) model is established to estimate the 17 different cost and rate values as input data. An ABC analysis is applied to input factors that only 5 of 17 are more important. Then, SVR model is estimated to output with a sensitivity analysis. The results are shown with the estimation error exceeds the defined lower and upper limits in approximately 7 data out of 60 data. At the same time, it is observed that the number of support vectors has decreased to 3. Consequently, the effective solutions with reasonable solution times are presented in this study, thus, the machine learning, deep learning and metaheuristics methods with SVR might be applicable as further research for the different industrial problems together.

Keywords – Production Function, Input-Output, Support Vector Machines, Forecasting, Cobb-Douglas

I. Introduction

The goods and services are converted by using the production factors. In economic theory, the inputs-outputs, and the firm profit are determined by production function with the profit maximization [1-4]. Then, the production factors which are human resources, labor and enterprise capabilities, technology, energy, capital, knowledge etc. are classified as capital and natural resources. At this phase, the Cobb-Douglas production function is a widely used application tool that gives the relationship between the factors of production and the amount of production, the independent effects of the factors of production on the amount of production, how intensively labor and capital are used in production, and returns to scale [4-6]:

$$Q = A * L^{\alpha} * K^{\beta} \tag{1}$$

In here, Q is product quantity, L is labor and and K is capital. Then, the parameter A is related to technology. The parameter α indicates how many percent a 1% increase in labor will increase total output when the amount of capital is fixed. Thus, α is the labor elasticity of production. Similarly, the parameter β shows how much production will increase in response to a 1% increase in capital when the amount of labor is fixed. Thus, β is the capital elasticity of production [6].

Another definition of Cobb-Douglas production function (Eq.2), profit definition (Eq.3) and the conditions of the profit maximization (Eq.4) [2]:

$$X = A * L^{\alpha_1} * K^{\alpha_2} \tag{2}$$

$$\pi = pX - wL - rK \tag{3}$$

$$\begin{aligned} \frac{\partial \pi}{\partial L} &= 0 \\ \frac{\partial \pi}{\partial K} &= 0 \end{aligned} \tag{4}$$

Here, π is profits; X represents the output quantities, L is labor and K is capital respectively. $p, w,$ and r are the cost/price coefficients.

Many of studies are presented in the scientific literature with Cobb-Douglas production function and its applications since 1928 [1]. In this paper, the last two decades are reviewed from the literature [7]. In China, labor and capital analysis [8]; labor, capital and transportation purchases analysis in Spain [9]; labor, capital and energy consumption analysis in China [10], labor and information technology analysis with regression analysis in Australia [11], and labor, capital, technology transfer and innovations analysis in South Africa [12].

So, in this paper, the Support Vector Machines (SVMs) is used the production function analysis to input-output factors analysis in textile industry. SVMs is widely used methodology regression classification and prediction approach. The near literature of SVMs applications is given as follows. The exchange rate prediction [13], failure and reliability prediction [14], electricity consumption forecasting [15], and stock market prediction [16].

The organization of this paper, Section 2 presents materials and method with production data and SVM method. In Section 3, the experimental results are obtained and in Section 4, the discussions are given and the conclusions are given with production factors in Section 5.

II. Materials and Method

In this research, instead of the classical regression-based estimation of the Cobb-Douglas production function, regression methods with Support Vector Machines (SVM) were used for export estimation using the data of an exporting factory in Denizli. The reason why these methods are preferred is to enable the non-linear relations between inputs and outputs to be solved directly through a model without the need to determine any mathematical relation. Based on this obtained model, it is to develop a model proposal in order to predict the monthly export amounts of the firm in the future.

The exporter company supplies yarn, which is its main raw material, by importing and/or purchasing from the domestic market. After completing the sizing, weaving, dyeing and garment stages, the purchased yarn is packaged and made ready for export. The company manufactures in line with the order demands from foreign markets. The production process of the company is shown in Figure 1 below.

A. Support Vector Machines

Cortes and Vapnik firstly developed the Support Vector Machines (SVMs) at the AT&T Bell Laboratories in 1995 as a popular form. They introduce that SVMs is a new learning machine for two-group classification problems. The machine conceptually implements the following idea: input vectors are non-linearly mapped to a very high-dimension feature space [17].

Initial applications focused on binary classification of test instances and pattern recognition. Several introductory articles surfaced and constituted the foundation with the rapidly increasing attention for SVMs [18, 19].

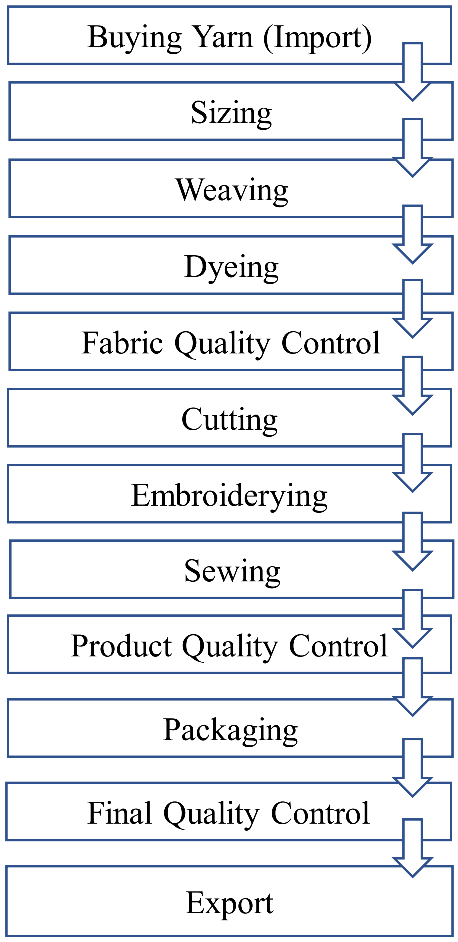

Fig. 1 Product work-flow scheme

In general, SVMs employ a model to construct a decision surface by mapping the input vectors into a high-dimensional (or infinite-dimensional) feature space. Next, a linear regression is executed in the high-dimensional feature space. This mapping operation is necessary because most of the time, the relation between a multidimensional input vector x and the output y are unknown and very likely to be non-linear. Support Vector Machine Regression (SVR) aims at finding a linear hyperplane, which fits the multidimensional input vectors to output values. The outcome is then used to predict future output values that are contained in a test set. Let us define a set of data points $P = (x_i, a_i), i = 1,\ldots n$ with x_i the input vector of data point i, ai the actual value and n the number of data points. For linear functions f, the hyperplane that is constructed by the SVR is determined as follows [17, 20, 21]:

$$f(x) = wx + b \qquad (5)$$

In here, the predicted value, *f(x),* depends on a slope w and an intercept b. Thus, Eq. (5) shows the similarities to a linear regression model. Then, the common kernel functions are given in Table 1 with γ, r and d are parameters that are kernel-specific [21]. Table 2 shows the Input data descriptions of analysis.

Table 1. Common kernel functions

Kernel name	*Formula*
Linear	$x_i^T x$
Polynomial	$(\gamma x_i^T x + r)^d$
Radial basis	$e^{-\gamma(\lVert x_i - x \rVert^2)}$
Sigmoidal	$tanh(\gamma x_i^T x + r)$

Table 2. Input data descriptions of analysis

#	*Input Factors*
1	Raw Material Cost (USD)
2	Outsource Weaving Cost (USD)
3	Outsource Confection Cost (USD)
4	Confection Cost - Plant (USD)
5	Weaving Cost - Plant (USD)
6	Dyehouse Cost (USD)
7	Labor Force Cost (USD)
8	Management Cost (USD)
9	Weaving Electricity Cost (USD)
10	Confection Electricity Cost (USD)
11	Confection Intermediate Input Cost (USD)
12	Weaving Intermediate Input Cost (USD)
13	Weaving Amortization (USD)
14	Confection Amortization (USD)
15	Nominal Exchange Rate
16	Real Exchange Rate
17	Producer Price Indices

The data set was first normalized to the range [0,1]. After this process, the Support Vector Regression (SVR) model was first developed. Polynomial function is used as kernel function. The first 30 months of the 60-month data set were used as the training data, and the second 30-month data set was used as the test data as 50% training and 50% test data. After the first run of the model, sensitivity analysis was performed. Based on this sensitivity analysis, the model was reconstructed.

III. Results

The computational study is applied with i7 CPU, 16 GB RAM and using MATLAB@ Academic Use scientific computing software. In the first analysis with SVR, the model was estimated with 12 support vectors. As a result of the sensitivity analysis on this model, it was determined that only 5 of the 17 input factors were the main factors determining the output. The graph of the input sensitivity analysis is shown in Fig. 2.

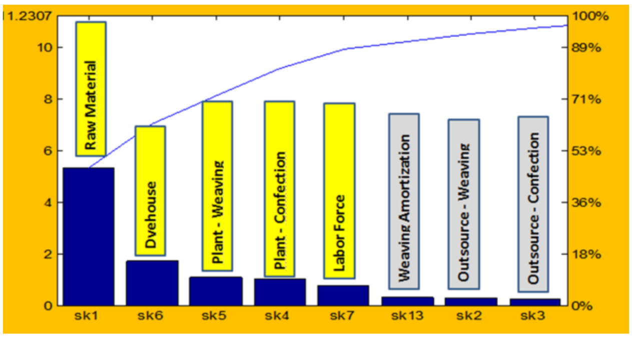

Fig. 2 ABC analysis of input factors

According to the Fig. 2, 89% of the output is explained by the input factors which are raw material cost, dyehouse cost, weaving cost, confection cost and labor cost. Then, based on this analysis, the SVR model was reconstructed and analyzed by considering only those that were significant in the sensitivity analysis. In this case, the number of support vectors has decreased to 3.

The graph of the real and predicted outputs of the new model obtained is shown in Fig. 3.

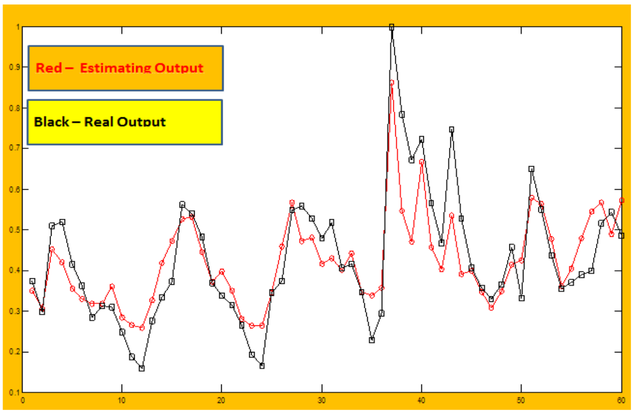

Fig. 3 Sensitivity analysis of new SVR model

Then, the estimation errors were calculated on Fig. 3 by using real and estimated values. These errors and support vectors are shown in Fig. 4. According to this, much of the estimation values are changed as ± 10%. These errors are calculated using the lower and upper limits for the SVR. When the error graph is examined, it is seen that the error exceeds the defined lower and upper limits in approximately 7 data out of 60 data. At the same time, it is observed in the graph that the number of support vectors has decreased to 3.

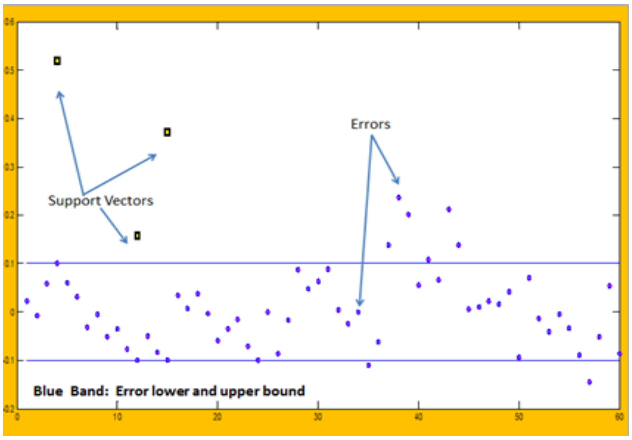

Fig. 4 Error analysis and support vectors of new SVR model

IV. DISCUSSION

According to computational results, the 5 main factors are determined on the 17 input factors via SVR solution at first. The significance of the results of the work is decreased the number of examining input factors as the important factors using the explanation. Finally, 3 support vectors are obtained for the effective solution with estimation error analysis.

V. CONCLUSION

This paper examines the Cobb-Douglas production function to input-output relations on a textile industry by support vector regression analysis. The effective results of SVR are obtained by decreasing the support vectors and the acceptable estimation errors. The calculations are gained with higher reliability and the reasonable solution times. The results can be shown different analyzes can be applied on different industrial production systems, comparisons can be made with SVM studies and different techniques such as classical regression analyzes, machine learning, deep learning and also metaheuristics as the future works.

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