On The Geometry of the Cotangent Bundle of a Hamilton Manifold with Pseudo Riemann Metric C^n

Ismet AYHAN

Department of Secondary School Science and Mathematics Education, Education Faculty, Pamukkale University., , 20070, Denizli, Turkey. iayhan@pau.edu.tr

Abstract

In this paper, it is obtained the geometrical interpretation of the mechanic concepts, well known in classic mechanic. It is found geodesics in the cotangent bundle of a Hamilton manifold with pseudo-Riemann metric c^g by using the Hamilton formalism. In two dimensional De Sitter space, time-like, space-like, light-like geodesic equations are obtained by using Hamilton equations. It is calculated the geodesic equations in the cotangent bundle of two dimensional De Sitter space with pseudo Riemann metric c^g

Keywords:*The Canonic Hamilton Equations, Geodesics in the Cotangent bundle of De Sitter Space*

1. Introduction

The geometry of the cotangent bundle is one of the most important subjects attracted the attention of mathematicians as well as physicists and mechanic scientists.

A Hamilton mechanical system is given by a manifold and function on its cotangent bundle, i.e. the Hamiltonian. In addition gravitational field is also given by a (pseudo) Riemann metric. The path of moving a point vise particle under the effect of the gravitational field is describe a geodesic, which is the shortest one among the curves passing from one point to another in (pseudo-)Riemann manifold.

In calculus of variations, the canonic Hamilton equations which are equivalent the Euler-Lagrange equations are the differential equations system whose solutions are the one parameters curves family. But there is only special solution that provides the initial values, which is the shortest one among curves passing through by different two points, i.e. geodesic.

The aim of this paper, firstly, it is obtain the geometrical interpretation of the mechanic concepts as the movement of the particles by the effect of the gravitational field, the law of conservation of momentum or energy, well known in classic mechanic.

Furthermore, having obtained geodesics in the cotangent bundle $(T^*M, {}^Cg)$ by using the Hamilton formalism, it must examine relation between geodesics of manifold (M, g) and the integral curves of Hamilton vector field in its cotangent bundle T^*M.

Finally, having obtained geodesics in the two dimensional De Sitter Space by using the Hamilton equations, it must calculate geodesics in the cotangent bundle of two dimensional De Sitter Space.

2. Materials and methods

The creation of this study, firstly, it was benefitted form references of [2], [3] and [4]. In these references, it was examined the movement of particles in the gravitational field, total energy function in Hamilton mechanic system, kinetic energy

of a moving particle, the law of conservation of momentum and the law of conservation of energy.

Later, it was considered the geodesic equations in the cotangent bundle with pseudo Riemann metric ${}^Cg$ to be Riemann extension of Levi Civita connection in Yano and Ishihara's book [5]

From [1], it was examined that how obtained space like, time like and null geodesic of two dimensional De Sitter Space by using the Euler Lagrange formalism.

3. Results and discussion

This study is contained four sections.

In the first section, it was given the geometrical interpretation of the mechanic concepts as the movement of the particles by the effect of the gravitational field, the law of conservation of momentum or energy, well known in classic mechanic.

In the second section, it was found geodesics in the cotangent bundle $(T^*M, {}^Cg)$ by using the Hamilton formalism and it was showed to be the projected curves to T^*M of integral curves of the Hamilton vector field $\tilde{X}_{\tilde{H}}$ depends on the Hamiltonian $\tilde{H}$ on T^*T^*M were geodesics of T^*M with the pseudo Riemann metric ${}^Cg$.

In the third section, it was obtained time-like, space-like, light-like geodesics equations with respect to Cartesian coordinate system and generalized coordinates of the two dimensional De Sitter Space S_1^2 by using the Hamilton equations.

Finally, It was found out geodesics in the cotangent bundle of two dimensional De Sitter Space $(T^*S_1^2, {}^Cg)$ by using general formula of geodesics contained Christoffel symbols and it was showed the geodesic equations of $(T^*S_1^2, {}^Cg)$ were also provided differential equations of geodesics to be projected integral curves of Hamilton vector field in the second section.

References

1. Free P., 2003, http://personalpages.to.infn.it/~fre/PPT/virgolect.ppt.
2. Polnarev A., 2011, Motion of a test particle in a Gravitational Field and Hamilton Jacobi Equations, Relativity and Gravitation, Lectures Notes 5.
3. Waner S., Levine G. C., 2005, Introduction to Differential Geometry and General Relativity, Lectures Notes.
4. Wonk, C. K. (25-Nov-2009), http://ckw.phys.ncku.edu.tw/public/pub/Notes/TheoreticalPhysics/pdf/3._ClassicalPhysics.pdf.
5. Yano K., 1973, Ishihara S., Tangent and Cotangent Bundles, Marcel Decker, Inc., New York.