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New semi-analytical solution of fractional Newell–Whitehead–Segel equation arising in nonlinear optics with non-singular and non-local kernel derivative

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Abstract

In this paper, a combined form of Laplace transform is applied with the Adomian Decomposition technique for the first time to obtain new semi-analytical solutions of the fractional Newell–Whitehead–Segel equation which is a model arising in nonlinear optics with Caputo–Fabrizio derivative which involves non-singular and non-local kernels in its definition. The obtained results by the suggested method are compared with exact solutions, as a result of remarkable concurrence between the acquired results and the exact proposed method and the exacted solutions. Plotted graphs and given tables illustrate the efficiency and accuracy of the proposed technique. All the calculations are made by the computer software called MAPLE and Mathematica.

Keywords Newell–Whitehead–Segel equation · Caputo–Fabrizio derivative · Adomian decomposition method · Laplace transformation · Semi-analytical solutions

1 Introduction

Fractional calculus plays a significant role in understanding the dynamics of complex real-world problems. It is non-local operators that provide a more accurate representation of various natural phenomena. The use of fractional derivatives is highly advantageous for researchers, engineers, mathematicians, and scientists working with real-life phenomena (Miller and Ross 1993; Kumar and Baleanu 2019; Ali et al. 2019; Jaradat et al. 2018; Alquran et al. 2021). It is used in various significant areas including electro-magnetic waves (Gómez-Aguilar et al. 2016), diffusion equations (Shaikh et al. 2019), viscoelasticity (Bagley and Torvik 1983) polarization (Berezovsky and Cheremnykh 2018; Eslami et al. 2021), electrode–electrolyte (L'vov et al. 2021), heat transfer (Yang et al. 2019), control

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theory (Kilbas et al. 2006), finance (Raberto et al. 2002), biomedical engineering, and biology (Prakash and Verma 2019).

Nonlinear mathematical models involving fractional terms are more compatible with the real-world data. Due to non- local characteristics can be used for modeling systems with memory. Fractional calculus comes through the critical trouble of integer calculus that the theoretical model results often fail to match up with the experimental results. These circumstances make the fractional derivative one step ahead of the integer order derivative. As a result, nonlinear fractional models are more useful than integer ones.

Therefore, the field of fractional calculus has recently undergone transformative developments, opening doors to profound insights and fostering innovative applications across an array of scientific domains (see references Liu et al. 2022,2023; Liu and Yang 2023).

A fractional derivative, denoted by D^{α}, is a mathematical operator that extends the concept of the ordinary derivative. The history of fractional derivatives can be traced back to 1695 when L'Hopital wrote a letter to Leibniz inquiring about the interpretation of the expression

$$D^{n}f(t) = \frac{d^{n}}{dt^{n}}f(t), \tag{1}$$

when *n* is non-integer. The question led to the development of the theory of fractional calculus. Although the concept of non-integer derivatives and integrals as a generalization of the traditional integer order differential and integral calculus was mentioned earlier, the first definition of the fractional derivative was introduced by Liouville and Riemann at the end of the nineteenth century (Girejko et al. 2011; Nchama et al. 2020).

Thenceforth, various notions of fractional order derivatives have been available for solving fractional differential equations, including well-known derivatives such as Riemann–Liouville and Caputo, also more recent derivatives such as Atangana-Baleanu and Caputo–Fabrizio, among others (Diethelm 2010; Atangana and Baleanu 2016; Caputo and Fabrizio 2015; Ilie et al. 2018a; 2018b; Alquran 2023; Alquran et al. 2020; Alquran and Jaradat 2019).

Because different researchers aim to preserve different features of the classical integer order derivative, the fractional derivative has been defined in various ways, each with its own advantages and disadvantages. Furthermore, these definitions do not generally coincide and often do not lead to the same result, even for smooth functions. Due to these incompatible definitions, it is essential to explicitly state which definition is being used.

In 2015, Caputo and Fabrizio proposed a new fractional derivative without a singular kernel. The previously used singular power-law kernel in Riemann–Liouville and Caputo was replaced with a non-singular exponential kernel, which is the main advantage of this proposed approach (Rosales García et al. 2018).

The motivation for proposing this new definition with a regular kernel came from the need to describe a class of non-local systems that describe material heterogeneities and fluctuations of different scales which cannot be adequately modeled using classical local theories or fractional models with a singular kernel (Nchama et al. 2020). Further, using it has resulted in the ease of theoretical analysis, numerical calculations, and real-world applications (Rosales García et al. 2018).

The Newell–Whitehead–Segel equation is one of the most important amplitude equations which describes the appearance of the stripe pattern in two-dimensional systems. This equation, a specialized form of the reaction–diffusion equation, belongs to a broader class of mathematical models elucidating the interplay between chemical reactions and diffusion processes (Elgazery 2020). Its applications span diverse systems such as Rayleigh–Benard convection, Faraday instability, nonlinear optics, chemical reactions, and biological systems (Ayata and Ozkan 2020; Prakash et al. 2019; Prakash and Kumar 2016; Elgazery 2020). Therefore, solving this equation is of paramount importance (Kanchana et al. 2020; Newell and Whitehead 1969; Latif et al. 2020).

The fractional model of the Newell–Whitehead–Segel equation is given by:

$$\frac{\partial^{\alpha}U}{\partial t^{\alpha}} = k\frac{\partial^2 U}{\partial x^2} + cU - dU^r, \quad 0 < \alpha \le 1 \tag{2}$$

where α is a parameter that determines the order of the time-fractional derivative. When $\alpha = 1$, the fractional form of the equation reduces to the classical form shown below:

$$\frac{\partial U}{\partial t} = k\frac{\partial^2 U}{\partial x^2} + cU - dU^r, \tag{3}$$

Here the constants $c, d, k \in R$, with r being a positive integer and $k > 0$, the first term on the left side, $\frac{\partial U}{\partial t}$, indicates the temporal variations of $U(x, t)$ at a fixed location, while the first term on the right side, $\frac{\partial^2 U}{\partial x^2}$, represents the spatial variations of $U(x, t)$ at a specific time. The remaining terms on the right side, $cU - dU^r$, account for the influence of the source term. In this equation, $U(x, t)$ is a function of the spatial variable variable x, where $t \ge 0, x \in R$. The function $U(x, t)$ can be interpreted as the (nonlinear) distribution of temperature in an infinitely thin and long rod or as the flow velocity of a fluid in an infinitely long pipe with a small diameter.

In conclusion, the Newell–Whitehead–Segel equation, within the realm of reaction–diffusion equations, plays a crucial role in understanding various line patterns observed in natural phenomena. Its versatile applications span across mathematical, chemical, and mechanical physics, as well as in bioengineering and fluid mechanics.

Over the past few decades, various methods have been employed to investigate the classical Newell–Whitehead–Segel equation, including differential transform (Aasaraai 2011), reduced differential transform (Keskin and Oturanç 2009), Adomian decomposition (Pue-on 2013), Homotopy perturbation (Singh and Kumar 2012), variational iteration (Prakash et al. 2019), finite difference scheme (Hilal et al. 2020), and many others. Furthermore, several analytical (Areshi et al. 2022) and approximate solutions have been proposed for solving this fractional equation.

In this study, we apply a method that consists of using Caputo–Fabrizio operator in combination with the Adomian decomposition method and the Laplace transform to solve the fractional form of the Newell–Whitehead–Segel equation. This new method overcomes the diffuculties of applying the methods on fractional terms. By the help of Laplace transformation, we don't need any discretization or normalization functions. With contrast to other methods this procedure makes evaluations easier and more convenient. Combination of the Adomian polynomials and Laplace transformation gives us a great chance to get the semi analytical solutions of nonlinear fractional partial differential equations. In general, this method can be used to find approximate analytical solutions for many linear and nonlinear fractional PDEs.

Definition 2.1 Let $a, b, \alpha \in R, b > a, a \in [-\infty, t), f \in H^1(a, b), \alpha \in [0, 1]$ (Al-Refai and Pal 2019). The Caputo–Fabrizio fractional derivative of order α respect to time variable is defined by

$$ {}^{CF}D_t^{\alpha} f(t) = \frac{N(\alpha)}{1-\alpha}\int_a^t f'(\tau) e^{-\frac{\alpha(t-\tau)}{1-\alpha}} d\tau, \tag{4} $$

where $N(\alpha)$ is a normalization function such that $N(0) = N(1) = 1$. This derivative will be zero if $f(t)$ is constant, as in Caputo–Fabrizio fractional derivative, the kernel does not have a singularity for $t = \tau$.

Definition 2.2 The Laplace transform of Caputo–Fabrizio derivative, is given as:

$$ \mathcal{L}\left[{}^{CF}D_t^{\alpha} f(t)\right] = \frac{1}{1-\alpha}\int_0^{\infty} e^{-st}\int_0^t f'(\tau) e^{-\frac{\alpha(t-\tau)}{1-\alpha}} d\tau dt. \tag{5} $$

Hence, from the convolution property of the Laplace transform (Caputo and Fabrizio 2015), we have:

$$ \begin{aligned} \mathcal{L}\left[{}^{CF}D_t^{\alpha} f(t)\right] &= \frac{1}{1-\alpha}\mathcal{L}\left[f'(\tau)\right]\mathcal{L}\left[e^{-\frac{\alpha t}{1-\alpha}}\right] \\ &= \frac{s\mathcal{L}[f(t)] - f(0)}{s+\alpha(1-s)}. \end{aligned} \tag{6} $$

2 Brief description of the considered method

Let us illustrate the main steps of this method on the following fractional PDE's in general operator form

$$ \begin{array}{ll} {}^{CF}D_t^{\alpha} u(x,t) + D_x^n u(x,t) + R(u(x,t)) + N(u(x,t)) = g(x,t) & t > 0, x > 0, \\ 0 < \alpha \le 1, & u(x,0) = h(x), \end{array} \tag{7} $$

where ${}^{CF}D_t^{\alpha}$ is the Caputo–Fabrizio fractional differential operator of order α in t, D_x^n is the highest order linear classical derivative operator in x, respectively, $R(u(x,t))$, $N(u(x,t))$ corresponds to linear, nonlinear operator in x, and and $g(x,t)$ is the non-homogenous term.

Applying the Laplace transform to Eq. (7) with respect to t, it becomes,

$$ \mathcal{L}\left[{}^{CF}D_t^{\alpha} u\right] = \mathcal{L}\left[g(x,t)\right] - \mathcal{L}\left[D_x^n u\right] - \mathcal{L}[R(u) + N(u)]. \tag{8} $$

From the Laplace transformation of Caputo–Fabrizio fractional derivatives, Eq. (8), yields the following equation:

$$ \frac{s\mathcal{L}[u] - u(0)}{s+\alpha(1-s)} = \mathcal{L}\left[g(x,t)\right] - \mathcal{L}\left[D_x^n u\right] - \mathcal{L}[R(u) + N(u)]. \tag{9} $$

If the above Eq. (9) is simplified, then

$$ \mathcal{L}[u] = \frac{1}{s}u(x,0) + \frac{s+\alpha(1-s)}{s}\left(\mathcal{L}\left[g(x,t)\right] - \mathcal{L}\left[D_x^n u\right] - \mathcal{L}[R(u) + N(u)]\right). \tag{10} $$

Taking inverse Laplace transform on both sides of Eq. (10), we get

$$u(x,t)=\mathcal{L}^{-1}\left[\frac{1}{s}u(x,0)\right]+\mathcal{L}^{-1}\left[\frac{s+\alpha(1-s)}{s}\left(\mathcal{L}\left[g(x,t)\right]-\mathcal{L}\left[D_x^n u\right]-\mathcal{L}[R(u)+N(u)]\right)\right]. \tag{11}$$

Now, according to the ADM, we represent solution as an infinite series given below.

$$u(x,t)=\sum_{m=0}^{\infty}u_m(x,t), \tag{12}$$

and the nonlinear part can be decomposed as

$$N(u(x,t))=\sum_{m=0}^{\infty}A_m. \tag{13}$$

Here,

$$A_m=\frac{1}{m!}\frac{d^m}{d\lambda^m}\left[N\left(\sum_{i=0}^{m}\lambda^i u_i\right)\right]\Bigg|_{\lambda=0},\ m=0,1,2,3,\ldots \tag{14}$$

By substituting Eqs. (12)–(14) in Eq. (11), we have:

$$\begin{aligned}\sum_{m=0}^{\infty}u_m=&\ \mathcal{L}^{-1}\left[\frac{1}{s}u(x,0)\right]+\mathcal{L}^{-1}\left[\frac{s+\alpha(1-s)}{s}\mathcal{L}\left[g(x,t)\right]\right]-\mathcal{L}^{-1}\left[\frac{s+\alpha(1-s)}{s}\mathcal{L}\left[D_x^n\sum_{m=0}^{\infty}u_m\right]\right]\\&-\mathcal{L}^{-1}\left[\frac{s+\alpha(1-s)}{s}\mathcal{L}\left[R\left(\sum_{m=0}^{\infty}u_m\right)+\sum_{m=0}^{\infty}A_m\right]\right].\end{aligned} \tag{15}$$

By comparing both sides of the Eq. (15), we get the following iterative algorithm

$$u_0=\mathcal{L}^{-1}\left[\frac{1}{s}u(x,0)\right] \tag{16}$$

$$\begin{aligned}u_{m+1}=&\ \mathcal{L}^{-1}\left[\frac{s+\alpha(1-s)}{s}\mathcal{L}\left[g(x,t)\right]\right]-\mathcal{L}^{-1}\left[\frac{s+\alpha(1-s)}{s}\mathcal{L}\left[D_x^n u_m\right]\right]\\&-\mathcal{L}^{-1}\left[\frac{s+\alpha(1-s)}{s}\mathcal{L}\left[R\left(u_m\right)+A_m\right]\right],\ m=0,1,2,3,\ldots.\end{aligned} \tag{17}$$

As it is seen from the brief description of the method, we don't need any discretization or normalization for fractional term. This procedure makes calculations less complicated and we don't make one more evaluation for reduction of fractional term.

Laplace decomposition method is used to get the solutions of nonlinear homogeneous and non-homogenous advection equations by Khan and Austin (Khan and Austin 2010). Jafari et al. (2011) employed Laplace decomposition method to achieve the solutions for fractional diffusion–wave equations. Also, Kumar et al. used (Kumar et al. 2014) modified version of Laplace decomposition method to get the exact solution of fractional Navier–Stokes equation. Numerous studies have been conducted to evaluate the utility and precision of this derivative (Caputo and Fabrizio 2015; Rosales García et al. 2018; Kanchana et al. 2020; Newell and Whitehead 1969; Latif et al. 2020; Ayata and Ozkan 2020; Prakash et al. 2019; Aasaraai 2011; Keskin and Oturanç 2009; Pue-on 2013; Singh and Kumar 2012; Hilal et al. 2020; Areshi et al. 2022; Al-Refai and Pal 2019; Ilie et al. 2018a, 2018b; Eslami et al. 2021; Ali et al. 2019; Jaradat et al. 2018; Alquran et al. 2021, 2020; Alquran 2023; Alquran and Jaradat 2019; Cabré and Cinti 2014; Rahimkhani and Ordokhani 2020; Shatnawi et al. 2021; Khudair 2013; Jassim 2015; Khan et al. 2019; Eltayeb and Mesloub 2020; Geng and Cui 2011; Nourazar et al. 2013; Losada and Nieto 2015; Kanth and Garg 2018; Prakash and Kumar 2016). In order to enhance accuracy and convergence in solving equations, the literature encompasses various modifications and hybrid methods, incorporating different transforms. Examples include the conformable Laplace decomposition method (Ayata and Ozkan 2020), double Laplace (Khan et al. 2019; Eltayeb and Mesloub 2020), a combination of reproducing kernel method and Adomian (Geng and Cui 2011), Fourier Transform Adomian decomposition (Nourazar et al. 2013), and a combination of Adomian and reproducing kernel method (Geng and Cui 2011). In this study, we apply the Laplace transform in conjunction with the Adomian method to address linear and nonlinear fractional equations, with the aim of achieving improved results.

3 New application of the considered method

The effectiveness of this algorithm will be demonstrated by using some examples of linear and nonlinear version of fractional Newell–Whitehead–Segel equation.

Example 3.1 Consider the homogeneous time-fractional Newell–Whitehead–Segel equation below (Prakash and Verma 2019),

$$D_t^{\alpha}u(x,t) = D_x^2u(x,t) - 2u(x,t), \quad 0 < \alpha \le 1 \tag{18}$$

with initial condition

$$u(x,0) = e^x. \tag{19}$$

Applying the Caputo–Fabrizio fractional operator of order $0 < \alpha \le 1$, and taking the Laplace transform with respect to t,

$$\mathcal{L}\left[{}^{CF}D_t^{\alpha}u(x,t)\right] = \mathcal{L}\left[D_x^2(x,t)\right] - \mathcal{L}[2u(x,t)]. \tag{20}$$

Base on the Def. (2.2) about the Laplace transformation of Caputo–Fabrizio fractional derivatives, it turns into,

$$\frac{s\mathcal{L}[u(x,t)] - u(x,0)}{s(1-\alpha)+\alpha} + 2\mathcal{L}[u(x,t)] = \mathcal{L}\left[D_x^2(x,t)\right]. \tag{21}$$

By simplifying the Eq. (21), it becomes,

$$\mathcal{L}[u(x,t)] = \frac{e^x}{s(3-2\alpha)+2\alpha} + \frac{s(1-\alpha)+\alpha}{s(3-2\alpha)+2\alpha}\mathcal{L}\left[D_x^2(x,t)\right]. \tag{22}$$

Applying inverse Laplace transform for Eq. (22), we get

$$u(x,t) = \frac{e^x e^{-\frac{2\alpha t}{3-2\alpha}}}{3-2\alpha} + \mathcal{L}^{-1}\left[\frac{s(1-\alpha)+\alpha}{s(3-2\alpha)+2\alpha}\mathcal{L}\left[D_x^2(x,t)\right]\right]. \tag{23}$$

Now, by applying the Adomian Decomposition method, as explained in Eq. (12), the Eq. (23) transforms into as follow.

$$\sum_{m=0}^{\infty} u_m = \frac{e^x e^{-\frac{2\alpha t}{3-2\alpha}}}{3-2\alpha} + \mathcal{L}^{-1}\left[\frac{s(1-\alpha)+\alpha}{s(3-2\alpha)+2\alpha}\mathcal{L}\left[D_x^2 \sum_{m=0}^{\infty} u_m\right]\right]. \tag{24}$$

By comparing both sides and taking the iterative algorithm, in Eq. (16) and Eq. (17), it becomes:

$$u_0 = \frac{e^{x-\frac{2\alpha t}{3-2\alpha}}}{3-2\alpha}, \tag{25}$$

$$u_1 = t\frac{e^{x-\frac{2\alpha t}{3-2\alpha}}}{(3-2\alpha)^2}, \tag{26}$$

$$u_2 = \frac{1}{2!}t^2\frac{e^{x-\frac{2\alpha t}{3-2\alpha}}}{(3-2\alpha)^3}, \tag{27}$$

$$u_3 = \frac{1}{3!}t^3\frac{e^{x-\frac{2\alpha t}{3-2\alpha}}}{(3-2\alpha)^4}. \tag{28}$$

Hence, the u(x, t) can be expressed as:

$$\begin{aligned} u(x,t) &= u_0 + u_1 + u_2 + u_3 + \dots \\ &= \frac{e^{x-\frac{2\alpha t}{3-2\alpha}}}{3-2\alpha} + t\frac{e^{x-\frac{2\alpha t}{3-2\alpha}}}{(3-2\alpha)^2} + \frac{1}{2!}t^2\frac{e^{x-\frac{2\alpha t}{3-2\alpha}}}{(3-2\alpha)^3} + \frac{1}{3!}t^3\frac{e^{x-\frac{2\alpha t}{3-2\alpha}}}{(3-2\alpha)^4} + \dots \\ &= \frac{e^{x-\frac{2\alpha t}{3-2\alpha}}}{3-2\alpha}\left[1 + \frac{t}{3-2\alpha} + \frac{t^2}{2!(3-2\alpha)^2} + \frac{t^3}{3!(3-2\alpha)^3} + \dots\right] \\ &= \frac{1}{3-2\alpha}e^{x-\frac{2\alpha t}{3-2\alpha}}e^{\frac{t}{3-2\alpha}} \\ &= \frac{e^{x-\frac{t(2\alpha-1)}{3-2\alpha}}}{3-2\alpha}. \end{aligned} \tag{29}$$

Example 3.2 Consider the following fractional Newell–Whitehead–Segel equation with quadric nonlinearity (Jassim 2015)

$$D_t^{\alpha}u(x,t) = 5D_x^2u(x,t) + 2u(x,t) + u^2(x,t), \quad 0 < \alpha \le 1. \tag{30}$$

with the initial condition

$$u(x,0) = \rho, \tag{31}$$

where ρ is arbitrary constant, and D_t^{α} is the Caputo–Fabrizio fractional operator of order $0 < \alpha \le 1$.

By applying the Laplace transform of Caputo–Fabrizio fractional Eq. (30),

$$\mathcal{L}\left[{}^{CF}D_t^{\alpha}u(x,t)\right] = \mathcal{L}\left[5D_x^2(x,t)\right] + \mathcal{L}[2u(x,t)] + \mathcal{L}\left[u^2(x,t)\right], \tag{32}$$

while considering the initial condition Eq. (31), we obtain

$$\frac{s\mathcal{L}[u(x,t)]-u(x,0)}{s(1-\alpha)+\alpha}=\mathcal{L}\left[5D_x^2(x,t)\right]+\mathcal{L}[2u(x,t)]+\mathcal{L}\left[u^2(x,t)\right]. \tag{33}$$

By simplifying Eq. (33), we get

$$\mathcal{L}[u(x,t)]=\frac{\rho}{s(2\alpha-1)-2\alpha}+\frac{5(s(1-\alpha)+\alpha)}{s(2\alpha-1)-2\alpha}\mathcal{L}\left[D_x^2u\right]+\frac{s(1-\alpha)+\alpha}{s(2\alpha-1)-2\alpha}\mathcal{L}\left[u^2(x,t)\right]. \tag{34}$$

Taking inverse Laplace transform for Eq. (34), we get

$$u(x,t)=\frac{\rho e^{\frac{2\alpha t}{2\alpha-1}}}{2\alpha-1}+\mathcal{L}^{-1}\left[\frac{5(s(1-\alpha)+\alpha)}{s(2\alpha-1)-2\alpha}\mathcal{L}\left[D_x^2u\right]\right]+\mathcal{L}^{-1}\left[\frac{s(1-\alpha)+\alpha}{s(2\alpha-1)-2\alpha}\mathcal{L}[u^2(x,t)]\right]. \tag{35}$$

Following the ADM method in Sect. (2.3), the following relation is obtained,

$$\sum_{m=0}^{\infty}u_m=\frac{\rho e^{\frac{2\alpha t}{2\alpha-1}}}{2\alpha-1}+\mathcal{L}^{-1}\left[\frac{5(s(1-\alpha)+\alpha)}{s(2\alpha-1)-2\alpha}\mathcal{L}\left[D_x^2\sum_{m=0}^{\infty}u_m\right]\right]+\mathcal{L}^{-1}\left[\frac{s(1-\alpha)+\alpha}{s(2\alpha-1)-2\alpha}\mathcal{L}\left[\sum_{m=0}^{\infty}A_m\right]\right], \tag{36}$$

where $A_m(x)$ is transformed form of the nonlinear terms, $u^2(x,t)$, and the first nonlinear terms are given as

$$\begin{aligned} A_0 &= u_0^2,\\ A_1 &= 2u_0u_1,\\ A_2 &= 2u_0u_2+u_1^2,\\ A_3 &= 2u_0u_3+2u_1u_2 \end{aligned} \tag{37}$$

and so on.

Now, applying the iterative algorithm in Eq. (16) and Eq. (17) for Eq. (36), it gives

$$u_0=\rho\frac{\rho e^{\frac{2\alpha t}{2\alpha-1}}}{2\alpha-1}, \tag{38}$$

$$u_1=-\left(\frac{1}{2}\right)\rho^2\frac{e^{\frac{2\alpha t}{2\alpha-1}}}{(2\alpha-1)^3}\left[(2\alpha-3)e^{\frac{2\alpha t}{2\alpha-1}}+1\right], \tag{39}$$

$$u_2=\left(\frac{1}{2}\right)^2\rho^3\frac{e^{\frac{2\alpha t}{2\alpha-1}}}{(2\alpha-1)^5}\left[(2\alpha-3)(4\alpha-5)e^{\frac{4\alpha t}{2\alpha-1}}+2((2\alpha-3))e^{\frac{2\alpha t}{2\alpha-1}}+(2\alpha-1)\right], \tag{40}$$

$$\begin{aligned} u_3=-\left(\frac{1}{2}\right)^3\rho^4\frac{e^{\frac{2\alpha t}{2\alpha-1}}}{(2\alpha-1)^7}&\left[\frac{(2\alpha-3)(6\alpha-7)(10\alpha-13)}{3}\,e^{\frac{6\alpha t}{2\alpha-1}}+3(2\alpha-3)(4\alpha-5)\,e^{\frac{4\alpha t}{2\alpha-1}}\right.\\ &\left.+\frac{(2\alpha-3)(10\alpha-13)+30(\alpha-1)}{3}\right] \end{aligned} \tag{41}$$

Therefore, the series solution is given as

$$\begin{aligned}
u(x,t) &= u_0 + u_1 + u_2 + u_3 + \dots \\
&= \rho \frac{e^{\frac{2\alpha t}{2\alpha-1}}}{2\alpha-1} - \left(\frac{1}{2}\right)\rho^2 \frac{e^{\frac{2\alpha t}{2\alpha-1}}}{(2\alpha-1)^3}\left[(2\alpha-3)e^{\frac{2\alpha t}{2\alpha-1}} + 1\right] \\
&+ \left(\frac{1}{2}\right)^2 \rho^3 \frac{e^{\frac{2\alpha t}{2\alpha-1}}}{(2\alpha-1)^5}\left[(2\alpha-3)(4\alpha-5)e^{\frac{4\alpha t}{2\alpha-1}} + 2(2\alpha-3)e^{\frac{2\alpha t}{2\alpha-1}} + 2\alpha - 1\right] \\
&- \left(\frac{1}{2}\right)^3 \rho^4 \frac{e^{\frac{2\alpha t}{2\alpha-1}}}{(2\alpha-1)^7}\left[\frac{(2\alpha-3)(6\alpha-7)(10\alpha-13)}{3} e^{\frac{6\alpha t}{2\alpha-1}} + 3(2\alpha-3)(4\alpha-5)e^{\frac{4\alpha t}{2\alpha-1}}\right. \\
&\left. + \frac{(2\alpha-3)(10\alpha-13) + 30(\alpha-1)}{3}\right] + \dots
\end{aligned} \tag{42}$$

4 Results and discussion

In Table 1, the 4-order approximate solutions of the fractional Newell–Whitehead–Segel equation, the Ex. (3.1), are numerically compared with the exact solution in (Prakash et al. 2019) for $\alpha = 1$ at $x = 1$.

Also, in Ex. (3.2) By letting $\alpha = 1$ and $\rho = 1$, the fifth and sixth the solutions are:

$$u_4 = \frac{e^{2t}}{16} - \frac{e^{4t}}{4} + \frac{3e^{6t}}{8} - \frac{e^{8t}}{4} + \frac{e^{10t}}{16} \tag{43}$$

$$u_5 = -\frac{e^{2t}}{32} + \frac{5e^{4t}}{32} - \frac{5e^{6t}}{16} + \frac{5e^{8t}}{16} - \frac{5e^{10t}}{32} + \frac{e^{12t}}{32} \tag{44}$$

Then, in Table 2, the solution of our implemented method has been compared with the exact solution in (Singh and Kumar 2012) by considering $\alpha = 1$ at $\rho = 1$.

The given tables show that the error values are in acceptable limits.

The graphs represent 3-step semi-analytical solutions of $u(x,t)$ for various values of α.

Table 1 The numerical values of the Exact solution and 4-term semi-analytical solutions when $x = 1$ and $\alpha = 1$

t	By implemented method	Exact solution	$\left\lvert u_{exact} - u_{expressedmethod} \right\rvert$
$\alpha = 1$			
0.01	2.691234471	2.691234472	1.1124×10^{-9}
0.02	2.664456224	2.664456241	1.7481×10^{-8}
0.03	2.637944372	2.637944459	8.6920×10^{-8}
0.04	2.611696203	2.611696473	2.6981×10^{-7}
0.05	2.585709012	2.585709659	6.4698×10^{-7}

Table 2 The numerical values of the Exact solution and 4-term semi-analytical solutions when ρ = 1 and α = 1

t	By implemented method	Exact solution	$\left\|u_{exact} - u_{expressedmethod}\right\|$
$\alpha = 1$			
0.01	1.030611204	1.030611203	2.9250×10^{-10}
0.02	1.062491317	1.062491320	3.8813×10^{-9}
0.03	1.095714104	1.095714135	3.16191×10^{-8}
0.04	1.130359080	1.130359220	1.4010×10^{-7}
0.05	1.166512045	1.166512514	4.6910×10^{-7}

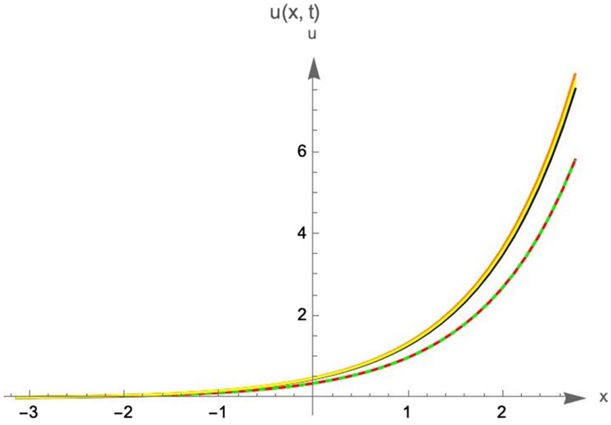


Fig. 1. 2D-Plots graphs of the 3-term semi analytical solutions and exact solution for Eqs. (18) when $t = 1$

Figure 1, which pertains to Example (3.1), compares our semi analytical solution for α = 1 with the exact solution for Eqs. (18) at t = 1. Additionally, Fig. 2 presents graphs comparing solutions for Eqs. (18) at x = 1.

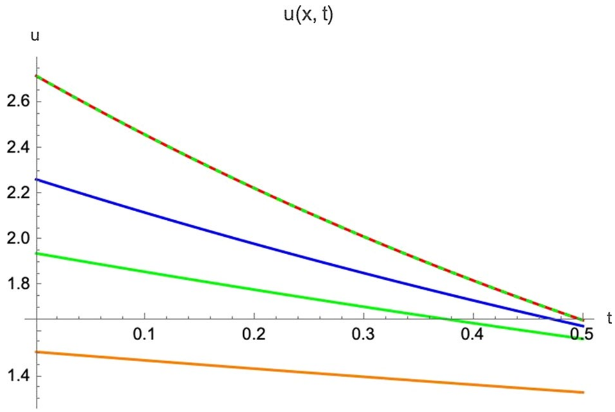
Figure 3, related to Ex. (3.2), compares our semi analytical solution and exact solution for Eqs. (30) when α = 1 and ρ = 1.

These tables and figures assess the accuracy of the derived solution for $u(x, t)$ through graphical validation and illustrate how the physical behavior of the solution is influenced when orders of derivatives change. Additionally, they demonstrate that the exact solution obtained from (Prakash et al. 2019; Singh and Kumar 2012) and the semi-analytical results gained through this implemented method overlap with each other when $\alpha = 1$. This implies that as α converges to the first derivative, the obtained approximate solution seamlessly aligns with the exact solution.

Figure 4, related to Ex. (3.1), illustrates how the physical behavior of the solution is affected by changes in derivative orders. The graphs represent 3-step semi-analytical solutions of u(x, t) for various values of α. It indicates that obtained exact solution from (Prakash et al. 2019) and gained semi-analytical results by this implemented method overlap with each other when α = 1.

Figure 5, related to Ex. (3.1), compares our semi analytical solution for α = 1 and exact solution for Eqs. (18) when x = 1 and α = 1

Fig. 2. 2D-Plots graphs of the 3-term semi analytical solutions and exact solution for Eqs. (18) for $x = 1$



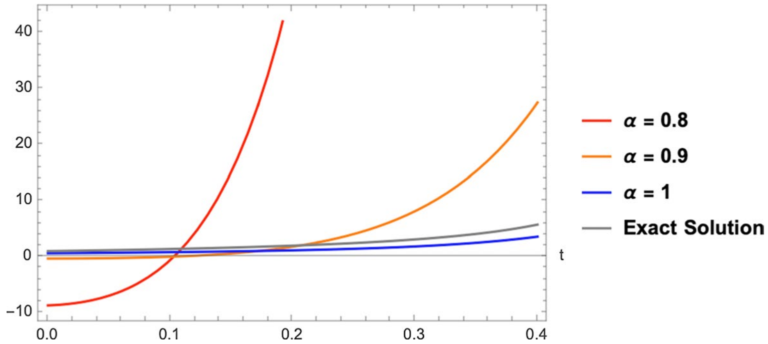


Fig. 3. 2D-Plots graphs of the 3-term semi analytical solutions and exact solution for Eqs. (30) for $\rho = 1$

Figure 6, related to Ex. (3.2), illustrates how the physical behavior of the solution is affected by changes in derivative orders. The graphs represent semi-analytical solutions of $u(x, t)$ for α and $\rho = 12$. It indicates that obtained exact solution from in (Singh and Kumar 2012) and our semi-analytical results by this implemented method overlap with each other when $\alpha = 1$.

Figure 7, related to Ex. (3.2), compares our semi analytical solution and exact solution for Eqs. (30) when $\alpha = 1$ and $\rho = 1, 12$.

5 Conclusion

In this work, Laplace transform with the Adomian Decomposition technique was successfully applied for solving fractional Newell–Whitehead–Segel equation. Indeed, the results show that using Caputo–Fabrizio derivative combined with Laplace transform and Adomian is a powerful and easy mathematical tool for finding semi-analytical solutions which are accurate. The obtained solutions were compared with exact solutions and also with other existing solutions in the references. The performance of the approach showed the obtained solutions for the first four terms was very precise and converged rapidly to the exact solutions.

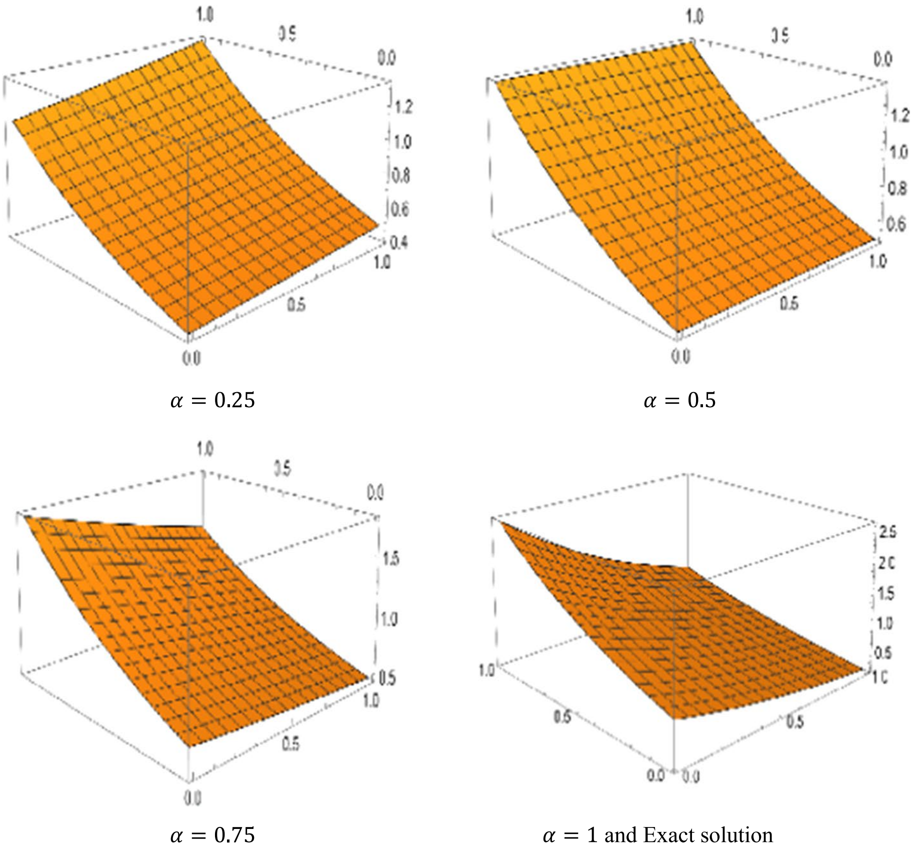
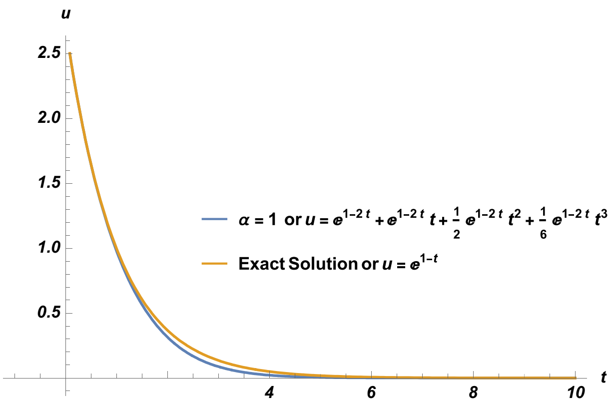


Fig. 4. 3D-Plots graphs of the 3-term semi analytical solutions and exact solution for Eqs. (18)

Fig. 5. 2D-Plots graphs of comparison solutions of Eqs. (18) for $x = 1$ and $\alpha = 1$



It assures us that the implemented technique is a reliable and promising method for solving other fractional PDEs that have been favored domains for scientists working on mathematical models arising in physics, biology, and other real-life phenomena.

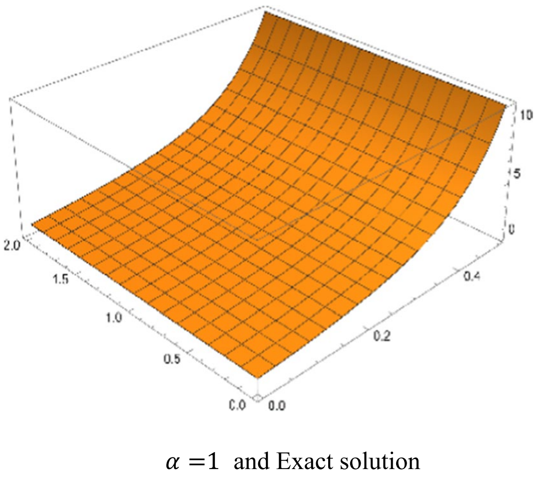


Fig. 6. 3D-Plots graphs of analytical solutions and exact solution for Eqs. (30)

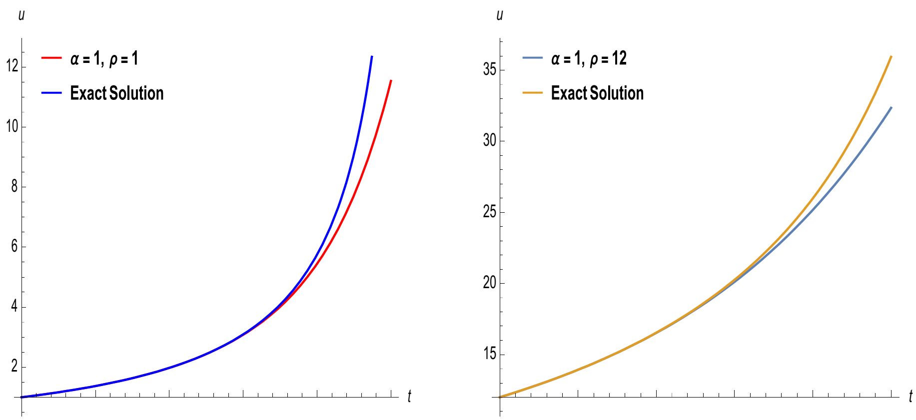


Fig. 7. 2D-Plots graphs of comparison solutions of Eqs. (30) when $\alpha = 1$

As a future research direction, the utilized method can be further developed to handle fractional order problems in various linear and nonlinear phenomena. We are keen on applying this method to other fractional equations, including equations of diffusion equations (Cabré and Cinti 2014), viscoelastic (Rahimkhani and Ordokhani 2020), oscillator (Shatnawi et al. 2021), Euler equations (Khudair 2013), and more.

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Data availability All the data used in this manuscript are explicitly explained within the manuscript.

Declarations

Competing interests The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Ethical Approval Not applicable.

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