# FREE VIBRATION AND BUCKLING ANALYSIS OF THE LAMINATED COMPOSITE BEAMS BY USING GDQM

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**ABSTRACT**

In this study, effects of stacking sequences of composite laminated beams on natural frequencies and buckling behaviour have been analyzed by Generalized Differential Quadrature Method (GDQM) and Finite Element Method (FEM). Mode shapes were also investigated for one mode of buckling and three modes of free vibration analyses. In addition, variations of mode shapes for different boundary conditions were presented in details. Numerical results show that the effective stiffness of the laminated composite beam can be altered through an adjustment in the stacking sequence. Thus, such an adjustment in stacking sequences allows operations in desired natural frequencies and load carrying capacity without changing its geometry drastically or without changing its weight.

***Keywords:*** GDQM, Laminated composite beam, Natural frequency, Buckling analysis.

## 1. INTRODUCTION

The Differential Quadrature Method (DQM) was first proposed by Bellman and his associates [1, 2]. Since 1970s, DQM has been applied many areas in engineering problems. Bert and Malik [3] have mentioned examples to deal with the high order differential equations of Euler beam, whose governing equation is a forth order one with double boundary conditions at each boundary. The main difficulty for such forth order problems as Euler beams is that there are multiple boundary conditions but only one variable at each boundary. Applying multiple boundary condition at the same location is a big problem that to be dealt with in DQM. For this reason, Jang et al. [4] have proposed a -point approximation. A point very adjacent to the real boundary has been inserted to impose the second condition on there. Bert and Malik [3] have applied the -technique in the solution of linear structural problems. For differential equations involving more than one boundary condition at one point, the DQM has no generally effective technique to solve them without use to the current $\delta$ -point technique.

The Generalize Differential Quadrature Method (GDQM) was proposed to solve the kind of differential equations which involve more than one boundary or initial condition at one point by Wu and Liu [5]. The problem of a sixth order differential equation [6], which has coped with using $\delta$ -point at each end in the DQM, has been solved [7] without any difficulties using the GDQM. The forth order equation of beam [8, 9] problems were also solved in the GDQM.

Moradi and Taheri [10] analyzed the postbuckling response of a one dimensional delaminated composite beam under axial compression using with DQM. Ramtekkar et al. [11] developed a six-node, plane-stress mixed FEM by using Hamilton's energy principle for the natural vibrations of laminated composite beams. Adam [12] analyzed moderately large amplitude vibrations of a polygonal shaped composite plate with thick layers. Chen et al. [13, 14] presented a new method of state-space-based differential quadrature for free vibration of generally laminated beams, based on the orthotropic elasticity equations for plane stress problems. Della and Shu [15] solved analytically the free vibration of composite beams with two overlapping delaminations without resorting to numerical approximation. Shrivastava and Kumar [16] developed a finite element formulation based on higher order shear deformation theory (HSDT) and Hamilton's principle to study the free vibration response of thick square composite plates having a central rectangular cut-out, with and without the presence of a delamination around the cut-out.

In this study, natural frequency and buckling analysis of composite laminated beam were investigated by using GDQM and ANSYS. The results obtained by using GDQM were compared with ANSYS solutions. Mode shapes were also investigated for one mode of buckling and three modes of free vibration analyses. It has been concluded that results obtained from GDQM are very close to results obtained from ANSYS. As the stacking sequence changes, the effective stiffness of a laminated composite beam also changes. That allows arranging of the stacking sequence to achieve desired natural frequencies and load carrying capacity without changing its geometry drastically or without changing its weight.

## 2. CALCULATING WEIGHTING COEFFICIENT FOR GDQM

The computational of the weighting coefficients in DQM was initially made by Bellman et al. [1, 2]. They suggested two ways to determine the weighting coefficients for the first order derivative. The first way involves the solution of an algebraic equation system. The second uses a simple algebraic formulation, but with the coordinates of the grid points chosen as the roots of the test function. Bellman's first approach is to obtain the weighting coefficients because it allows the coordinate of the grid points to be chosen arbitrarily. However, when the order of the algebraic equation system is large, the matrix is ill-conditioned because of Vandermonde matrix. The second approach, assumes the test function to be an Nth-order Legendre polynomial. However, it requires $\chi_i$ $(i = 1,2,...,N)$ have to be the roots of the shifted Legendre polynomial.

Shu [17] presented simple algebraic formulation to compute the weighting coefficients of the first order derivative without any restriction on the choice of the grid point, and a recurrence relationship to compute the weighting coefficients of the second and higher order derivatives.

The weighting coefficients of the first-order derivatives are:

$$\frac{df(x_i)}{dx} = \sum_{j=1}^{N} a_{ij}^{(1)} f(x_i) \qquad (i = 1,2,...,N) \tag{1}$$

In GDQM, the test functions are supposed to be the Lagrange interpolated polynomial at point $X_k$ as follow [17]:

$$L_k(x) = \frac{M(x)}{(x - x_k) M^{(1)}(x_k)}, \qquad (k = 1,2,...,N) \tag{2}$$

where:

$$M(x) = \prod_{j=1}^{N} (x - x_j) \quad and$$
$$M^{(1)}(x_k) = \frac{dM(x_i)}{dx} = \prod_{j=1, j\neq i}^{N} (x_i - x_j). \tag{3}$$

Using the Lagrange interpolation function, weighting coefficient obtained as follow:

$$a_{ij}^{(1)} = \frac{M^{(1)}(x_i)}{(x_i - x_j) M^{(1)}(x_j)} \quad (i, j = 1,2,...,N,\ i \neq j) \tag{4}$$

and

$$a_{ii}^{(1)} = -\sum_{j=1, j\neq i}^{N} a_{ij}^{(1)}, \qquad (i = 1,2,...,N) \tag{5}$$

Similarly, the weighting coefficients for second and higher-order derivatives can be computed. The rth-order derivative can be expressed as:

$$\frac{d^r f(x_i)}{dx^r} = \sum_{j=1}^{N} a_{ij}^{(r)} f(x_i), \qquad (i = 1,2,...,N) \tag{6}$$

A recurrence relationship can be established for the rth-order weighting coefficients $\alpha_{ij}^{(r)}$ :

$$a_{ij}^{(r)} = r\left( a_{ii}^{(r-1)} a_{ij}^{(1)} - \frac{a_{ij}^{(m-1)}}{(x_i - x_j)} \right) \tag{7}$$

$$(j = 1,2,...,N,\ r = 2,3,...,(N-1),\ i \neq j)$$

and $\alpha_{ij}^{(r)}$ can be obtained from similar to Eq. (5):

$$a_{ii}^{(r)} = -\sum_{j=1, j\neq i}^{N} a_{ij}^{(r)} \qquad (i = 1,2,...,N) \tag{8}$$

From Eq. (2) and Eq. (3), it is clear that the weighting coefficients are functions of the sampling points only. The selection of location of the sampling points plays a significant role in the accuracy of the solution of differential equations. Using equal spaced grids can be considered to be a convenient and easy selection method. For a domain discretized by N points, the coordinate of any point $i$ can be evaluated by:

$$x_i = \frac{i-1}{N-1}, \qquad (i = 1,2,...,N) \tag{9}$$

## 3. TRANSVERSE FREE VIBRATION ANALYSIS

The bending moment $M$ on a composite laminated beam is shown in Fig. 1 and can be written as [18]:

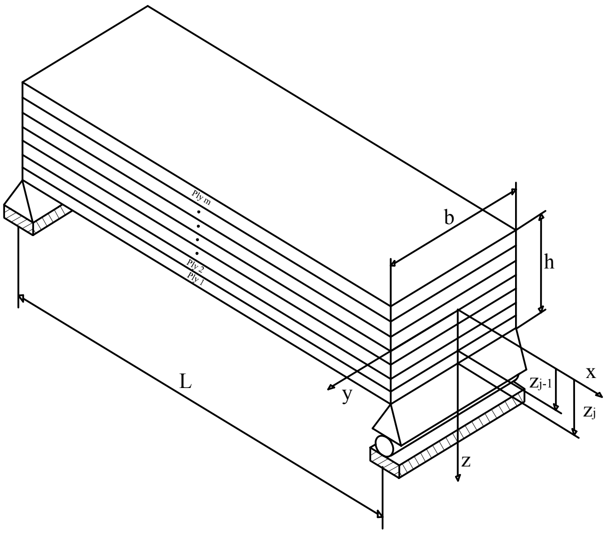


**Fig. 1:** Simply supported composite laminated beam

$$M = \frac{2b}{3\rho}\sum_{j=1}^{\frac{m}{2}}(E_x)_j(z_j^3 - z_{j-1}^3) \tag{10}$$

where $b$ is width of the beam, $\rho$ is curvature of the beam, m is the number of layer and $z_j$ is distance between the outer face of $j$th layer and the neutral plane, respectively. The relationship between the bending moment and the curvature can be written as follow:

$$M = \frac{E_{ef} I_{yy}}{\rho} = E_{ef} I_{yy}\frac{d^2 w}{dx^2} \tag{11}$$

and

$$E_{ef} = \frac{8}{h^3}\sum_{j=1}^{\frac{m}{2}}(E_x)_j(z_j^3 - z_{j-1}^3) \tag{12}$$

where $E_{ef}$ is the effective elasticity modulus and $I_{yy}$ is the cross-sectional inertia moment about the neutral axis of the beam.

The rate of change of shear along the length of the beam is equal to the loading per unit length, and the rate of change of the moment along the beam is equal to the shear as below [19]:

$$\frac{d^2 M}{dx^2} = \frac{dV}{dx} = q(x) \tag{13}$$

where $V$ represents shear force and $q(x)$ represents load in unit length. By substituting Eq. (11) into Eq. (13):

$$\frac{d^2}{dx^2}\left(E_{ef} I_{yy}\frac{d^2 w}{dx^2}\right) = q(x) \tag{14}$$

From Eq. (14), an expression for the beam exposed to static loading is obtained. Regarding dynamic loading, D'Alembert principle is used where mass and acceleration terms are added to the expression above [20].

$$E_{ef} I_{yy}\frac{\partial^4 w(x,t)}{\partial x^4} = q(x,t) - \frac{\rho_m A \partial^2 w(x,t)}{\partial t^2} \tag{15}$$

where $w$ and $q$ become functions of time and domain. Thus, derivatives become partial derivatives where $\rho_m$ is the mass density of the beam material and $A$ is the beam cross-sectional area.

The natural frequency of the beam is the function of the material properties and the geometry. Therefore, they are not affected by the forcing functions: it means that for this study $q(x,t)$ can be taken zero. Thus, Equation (15) becomes:

$$E_{ef} I_{yy}\frac{\partial^4 w}{\partial x^4} + \rho_m A\frac{\partial^2 w}{\partial t^2} = 0 \tag{16}$$

As a solution of Eq. (16), it can be used a separation of variables solution for harmonic free vibration:

$$w(x,t) = \sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{L}\cos\omega_t t \tag{17}$$

where $\omega_n$ is the frequency, $A_n$ is an amplitude of the beam and L is the length of beam. Substitution of this solution into Eq. (16) eliminates the time dependency and normalizing the equation yields the following characteristic problem as below:

$$\frac{d^4W}{dX^4} - \lambda^2 W = 0 \tag{18}$$

where $\lambda_2 = \rho_m A\omega^2_n L^4 / E_{ef} I_{yy}$ is the dimensionless frequency parameter and $X = x / L$ is the normalized form of $x$. Since Eq. (18) is the forth order, four boundary conditions are required.

In this work three types of boundary conditions are considered. For simplicity, the boundary conditions of clamped, simply supported and free types are denoted as C, SS and F respectively, as follows:

$$W = \frac{dW}{dX} = 0,\ W = \frac{d^2W}{dX^2} = 0,\ \frac{d^2W}{dX^2} = \frac{d^3W}{dX^3} = 0, \text{ at } X = 0 \text{ or } 1 \tag{19}$$

It is noted that for two ends which are $X=0$ and $X=1$ for dimensionless form, there are four boundary conditions such as C-C, C-SS, SS-SS and C-F. Eq. (18) should be applied at $(N\text{-}4)$ interior points and equation can be discretized as below:

$$\sum_{j=1}^{N} a_{ij}^{(4)} W_j = \lambda^2 W_i, \qquad i = 3,4,\ldots,(N-2) \tag{20}$$

The discretized forms of C-F boundary conditions are:

$$W_1 = 0:\ \sum_{j=1}^{N} a_{1j}^{(1)} W_j = 0: \text{ at } X = 0 \tag{21a}$$

$$\sum_{j=1}^{N} a_{Nj}^{(2)} W_j = 0:\ \sum_{j=1}^{N} a_{Nj}^{(3)} W_j = 0: \text{ at } X = 1 \tag{21b}$$

Eq. (20) can be rearranged as below:

$$\begin{bmatrix} S_{bb} & S_{bd} \\ S_{db} & S_{dd} \end{bmatrix} \begin{Bmatrix} W_b \\ W_d \end{Bmatrix} = \begin{Bmatrix} 0 \\ \lambda^2 W_d \end{Bmatrix} \tag{22}$$

where the subscripts $b$ and $d$ indicate the known and unknown grid points, respectively. As Eq. (22) rearranges, following two equations can be obtained:

$$[S_{bb}]\{W_b\} + [S_{bd}]\{W_d\} = \{0\} \tag{23}$$

$$[S_{db}]\{W_b\} + [S_{dd}]\{W_d\} = \lambda^2 \{W_d\} \tag{24}$$

From Eq. (23), one obtains:

$$\{W_b\} = -[S_{bb}]^{-1}[S_{bd}]\{W_d\} \tag{25}$$

Back-substituting Eq. (25) into Eq. (24), one gets:

$$[S_{db}]\left(-[S_{bb}]^{-1}[S_{bd}]\{W_d\}\right) + [S_{dd}]\{W_d\} = \lambda^2 \{W_d\} \tag{26}$$

and

$$[S]\{W_d\} - \lambda^2 [I]\{W_d\} = \{0\} \tag{27}$$

where $[S]$ is of order $(N\text{-}4)\mathbf{x}(N\text{-}4)$ and given by:

$$[S] = [S_{dd}] - [S_{db}][S_{bb}]^{-1}[S_{bd}] \tag{28}$$

The eigenvalue problem represented by Eq. (27) can be solved easily. Hence, one can obtain the eigenvalues (frequency squared values), and the eigenvectors (mode shapes).

## 4. BUCKLING ANALYSIS

The governing equation for the buckling behaviour of an elastic beam is given by [21]:

$$\frac{d^4w}{dx^4} + \frac{P}{E_{ef} I_{yy}} \frac{d^2w}{dx^2} = 0 \tag{29}$$

where $w$ is the lateral deflection and $P$ is the applied buckling load.

The buckling loads, like the natural frequencies in vibration are independent of the lateral loads which will be disregarded in what follows. However, in actual structural analysis, the effect of lateral loads, along with the in-plane loads could causes overstressing and failure before the in-plane buckling load is reached.

The buckling mode shape $w(x)$ for the beam, simply supported at each end, one as follow:

$$w(x) = \sum_{i=1}^{\infty} A_n \sin\frac{n\pi x}{L} \tag{30}$$

Substitution Eq. (30) into Eq. (29) yields:

$$\sum_{i=1}^{\infty} A_n \left[ E_{ef} I_{yy} \frac{n^4\pi^4}{L^4} + P\frac{n^2\pi^2}{L^2} \right] \sin\frac{n\pi x}{L} = 0 \tag{31}$$

and for this to be true for all n, the critical values of $P$, called as $P_{cr}$:

$$P_{cr} = -\frac{n^2\pi^2}{L^2} E_{ef} I_{yy} \tag{32}$$

Here, only one giving value of $P_{cr}$ is important ($n=1$) because after buckling the beam is usually permanently deformed or fails, and finally, $P_{cr}$ is a negative value which means a compressive in-plane load.

The non-dimensional form of Eq. (29) can be obtained as:

$$\frac{d^4W}{dX^4}+\frac{PL^2}{E_{ef}I_{yy}}\frac{d^2W}{dX^2}=0 \quad (33)$$

Eq. (33) can be discretized as:

$$\sum_{j=1}^{N}a_{ij}^{(4)}W_j+\lambda\sum_{j=1}^{N}a_{ij}^{(2)}W_j=0,\quad i=1,2,...,N \quad (34)$$

where $X=x/L$, $W=w/L$ and $\lambda=\dfrac{PL^2}{E_{ef}I_{yy}}$.

Since Eq. (33) is the forth order, four boundary conditions are required. In this study, three types of boundary conditions are imposed, which have also been described in Eq. (19).
Eq. (34) can be written as matrix form:

$$\left[A^{(4)}\right]\{W\}+\lambda\left[A^{(2)}\right]\{W\}=0 \quad (35)$$

where $\{W\}=[W_1, W_2,..., W_N]^{(T)}$, $A^{(4)}$, and $A^{(2)}$ are forth and second-order weighting coefficient matrices, respectively. Forth and second order weighting coefficient matrices have been rearranged similar to Eq. (28). After this rearrangement, which is an eigenvalue problem, can be written as:

$$[Q]^{-1}[S]\{W\}+\lambda[I]\{W\}=0 \quad (36)$$

where $[S]$ is rearranged for matrix $A^{(4)}$ and matrix $[Q]$ is rearranged for matrix $A^{(2)}$.

As mentioned for Eq. (27), Eq. (36) is an eigenvalue problem and can be solved easily. Again as a result of this, one can obtain the eigenvalues (buckling load), and the eigenvectors (mode shapes).

## 5. RESULTS AND DISCUSSION

In this study, the natural frequencies and load carrying capacity of composite laminated beams were calculated using numerical methods. One of them was GDQM and the other one was ANSYS based FEM. The geometry and material properties of the laminated beam is given in Table 1. The natural frequencies and critical buckling loads have been taken non-dimensional form for solving GDQM and then converting dimensional form for comparing with FEM (ANSYS). The results obtained from the two methods for the laminated beam are given in Fig. s 2-5. The numerical results are very close to each other. The minor differences among the two methods come from the assumption that ANSYS-FEM model has a perfectly smooth surface and owing to the shear rotational effects take into account in the interlaminar regions of the beam. But the shear and rotational effects do not take into account in the interlaminar regions in the GDQM model owing to Euler-Bernoulli hypothesis is valid.

The effective stiffness of a laminated composite beam can be altered through a change in the stacking sequence, which allows arranging of the stacking sequence to achieve desired natural frequencies and load carrying capacity without changing its geometry drastically or without increasing its weight.

As seen from the Fig.s 2-5 that for all the modes the

**Table 1:** Material properties and dimensions of the laminated composite beams.

| Properties | Symbol | Present | Ref. [18] | Ref. [22] |
|---|---|---|---|---|
| Width | b (m ) | 0.020 | 0.036 | 0.00635 |
| Height | h (m) | 0.006 | 0.008 | 0.00635 |
| Length | L (m) | 0.400 | 0.320 | 0.762 |
| Density | $\rho$ (kg/m$^3$) | 2030 | 2030 | 1389.23 |
| Poisson's ratio | $\nu_{12}$ | 0.15 | 0.15 | 0.3 |
| Longitudinal elasticity modulus | $E_1$ (MPa) | 26950 | 26950 | 144800 |
| Transverse elasticity modulus | $E_2$ (MPa) | 21800 | 21800 | 9650 |
| Shear modulus | $G_{12}$ (MPa) | 7540 | 7540 | 4140 |

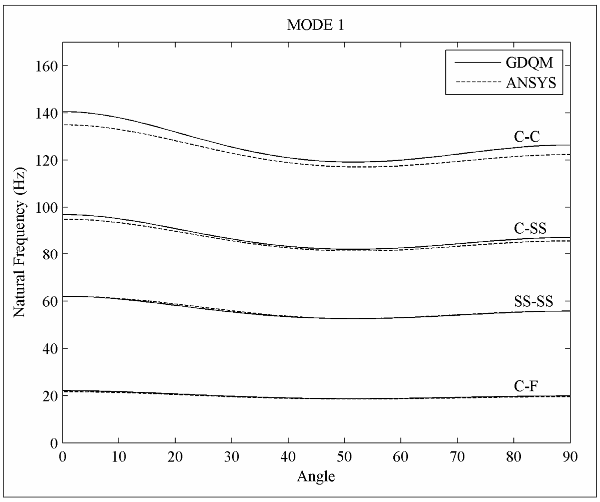


**Fig. 2:** Variation of natural frequency with stacking sequence for Mode 1

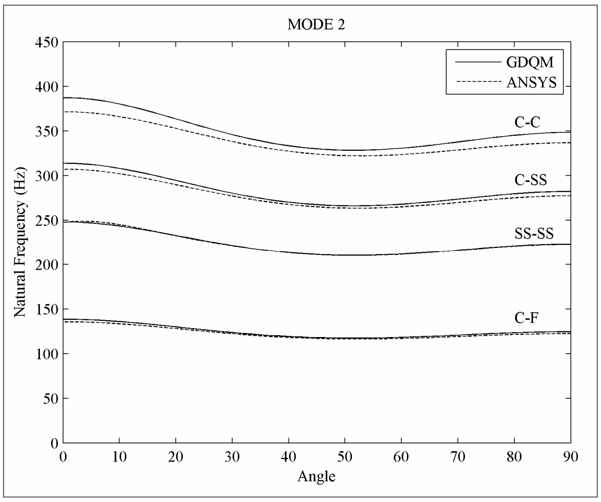


**Fig. 3:** Variation of natural frequency with stacking sequence for Mode 2

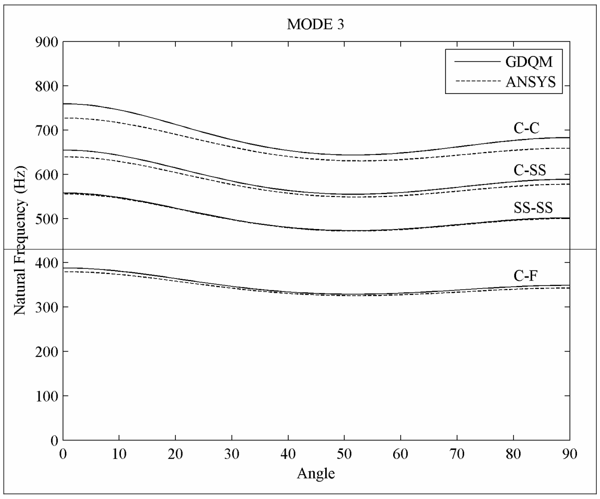


**Fig. 4:** Variation of natural frequency with stacking sequence for Mode 3

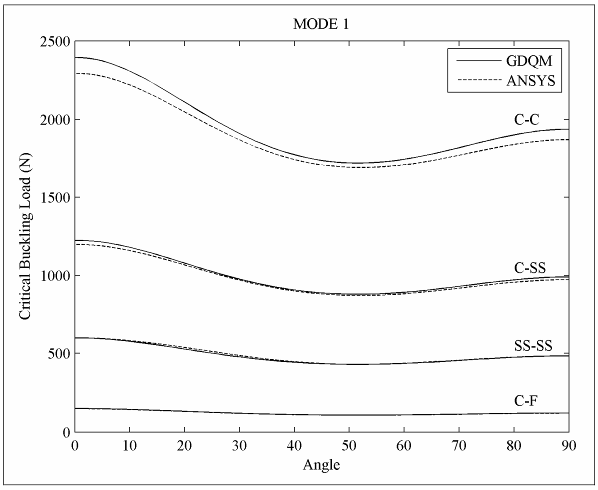


**Fig. 5:** Variation of load carrying capacity with stacking sequence for Mode1

**Table 2:** Comparison of GDQM natural frequencies values with values of Refs. [18, 22].

| Comparison Type | Application | | Mode Number 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| **Comp.1** | Present | GDQM (Hz) | 43.150 | 270.418 | 757.177 | |
| | | ANSYS (Hz) | 42.773 | 267.830 | 747.960 | |
| | Ref. [18] | Exptl. (Hz) | 43.152 | | | |
| | | Anlytcl. (Hz) | 43.150 | 270.360 | 757.210 | |
| **Comp.2** | Present | GDQM (kHz) | 0.051 | 0.202 | 0.454 | 0.808 |
| | Ref. [22] | Numrcl (kHz) | 0.051 | 0.202 | 0.451 | 0.795 |

natural frequency values and load carrying capacity values for the beam at [( 52o)16]s stacking sequence have the lowest values. Before and after this point stacking sequence takes higher values. The natural frequency and load carrying capacity alteration as a direct result of the change in the stacking sequence.

It can be also seen from Fig.s 2-5 that results obtained from two methods for C-F and C-SS boundary conditions are very close to each other. But the results from two methods for SS-SS and especially C-C boundary conditions are demonstrated a bit differences. Actually, error percentages for all boundary conditions are approximately equal.

The natural frequencies of composite laminated beams are calculated in order to compare with Ref. [18] and [22]. C-F and SS-SS boundary conditions were used in References [18] and [22] respectively. Material properties and geometries of the laminated beams used present work, [18] and [22] are given in Table 1. The present work results and others [18, 22] are very close to each other and they are presented in Table 2. Especially, analytical results of Ref. [18] and our results are very close to each other.

## 6. CONCLUSIONS

This work deals with the numerical natural frequency and load carrying capacity calculation on the symmetric laminated composite beam for different boundary conditions. The results obtained by GDQM were compared with references [18, 22] and ANSYS numerical results. As shown in Figs. 2-5 and Table 2, the present work and other results are very close to each other. The minor differences among the GDQM and ANSYS methods come from the assumption whether the shear and rotational effects in the interlaminar regions of the beam are taken into account or not. It has also been concluded that the effective stiffness of a laminated composite beam can be altered through a change in the stacking sequence, which allows arranging of the stacking sequence to achieve desired natural frequencies and load carrying capacity without changing its geometry drastically or without increasing its weight.

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