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OPTIMUM DESIGN OF STORM SEWER SYSTEMS BY USING HARMONY SEARCH OPTIMIZATION APPROACH

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ABSTRACT

The main objective of this study is to propose an optimization approach for solving the optimum storm sewer system design problems. In the proposed approach, the heuristic harmony search optimization algorithm is used to minimize the construction cost of the system by considering the slopes of the pipes as the decision variables. For the identified slopes, diameters of the pipes are selected from the discrete set of pipe diameters which is available in market. During the search process, all the physical and managerial constraints are considered by means of the penalty function approach. The applicability of the proposed simulation-optimization approach is evaluated on a benchmark example given in literature. Identified results indicated that the proposed simulation-optimization approach results with the better results than those obtained by using different deterministic and heuristic approaches in literature and can be effectively used to solve the optimum storm sewer system design problems.

Keywords: Storm Sewer System Design, Simulation, Optimization, Harmony Search

1 INTRODUCTION

Design of storm sewer systems is an important problem in water resources engineering. Building of these systems in big cities is an expensive task such that any reduction of the pipe and/or excavation costs usually results with a significant saving. Therefore, finding a feasible solution providing cost-effective results for the storm sewer systems becomes an important problem. In literature, different approaches were proposed to find the cost effective solutions for the storm sewer systems. Among them, one of the most widely used approaches is the optimization approach.

The current literature includes many studies that investigate the application of different simulation-optimization approaches to the solution of storm sewer system design problems. Although these approaches includes different simulation models, their main differences are due to the considered optimization approaches. Note that both deterministic and heuristic optimization methods were employed to the solution of the storm sewer system design problems. Deterministic methods include linear (Swamee and Sharma, 2012), nonlinear (Price, 1978), and dynamic programing (Mays and Wenzel, 1976) approaches. Note that these approaches are effective to solve the serially connected simple systems. However, they may not provide global optimum solutions for the problems with both continuous and discrete decision variables and high number of hydraulic constraints (Yeh and Chu, 2011). Therefore, heuristic optimization methods are used to solve such kind of problems.

There exists a large body of literature related with the optimum storm sewer system design problems by using the heuristic optimization approaches. The philosophy of these approaches is the mathematical application of some natural processes to the solution of the optimization problems. These approaches include the natural selection and evolution process in genetic algorithm (GA) (Afshar, 2012) and differential evolution (DE) (Liu et al. 2014), swarm intelligence process in particle swarm optimization algorithm (PSO) (Izquierdo, 2008), shortest path finding process in ant colony optimization (ACO) (Moeini and Afshar, 2012), etc. Although different heuristic approaches were used in literature, there is no any application of harmony search (HS) optimization method to solve the optimum storm sewer system design problems.

The main objective of this study is to propose a HS based optimization approach for solving the optimum storm sewer system design problems. In the proposed approach, total cost of the system including the pipe and excavation costs is considered as the objective function and this function is minimized by considering the pipe slopes as decision variables. After finding the optimum slopes for the pipes, the corresponding pipe diameters are selected from a discrete set of pipe diameters which is available in market. During the search process, all the managerial and physical constraints are included to the model by means of the penalty function approach. The performance of the proposed approach is evaluated by solving a benchmark example in literature. Identified results indicated that the proposed model can find better results than those obtained by using different deterministic and heuristic approaches in literature.

2 OPTIMUM SEWER SYSTEM DESIGN

The problem of optimum storm sewer system design can be mathematically solved based on the following optimization formulation which is adapted from Afshar (2012):

$$\Phi = \min\left\{\sum_{k=1}^{N} \mathcal{F}(d_k, L_k, \bar{Z}_k)\right\} \quad [1]$$

subject to

$$g_{k,1}: \ q_k \geq Q_k^* \quad [2]$$

$$g_{k,2}: \ V_k \leq V_{\max} \quad [3]$$

$$g_{k,3}: \ V_k \geq V_{\min} \quad [4]$$

$$g_{k,4}: \ \frac{y_k}{d_k} \leq \alpha \quad [5]$$

$$g_{k,5}: \ S_k \geq S_{\min} \quad [6]$$

$$g_{k,6}: \ E_k^u \leq E_{\max} \quad [7]$$

$$g_{k,7}: \ E_k^u \geq E_{\min} \quad [8]$$

$$g_{k,8}: \ E_k^d \leq E_{\max} \quad [9]$$

$$g_{k,9}: \ E_k^d \geq E_{\min} \quad [10]$$

where Φ is the objective function value, N is the number of pipes in the system, d_k is the diameter of the pipe k, L_k is the length of the pipe k, $\bar{Z}_k$ is the average excavation depth for the pipe k, $\mathcal{F}(\bullet)$ is the system cost including the pipe diameters, lengths, and excavation costs, q_k is the discharge in pipe k to generate the normal depth, Q_k^* is the peak design discharge in pipe k, V_k is the flow velocity in pipe k, $V_{\min}$ and $V_{\max}$ are the minimum and maximum flow velocities, y_k is the water depth in pipe k, α is the maximum allowable relative water depth over a pipe, S_k is the slope of pipe k, $S_{\min}$ is the minimum slope of the pipes, E_k^u and E_k^d are the upstream and downstream pipe covers for pipe k, and $E_{\min}$ and $E_{\max}$ are the minimum and maximum pipe covers, respectively.

As can be seen from the optimization formulation given above the flow value in the pipes should be greater than the design discharge ($g_{k,1}$). Flow velocity in the pipes should be between provided lower and upper bounds ($g_{k,2}$ and $g_{k,3}$). Also, there should be no pressured flow conditions in the pipes ($g_{k,4}$). All of these constraints should be satisfied during minimization of Eq. [1]. Note that the optimization formulation given above is the simplified version of the storm sewer design problem. In a more general version, the peak design discharge value of Q_k^* should be replaced with a storm hydrograph providing the variation of the discharge versus time. Furthermore, additional system components including drops, pumps, tanks, etc should also be included to the model.

3 HARMONY SEARCH (HS) OPTIMIZATION ALGORITHM

HS is a heuristic optimization algorithm which mimics the musicians' behavior during a musical improvisation process. In a musical performance, musicians try to find a musically pleasing or fantastic harmony by making several improvisations. During their trials, they play some notes in their memories. This process is analogous to optimization problems since the optimization process aims to find an optimum solution by making several iterations. During each iteration, the corresponding decision variables take some values in the memory. This analogy is first associated with the engineering optimization problems by Geem et al. (2001). In this adaptation, each musician corresponds to a decision variable and the notes in the musicians' memories correspond to the possible values of the decision variables. When the musicians find the fantastic harmony from their memories, it means, a global optimum solution is obtained using the corresponding decision variables. Note that the computational structure of the HS algorithm is based on the following three musical operations: i) Playing a note from the harmony memory; ii) Playing a note randomly from the possible note range; iii) Playing a note which is close to another one stored in memory. Adaptation of these musical operations to engineering optimization problems is as follows (Geem et al. 2001): i) New variable values are selected from harmony memory; ii) New variable values are randomly selected from possible random range; iii) New variable values are further replaced with other values which are close to the current values. Combination of these three operations allows searching a global optimum solution in an optimization framework. The following computational scheme describes the required solution steps for solving an optimization problem using HS:

Step 1: Generate random solution vectors $(\boldsymbol{x}^1, \boldsymbol{x}^2, \boldsymbol{x}^3, \cdots, \boldsymbol{x}^{HMS})$ as many as harmony memory size (HMS), then, store them in harmony memory (HM).

Step 2: Generate a new solution vector $(\boldsymbol{x}')$. For each element (x_i'):
- with probability of $HMCR$, (harmony memory considering rate), pick the stored value from HM such that $x_i' \leftarrow x_i^{\text{int}[r(0,1)\times HMS]+1}$ where $r(0,1)$ is the uniform random number.
- with probability of $1 - HMCR$, pick a random value within the allowed range.

Step 3: Perform additional adjustment if the value in Step 2 came from HM:
- with probability of PAR (pitch adjusting rate), change the value of x_i' by a small amount such that $x_i' \leftarrow x_i + fw \times r(-1,1)$ where fw is the fret width which can be defined as the amount of the maximum change in pitch adjusting process.
- with probability of $1 - PAR$, do nothing.

Step 4: If value of $\boldsymbol{x}'$ is better than the worst vector $\boldsymbol{x}^{\text{worst}}$ in HM, replace $\boldsymbol{x}^{\text{worst}}$ with $\boldsymbol{x}'$

Step 5: Repeat from Step 2 to Step 4 until termination.

The required solution parameters of HS are: harmony memory size (HMS), harmony memory considering rate $(HMCR)$, pitch adjusting rate (PAR), and fret width (fw). Among them, HM is a matrix where the decision variables and the corresponding objective function values are stored, $HMCR$ and PAR are the probabilities which are used to improve the solution globally and locally, and fw is used to perform pitch adjusting process. This computational procedure can be described in Figure 1 (Ayvaz, 2010).

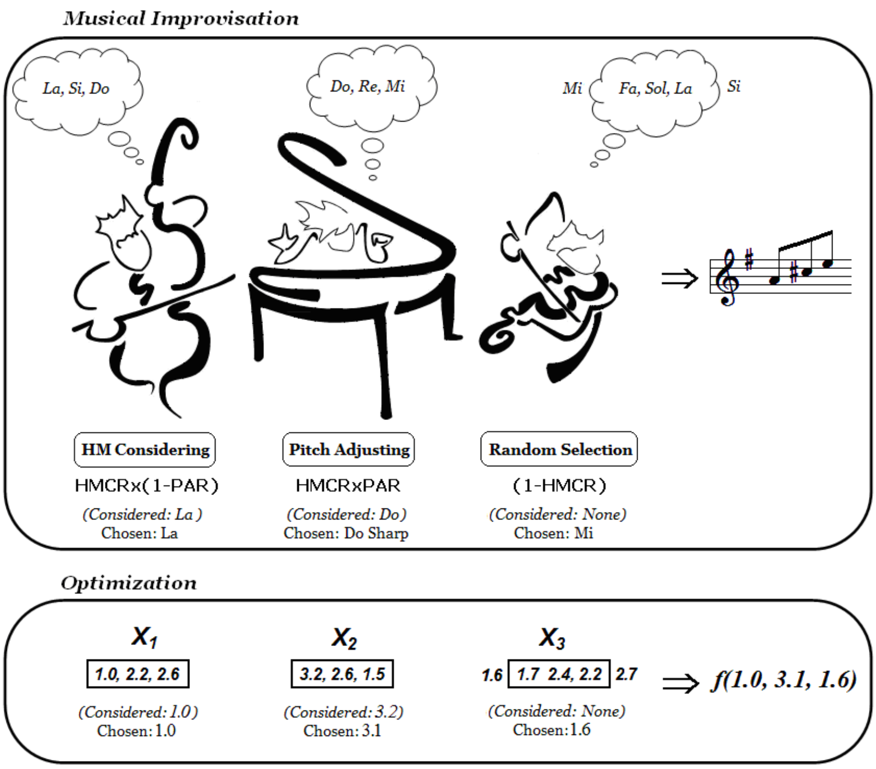

Figure 1. Analogy between musical improvisation and optimization (Ayvaz, 2010).

As can be seen in Figure 1, each musician has several notes in their HM. The main question to be asked here is “which notes will be played by the musicians to find a musically pleasing harmony?”. Depending on three musical operations indicated above, the answer of this question is given as follows:

i) *Memory Consideration:* The first musician in Figure 1 has three notes, {La, Si, Do} in his HM. With probability $HMCR \times (1 - PAR)$, he decides to choose and play {La}. Since {La, Si, Do} corresponds to {1.0, 2.2, 2.6} in the optimization process, choosing and playing {La} corresponds choosing and using {1.0} as the first decision variable.
ii) *Pitch Adjusting:* The second musician in Figure 1 has also three notes, {Do, Re, Mi} in his HM. Unlike the first musician, with probability $HMCR \times PAR$, he chooses {Do} and plays its neighbor {Do#}. Since {Do} corresponds to {3.2} in the optimization process, its neighbor {Do#} corresponds to {3.1} which is a small random amount neighbor to {3.2}.
iii) *Random Selection:* The third musician in Figure 1 has also three notes, {Fa, Sol, La} in his HM. Although his HM is used in the previous improvisations, due to his musical knowledge, he knows all the possible notes in the La Scale. Thus, he decides to choose and play a note randomly, for example {Mi} in this case. As differently from the possible data set stored in HM, {1.6} is randomly chosen and used in this case, even if it doesn't exist in HM.

After musicians' decision of what to play based on the memory consideration, pitch adjusting and random selection, the new harmony is composed as {La, Do#, Mi} which corresponds to {1.0, 3.1, 1.6} in the optimization process. This solution sequence is repeated until the given termination criterion is satisfied. Note that completing the given number of improvisations is used as the termination criterion in this study.

4 PROBLEM FORMULATION

The problem of optimum storm sewer design can be mathematically solved by using an optimization model. In this solution, main objective of the optimization model is to minimize the system cost given in Eq. [1] by satisfying the constraint set given between Eqs. [2] and [10]. Note that just like other heuristic algorithms, HS is also an unconstrained optimization method and cannot solve the constrained optimization problems in its basic form. Therefore, the constrained optimization problem given between Eqs. [1] and [10] need be converted to an unconstrained problem by means of the penalty function approach as follows:

$$\Phi' = \min\left\{\sum_{k=1}^{N} \mathcal{F}(d_k, L_k, \bar{Z}_k) + \sum_{l=1}^{9} \lambda_l \sum_{k=1}^{N} P\big(\hat{g}_{k,l}\big)\right\} \quad [11]$$

subject to

$$P\big(\hat{g}_{k,l}\big) = \begin{cases} 0 & \text{if} \quad \hat{g}_{k,l} \le 0 \\ \big(\hat{g}_{k,l}\big)^2 & \text{otherwise} \end{cases} \quad [12]$$

$$\hat{g}_{k,1}: \ \left(1 - \frac{q_k}{Q_k^*}\right) \le 0 \quad [13]$$

$$\hat{g}_{k,2}: \ \left(\frac{V_k}{V_{\max}} - 1\right) \le 0 \quad [14]$$

$$\hat{g}_{k,3}: \ \left(1 - \frac{V_k}{V_{\min}}\right) \le 0 \quad [15]$$

$$\hat{g}_{k,4}: \ \left(\frac{y_k}{\alpha \cdot d_k} - 1\right) \le 0 \quad [16]$$

$$\hat{g}_{k,5}: \ \left(1 - \frac{S_k}{S_{\min}}\right) \le 0 \quad [17]$$

$$\hat{g}_{k,6}: \ \left(\frac{E_k^u}{E_{\max}} - 1\right) \le 0 \quad [18]$$

$$\hat{g}_{k,7}: \ \left(1 - \frac{E_k^u}{E_{\min}}\right) \le 0 \quad [19]$$

$$\hat{g}_{k,8}: \ \left(\frac{E_k^d}{E_{\max}} - 1\right) \le 0 \quad [20]$$

$$\hat{g}_{k,9}: \left(1 - \frac{E_k^d}{E_{\min}}\right) \leq 0 \quad [21]$$

where Φ' is the penalized objective function value, $\hat{g}_{k,l}$ is the normalized constraint set for pipe k, $P(\hat{g}_{k,l})$ is the penalty function, and λ_l is the penalty coefficient for constraint l ($l = 1,2,3,\cdots,9$). As can be seen from Eqs. [11] and [12], the objective function gets penalty values if normalized constraints given between Eqs. [13] and [21] are not satisfied. One important thing in Eq. [11] is the selection of the penalty coefficients of λ_l. Since value of these parameters are mostly the problem dependent and there is no systematic way to find their values, it is required to conduct some trials before executing the optimization model.

5 NUMERICAL APPLICATION

The performance of the proposed optimization approach is evaluated on a benchmark system which is first designed by Mays and Wenzel (1976). The layout of the example system is given in Figure 2. As can be seen, the system has 21 nodes and these nodes are connected with 20 links with a total length of 2.62 km. The characteristic data of the network are given in Table 1. Note that this example is constrained to satisfy the minimum and maximum velocities of $V_{\min} = 0.6$ m/s and $V_{\max} = 3.6$ m/s, respectively. Similarly, minimum and maximum pipe cover should have the values of $E_{\min} = 2.40$ m and $E_{\max} = 6$ m, respectively.

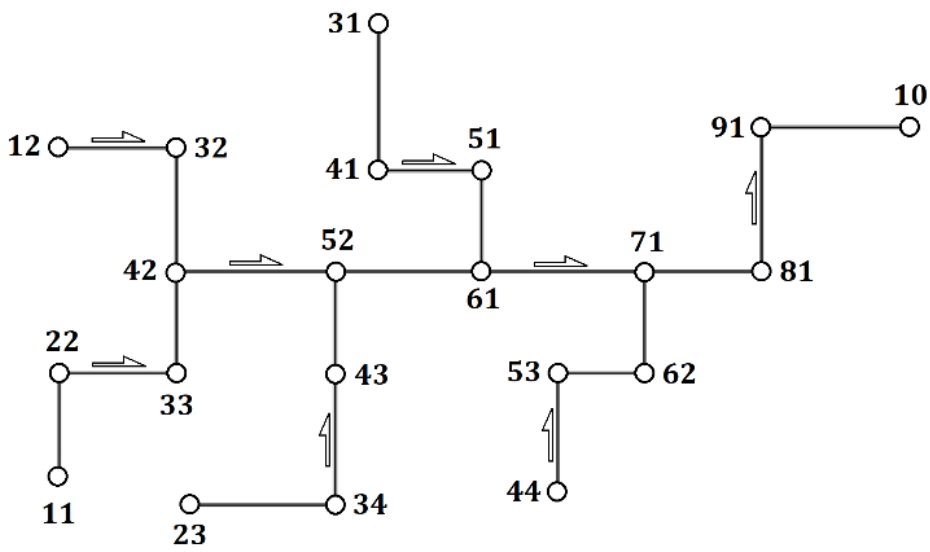

Figure 2. Layout of the example storm sewer system.

Table 1. Characteristics of the example storm sewer system.

Pipe	Ground elevation (m)		L (m)	Q^* (m^3/s)
	Upstream	Downstream		
11-22	152.40	150.88	106.68	0.1132
22-33	150.88	148.49	121.92	0.1982
33-42	148.49	146.30	106.68	0.2548
12-32	149.35	147.83	121.92	0.1132
32-42	147.83	146.30	131.08	0.2265
42-52	146.30	143.26	167.68	0.6229
23-34	149.35	147.83	147.64	0.2265
34-43	147.83	144.78	137.16	0.3398
43-52	144.78	143.26	106.68	0.4530
52-61	143.26	141.73	152.40	1.2459
31-41	147.83	144.78	152.40	0.2548
41-51	144.78	143.26	106.68	0.4530
51-61	143.26	141.73	106.68	0.5663
61-71	141.73	138.65	172.21	2.0104
44-53	142.65	141.43	121.92	0.1132
53-62	141.43	140.21	91.44	0.1699

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62-71	140.21	138.65	105.23	0.2548
71-81	138.65	137.46	121.92	2.4635
81-91	137.46	136.55	152.40	2.5201
91-10	136.55	135.64	186.54	2.6617

This example was first solved by Mays and Wenzel (1976) where differential dynamic programming based optimization approach were used. After them, various optimization approaches were applied to the solution of the same problem by different researchers (Robinson and Labadie, 1981; Afshar, 2012; Swamee and Sharma, 2012). All of these studies are conducted by considering the following cost function (Meredith, 1972):

$$C_p = \begin{cases} 10.98d + 0.80H - 5.98 & ; \ H < 10 \\ 5.94d + 1.17H + 0.50Hd - 9.64 & ; \ d \leq 3, H \geq 10 \\ 30.00d + 4.90H - 105.90 & ; \ d > 3 \end{cases} \quad [22]$$

$$C_m = 250 + h^2 \quad [23]$$

where C_p is the pipe cost (US $/ft), C_m is the manhole cost (US $), d is the pipe diameter (ft), H is the average invert depth (ft), and h is the manhole depth (ft). Note that the objective function to be minimized in Eq. [1] consists of the aggregation of C_p and C_m for the entire system. As indicated previously, this minimization is conducted by considering the pipe slopes as the decision variables of the optimization model. For such a design procedure, pipe diameters are determined implicitly by trying the available pipe diameters starting from the smallest one. The diameter which satisfies Eqs. [2] to [5] is selected as the diameter of the associated pipe. Note that pipe diameters are selected from the following diameter set which is available in market: $d \in \{304.8\text{ mm},\ 381.0\text{ mm},\ 457.2\text{ mm},\ 533.4\text{ mm},\ 762\text{ mm},\ 914.4\text{ mm},\ 1066.8\text{ mm},\ 1219.2\text{ mm}\}$. In order to obtain comparable results given in literature, the values of maximum allowable relative depth and Manning surface roughness values are taken as $\alpha = 0.82$ and $n = 0.013$, respectively. Note that the constraint set given between Eqs. [13] and [20] is integrated to the objective function by means of the penalty function given in Eq. [12]. This integration is conducted by using the penalty coefficients of $\lambda_l = 10^7$ $(l = 1,2,3,\cdots,9)$ for each constraint. By using these problem data, the proposed HS based optimization approach is executed by using the HS parameter values of $HMS = 10$, $HMCR = 0.95$, $PAR = 0.10$, and maximum improvisation number of 200,000. These values are selected based on the previous experiences and the recommendations given in literature. Furthermore, the model is executed 10 times by considering different random number seeds in order to evaluate the random number dependencies of the proposed HS based optimization approach. Figure 3 shows the convergence plots of these 10 model executions.

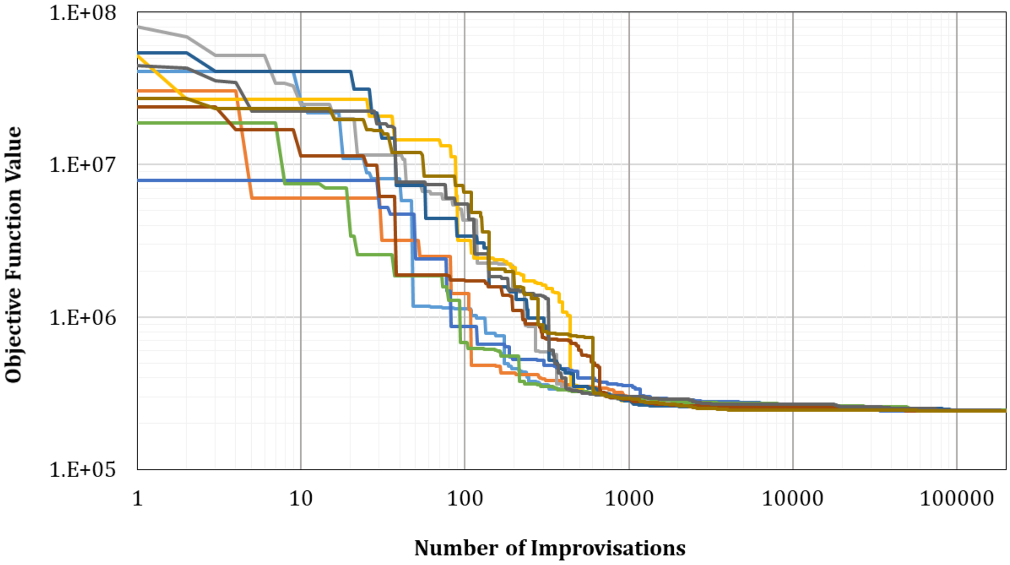

Figure 3. Convergence plots of 10 different model executions.

As seen in Figure 3, each solution starts the search process from different initial solutions since the values of random number seeds are different for each run. Although initial solutions are different, each solution converges around the similar objective function values. In early improvisations, it is observed very high objective function values since the constraints given in the model formulation are not satisfied. Therefore, value of the objective function is penalized depending on the magnitudes of the constraint violations. For the model results given in Figure 2, the statistical evaluation is given in Table 2. As can be seen, minimum, maximum and mean system costs are obtained as 240,981 US $, 243,594 US $, and 242,303 US $, respectively. The obtained results are scattered around mean value with a standard deviation of 879 US $. For the best solution, the identified model results are given in Table 3.

Table 2. Statistical evaluation of the ten model executions.

	Total system cost (US $)
Minimum	240,981
Maximum	243,594
Mean	242,303
St. Deviation	879

Table 3. Identified results for the best objective function value in the ten model executions.

Pipe	Slope	Diameter	Velocity	Relative depth	Pipe covers	
	S_k (m/m)	d_k (mm)	V_k (m/s)	y_k/d_k (m/m)	E_k^u (m)	E_k^d (m)
11-22	0.0168	0.3048	2.01	0.72	2.40	2.67
22-33	0.0175	0.3810	2.35	0.69	2.67	2.41
33-42	0.0205	0.3810	2.62	0.80	2.41	2.41
12-32	0.0126	0.3048	1.77	0.82	2.40	2.41
32-42	0.0116	0.4572	2.10	0.63	2.41	2.40
42-52	0.0193	0.5334	3.18	0.82	2.40	2.59
23-34	0.0106	0.4572	2.03	0.64	2.40	2.45
34-43	0.0219	0.4572	2.95	0.66	2.45	2.40
43-52	0.0142	0.5334	2.68	0.71	2.40	2.40
52-61	0.0115	0.7620	3.11	0.82	2.40	2.62
31-41	0.0200	0.3810	2.59	0.80	2.40	2.40
41-51	0.0175	0.5334	2.92	0.65	2.40	2.74
51-61	0.0205	0.5334	3.25	0.73	2.74	3.40
61-71	0.0121	0.9144	3.60	0.80	3.40	2.40
44-53	0.0129	0.3048	1.79	0.81	2.40	2.76
53-62	0.0097	0.3810	1.80	0.77	2.76	2.43
62-71	0.0146	0.4572	2.36	0.63	2.43	2.40
71-81	0.0097	1.0668	3.54	0.73	2.40	2.40
81-91	0.0078	1.0668	3.21	0.82	2.40	2.68
91-10	0.0087	1.0668	3.39	0.82	2.68	3.39

As can be seen from Table 3, all the velocity, relative depth, and pipe cover constraints are satisfied for the identified pipe slopes. This outcome indicates the constraint satisfaction ability of the proposed HS based optimization approach by means of the penalty function approach. As indicated previously, this example network is solved by different solution approaches in literature. Table 4 compares some results of them with those obtained by using the proposed HS based optimization approach. It is clearly seen that the proposed HS based optimization approach is resulted with the lower system cost compared to the both deterministic and heuristic approaches in literature. This result shows the applicability of the proposed approach to solve the storm sewer design optimization problems.

Table 4. Comparison of the identified system costs for different solution approaches.

	Solution Approach	Total system cost (US $)
Mays and Wenzel (1976)	Dynamic programming	265,775
Robinson and Labadie (1981)	Dynamic programming	275,218
Miles and Heaney (1988)	Spreadsheet	245,874

Afshar (2006)	Ant colony optimization	241,496
Afshar (2012)	Genetic algorithm	241,896
Proposed approach	Harmony search	240,981

6 CONCLUSIONS

In this study, an optimization approach is proposed to solve the storm sewer design optimization problems. In the proposed approach, the total system cost is used as the objective function and minimization of this function is aimed by means of the heuristic HS optimization approach. This is the first application of HS to the solution of storm sewer optimization problems. The proposed approach uses the pipe slopes as the decision variables. For each identified slope, the corresponding pipe diameters are calculated implicitly by following a solution sequence by satisfying a set of constraints. All the managerial and physical constraints are included to the optimization process by using the penalty function approach. The performance of the proposed HS based optimization approach is evaluated by solving a benchmark sewer system given in literature. Identified results indicated that the proposed HS based optimization approach satisfied all the constraints in the problem and resulted with a lower system cost compared to other deterministic and heuristic approaches in literature.

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