Eur. Phys. J. Plus (2012) **127**: 152
DOI 10.1140/epjp/i2012-12152-5

THE EUROPEAN
PHYSICAL JOURNAL PLUS

Regular Article

Gravity and electromagnetism with $Y(R)F^2$-type coupling and magnetic monopole solutions

Özcan Sert[a]

Department of Physics, Pamukkale University, 20070, Kınıklı, Denizli, Turkey

Received: 19 December 2011 / Revised: 16 November 2012
Published online: 12 December 2012 –

Abstract. We investigate $Y(R)F^2$-type coupling of electromagnetic fields to gravity. After we derive field equations by a first-order variational principle from the Lagrangian formulation of the non-minimally coupled theory, we look for static, spherically symmetric, magnetic monopole solutions. We point out that the solutions can provide possible geometries which may explain the flatness of the observed rotation curves of galaxies.

1 Introduction

We explore the gravitational model which involves a non-minimal coupling between a function of curvature scalar and electromagnetic fields in $Y(R)F^2$ form. The models with RF^2-type couplings were investigated in [1–13] in order to obtain more information about the modified gravity and electromagnetism. Later, they were extended to the R^mF^2-type couplings [14] to seed and grow the primordial magnetic fields in the universe. On the other hand, the late-time acceleration mechanism of the universe has not been well established in modified $f(R)$ gravity (for review, see [15]). Also, in the presence of the electromagnetic fields, the modified $f(R)$ gravity has some black-hole solutions which are not asymptotically flat [16]. Hence the natural extensions of $f(R)$ gravity to the models, $f(R)$-Maxwell [17,18], $f(G)$-Maxwell [19], $f(R)$-Yang-Mills [20] and $f(G)$-Yang-Mills gravity [21] were studied in order to explain the late-time acceleration and inflation of the universe. Moreover, there are works remarking that the rotational curves of test particles gravitating around galaxies may be realized by considering a general non-minimal $f(R)$-matter couplings [22–24].

Even though other modified $f(R)$ theories, involving a curvature scalar, exist, non-minimal couplings with electromagnetic field need further investigations, in order to show new relationships among gravity, electromagnetism and/or electromagnetic duality. Furthermore, it is important to observe that the non-minimal couplings arise from one-loop vacuum polarization effects in quantum electrodynamics of the photon effective action in a curved space-time [3]. Then, the non-minimal couplings that break the conformal invariance of electromagnetic field give rise to electromagnetic quantum fluctuations at the inflationary stage, which lead the inflation [25–29]. The scale of the fluctuations can be stretched towards outside the Hubble horizon because of the inflation at that time and they lead to classical fluctuations. Therefore, the non-minimal couplings can be the reason of inflation and the large-scale magnetic fields observed in clusters of galaxies [17,30–33]. Also, there are some exotic scenarios for producing the galactic magnetic field from the magnetic monopoles [33]. Hence, we emphasize that it is important to find spherically symmetric magnetic solutions consistent with observations from solar system to cosmological scales.

In the present paper, we proceed to investigate the non-minimal couplings of gravitational and electromagnetic fields obtaining the magnetic solutions. We first discuss a non-minimally coupled Einstein-Maxwell theory. Then we derive the gravitational field equations by a first-order variational principle using the method of Lagrange multipliers and algebra of exterior differential forms, which is in a way independent of the choice of local coordinates. Consequently, we find solutions recovering the metric functions in [34,35] in the presence of magnetic field. Thus, we point out that the solutions are asymptotically flat for some values of the parameters in the model and they can provide possible geometries which may explain the flatness of the observed rotational curves of galaxies.

[a] e-mail: osert@pau.edu.tr

2 Field equations of the non-minimally coupled theory

We will derive the field equations of the non-minimal theory by a variational principle from an action

$$I[e^a, \omega^a{}_b, F] = \int_M L = \int_M \mathcal{L}^* 1, \tag{1}$$

where $\{e^a\}$ and $\{\omega^a{}_b\}$ are the fundamental gravitational field variables and $F = \mathrm{d}A$ is the electromagnetic field 2-form. The space-time metric $g = \eta_{ab} e^a \otimes e^b$ has the signature $(-+++)$ and we fix the orientation by setting $*1 = e^0 \wedge e^1 \wedge e^2 \wedge e^3$. Torsion 2-forms T^a and curvature 2-forms $R^a{}_b$ of spacetime are given in the Cartan-Maurer structure equations

$$T^a = \mathrm{d}e^a + \omega^a{}_b \wedge e^b, \tag{2}$$

$$R^a{}_b = \mathrm{d}\omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b. \tag{3}$$

We consider the following Lagrangian density 4-form:

$$L = \frac{1}{2\kappa^2} R * 1 - \frac{1}{2} Y(R) F \wedge *F + T^a \wedge \lambda_a, \tag{4}$$

where $\kappa^2 = 8\pi G$ is Newton's universal gravitational constant ($c = 1$) and R is the curvature scalar which can be found by applying interior product ι_a twice to the curvature tensor R_{ab} 2-form. This Lagrangian density involves Lagrange multiplier 2-form λ_a whose variation imposes the zero-torsion constraint $T^a = 0$.

We use the shorthand notation $e^a \wedge e^b \wedge \cdots = e^{ab\cdots}$, and $\iota_a F = F_a$, $\iota_{ba} F = F_{ab}$, $\iota_a R^a{}_b = R_b$, $\iota_{ba} R^{ab} = R$. The field equations are obtained by considering the independent variations of the action with respect to $\{e^a\}$, $\{\omega^a{}_b\}$ and $\{F\}$. The electromagnetic field components are read from the expansion $F = \frac{1}{2} F_{ab} e^a \wedge e^b$. We will be working with the unique metric-compatible Levi-Civita connection.

The infinitesimal variations of the total Lagrangian density L (modulo a closed form) are given by

$$\begin{aligned}\dot{L} = {} & \frac{1}{2\kappa^2} \dot{e}^a \wedge R^{bc} \wedge *e_{abc} + \dot{e}^a \wedge \frac{1}{2} Y(R)(\iota_a F \wedge *F - F \wedge \iota_a * F) + \dot{e}^a \wedge D\lambda_a \\ & + \dot{e}^a \wedge Y_R(\iota_a R^b) \imath_b (F \wedge *F) + \frac{1}{2} \dot{\omega}_{ab} \wedge (e^b \wedge \lambda^a - e^a \wedge \lambda^b) \\ & + \dot{\omega}_{ab} \wedge \Sigma^{ab} - \dot{F} \wedge Y(R) * F + \dot{\lambda}_a \wedge T^a. \end{aligned} \tag{5}$$

where $Y_R = \frac{\mathrm{d}Y}{\mathrm{d}R}$, and the angular momentum tensor

$$\Sigma^{ab} = \frac{1}{2} D \imath^{ab} [Y_R F \wedge *F]. \tag{6}$$

The Lagrange multiplier 2-forms λ_a are solved uniquely from the connection variation equations

$$e_a \wedge \lambda_b - e_b \wedge \lambda_a = 2\Sigma_{ab}, \tag{7}$$

by applying the interior product operator twice as

$$\lambda^a = 2\imath_b \Sigma^{ba} + \frac{1}{2} \imath_{bc} \Sigma^{cb} \wedge e^a. \tag{8}$$

We substitute the λ_a's into the $\dot{e^a}$ equations and after some simplifications we find the Einstein field equations for the extended theory as

$$\frac{1}{2\kappa^2} R^{bc} \wedge *e_{abc} + \frac{1}{2} Y(\iota_a F \wedge *F - F \wedge \iota_a * F) + Y_R(\iota_a R^b)\imath_b(F \wedge *F) + \frac{1}{2} D[\iota^b D(Y_R F_{mn} F^{mn})] \wedge *e_{ab} = 0, \tag{9}$$

while the Maxwell equations are

$$\mathrm{d}(Y * F) = 0, \qquad \mathrm{d}F = 0. \tag{10}$$

We note that our action (4) and the field equations (9)–(10) when written out explicitly in any local coordinate system are equivalent to the action in [18]. However, our variational derivation involving Lagrange multipliers is given in a way independent of the choice of coordinates.

3 Static, spherically symmetric, magnetic solutions

We consider (1+3)-dimensional static, spherically symmetric solutions to the non-minimal model which are given by the metric

$$g = -f(r)^2 \mathrm{d}t^2 + f(r)^{-2}\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin(\theta)^2\mathrm{d}\phi^2, \tag{11}$$

and the following electromagnetic tensor F which has only magnetic component:

$$F = B(r) r^2 \sin(\theta)\mathrm{d}\theta \wedge \mathrm{d}\phi = B(r) e^2 \wedge e^3. \tag{12}$$

The non-minimally coupled field equations (9) give us the following system of equations for the metric (11) together with the magnetic field 2-form (12):

$$\begin{aligned}
\frac{1}{\kappa^2}\left(\frac{f^{2\prime}}{r} + \frac{f^2-1}{r^2}\right) + Y_R B^2 \left(\frac{f^{2\prime\prime}}{2} + \frac{f^{2\prime}}{r}\right) + \frac{1}{2} Y B^2 + [(B^2 Y_R)' f]' f + \frac{2}{r} f^2 (B^2 Y_R)' = 0,\\
\frac{1}{\kappa^2}\left(\frac{f^{2\prime}}{r} + \frac{f^2-1}{r^2}\right) + Y_R B^2 \left(\frac{f^{2\prime\prime}}{2} + \frac{f^{2\prime}}{r}\right) + \frac{1}{2} Y B^2 + (B^2 Y_R)' \left(\frac{f^{2\prime}}{2} + \frac{2f^2}{r}\right) = 0,\\
\frac{1}{\kappa^2}\left(\frac{f^{2\prime\prime}}{2} + \frac{f^{2\prime}}{r}\right) + Y_R B^2 \left(\frac{f^{2\prime}}{r} + \frac{f^2-1}{r^2}\right) - \frac{1}{2} Y B^2 + [(B^2 Y_R)' f]' f + (B^2 Y_R)' \left(\frac{f^{2\prime}}{2} + \frac{f^2}{r}\right) = 0.
\end{aligned} \tag{13}$$

Here the curvature scalar is calculated as

$$R = -f^{2\prime\prime} - \frac{4}{r} f^{2\prime} - \frac{2}{r^2}(f^2 - 1). \tag{14}$$

3.1 Exact solutions

We note that the system of equations are reduced to a simpler form when we assume the condition

$$Y_R B^2 = C, \tag{15}$$

where C is a constant to be fixed according to (17). Then, the field equations (13) turn out to be

$$\frac{1}{2}\left(f^{2\prime\prime} - \frac{2}{r^2}(f^2-1)\right)\left(C - \frac{1}{\kappa^2}\right) + Y B^2 = 0, \tag{16}$$

$$\frac{R}{2}\left(\frac{1}{\kappa^2} + C\right) = 0. \tag{17}$$

In this paper, to recover the previous metric functions obtained in [34, 35] we deal with certain non-minimal functions $Y(R)$ that involve a new non-minimal coupling constant λ which can be related to cosmological constant.

– First, we consider the inverse function of [34], *i.e.* the following non-minimal coupling [26, 36]:

$$Y(R) = 1 - a_1 \ln\left(\frac{R_\lambda}{R_0}\right), \tag{18}$$

where $R_\lambda(r) = R(r) + 12\lambda$, a_1 is a dimensionless coupling constant, R_0 is a constant with the same dimension as that of R. We note that the non-minimal coupling constants a_1 and R_0 are related to the strength of the electromagnetic field-gravity couplings.
As a phenomenological approach to take these forms of $Y(R)$, in [37], it has been demonstrated that a logarithmic non-minimal gravitational coupling like this type appears in the effective renormalization-group improved Lagrangian for an $SU(2)$ gauge theory in matter sector for a de Sitter background. For this model, the time variation of the fine structure constant has recently been examined in [38].
In this case we find the following geometry and magnetic field:

$$f^2(r) = 1 - \frac{2M}{r} + \frac{a_1\kappa^2 q^2}{r^2}\ln\frac{r}{r_0} + \frac{\kappa^2 q^2(1+5a_1)}{4r^2} + \lambda r^2, \qquad \text{for } a_1 \neq 0, \tag{19}$$

$$B(r) = \frac{q}{r^2}\,, \tag{20}$$

where r_0 is an integration constant satisfying the relation $r_0^4 = \frac{a_1\kappa^2 q^2}{R_0}$, q and M are, respectively, magnetic charge and mass of the gravitating object. Thus the curvature scalar becomes

$$R(r) = \frac{a_1\kappa^2 q^2}{r^4} - 12\lambda. \tag{21}$$

– Second, the inverse of [35], *i.e.* the following non-minimal coupling [3, 14, 26], which reads

$$Y(R) = 1 - \left(\frac{R_\lambda}{R_0}\right)^\beta. \tag{22}$$

We find the following metric function and magnetic field:

$$f^2(r) = 1 - \frac{2M}{r} + \frac{\kappa^2 q^2}{4r^2} - \frac{a_2(\beta-1)^2}{4\beta(3\beta+1)} r^{\frac{2\beta+2}{\beta-1}} + \lambda r^2, \qquad \text{for } R_0 \neq 0, \quad \beta \neq 0, 1, -\frac{1}{3}\,, \tag{23}$$

$$B(r) = \frac{q}{r^2}\,, \tag{24}$$

which give the curvature scalar

$$R(r) = a_2 r^{\frac{4}{\beta-1}} - 12\lambda, \tag{25}$$

where $a_2 = (\frac{R_0{}^\beta}{\kappa^2 q^2 \beta})^{\frac{1}{\beta-1}}$.

We note that this system of eqs. (15)–(17) has consistent solutions for only the magnetic field $B = \frac{q}{r^2}$ which can be obtained from the magnetic monopole (Dirac monopole) potential

$$A = q(1 - \cos(\theta))\mathrm{d}\phi, \tag{26}$$

which is determined by the Gauss integral

$$\frac{1}{4\pi}\int_{S^2} F = \frac{1}{4\pi}\int_{S^2} B(r) r^2 \sin\theta \mathrm{d}\theta \wedge \mathrm{d}\phi = q. \tag{27}$$

The analysis of the solutions such as horizons and asymptotic behaviors can be found in [34, 35]. The metric function (23) with $\beta = -3$ give us a Rindler acceleration term which is also obtained from a Dilaton-gravity model in [39–41] to explain some anomalies such as the rotation curves of spiral galaxies and Pioneer anomaly [42, 43]. Furthermore, the case with $0 < -\frac{2\beta+2}{\beta-1} < 1$ has the term which is more efficient in the large scale region. This case corresponds to an attractive force for $\frac{a_2(\beta-1)^2}{4\beta(3\beta+1)} > 0$. Thus, these solutions may explain the dark matter effects like the formation of the galaxies in these ranges $-\frac{1}{3} < \beta < 0$ and $a_2 < 0$ or $0 < \beta < 1$ and $a_2 > 0$. If dark matter is not an exotic matter, the non-minimal couplings give rise to such effects [44] for the values of the parameters in the above intervals depending on the physical system.

The metric (23) leads to the effective potential

$$V_{eff} = -\frac{M}{r} + \frac{\kappa^2 q^2}{8r^2} - \frac{a_2(\beta-1)^2}{8\beta(3\beta+1)} r^{\frac{2\beta+2}{\beta-1}}\,, \tag{28}$$

for vanishing angular momentum and cosmological constant. This effective potential can be rewritten in the following form:

$$V_{eff} = -\frac{M}{r} - \frac{r^{\tilde\beta-1}}{r_c{}^{\tilde\beta}} + \frac{\kappa^2 q^2}{8r^2}\,, \tag{29}$$

where $\tilde\beta = \frac{3\beta+1}{\beta-1}$. We notice that for large scale and $\tilde\beta > 0$ the last term vanishes and our potential reduces to the effective potential in [45, 46]. More comprehensive analysis of this case can be found there.

In order to give more indications on how confront our results with observational data we calculate the speed of rotational curves for the metric (23) from $v^2 = r\frac{\mathrm{d}V_{eff}}{\mathrm{d}r}$,

$$v = \sqrt{\frac{M}{r} - \frac{\kappa^2 q^2}{4r^2} - \frac{a_2(\beta^2-1)}{4\beta(3\beta+1)} r^{\frac{2\beta+2}{\beta-1}}}\,. \tag{30}$$

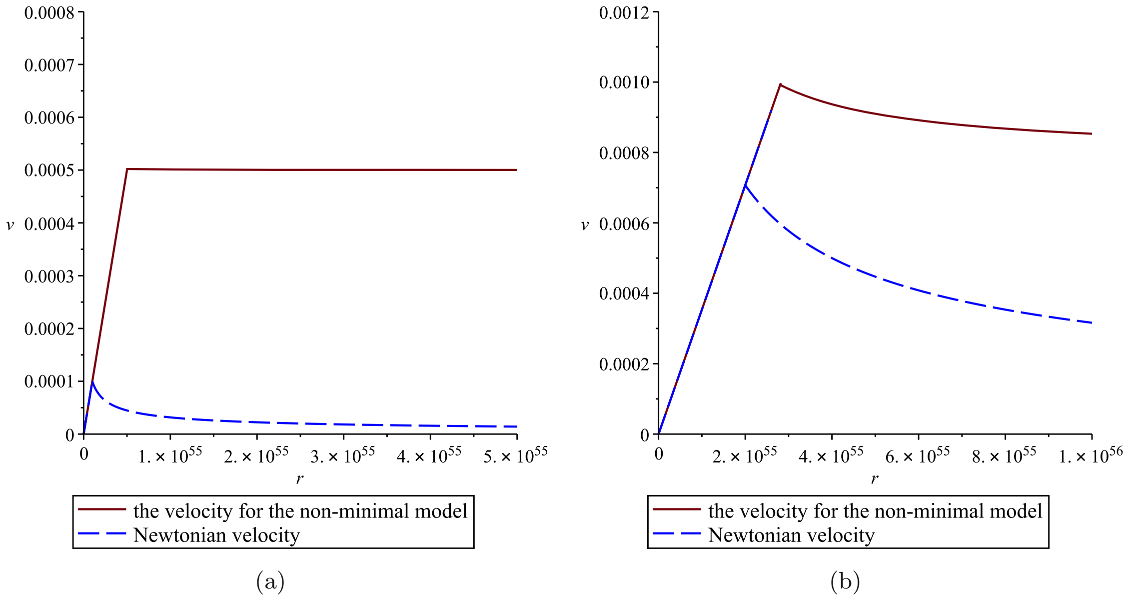

Fig. 1. The velocity profiles of rotational curves for (a) dwarf galaxy with $\beta = -1.000001 = a_2 = R_0$ and $q^2\kappa^2 = 1$, (b) large spiral galaxy with $\beta = -1.000002512 = a_2 = R_0$ and $q^2\kappa^2 = 1$.

The graphs of speed *vs.* radius are plotted in fig. 1(a) for small galaxies (10^8 solar masses $\simeq 10^{46}$ Planck mass) and in fig. 1(b) for large spiral galaxies (10^{11} solar masses $\simeq 10^{49}$ Planck mass), assuming that mass density is constant until $r = 10^{54}$ and $r = 2 \times 10^{55}$, respectively, then goes to zero. The graphs resemble to the observational velocity profiles see, *e.g.*, [47], where $10^{-3} = 300\,\text{km/s}$. Thus, the large scale magnetic field which is generated due to the breaking of the conformal invariance of the electromagnetic field can be reasons of the flatness of the rotational curves of galaxies for some parameter values.

It is relevant to point out that eqs. (19) and (23) show naked singularity solutions, for some values of the involved parameters (see, for example, [34]). Nevertheless, naked singularities, evaluated at some future space-time points, could be revealed by an observer, giving observational consequences [48–51]. The naked singularities and black holes could be observationally differentiated through their gravitational lensing features (number of images, their orientations, magnifications, and time delay, etc.) [52–55].

4 Conclusion

We have considered a non-minimally $Y(R)F^2$-coupled Einstein-Maxwell theory and looked for static, spherically symmetric and magnetic solutions. After we find the reduced field equations for the theory, we choose the non-minimal $Y(R)$ function to recover the metric solutions which is obtained from previous studies with electric field [34, 35]. In [34,35], those solutions are electrically charged and the electric field changes sign due to polarization effects. But, in this work, the magnetic monopole field is not modified by the non-minimal couplings. In addition, we obtain consistent magnetic monopole solutions only for the $Y(R)$ function which is equal to the inverse of that in [34, 35].

Meanwhile, the pp-wave solutions of the non-minimal couplings in RF^2 form can be found in [10]. Though it is difficult to find an exact electric monopole solution to the model, an exact magnetic monopole solution was found for a special case in [8]. In this case, Reissner-Nordstrom solution was modified by Q/r^4 term. But it is difficult to find more general modifications to the Reissner-Nordstrom-like solutions, such as r^α or $\ln(r)$. This work fills in this gap.

In the zero magnetic charge limit, the model with the non-minimal couplings reduces to the Einstein gravity which has some observational inconsistencies such as flatness of galactic rotation curves and Pioneer anomaly. But, in the case of non-zero magnetic charge, the model leads to new solutions (19) and (23). We can apply these new solutions to the behavior of a test object in gravitational potential of a galaxy (like a star) or the solar system (like a planet). In these cases, the effective potential of the test object is modified similarly as in [39–41, 44–47]. Here the new modifications are generated from the total magnetic charge in this closed region. Future efforts can be spent to investigate if our model can show a unified dark matter behavior (see, for instance, [56, 57]). Moreover, the solutions which have the naked singularities are important for their gravitational lensing features (number of images, their orientations, magnifications, time delay, etc.) and repulsive effects of gravity [48, 52–55].

The non-minimal coupling of the curvature scalar with magnetic fields may not be efficient in small scale, but it gives important modifications to the gravity in astrophysical scale. Since the value of the coupling parameters depends on the system we describe, our metric does not necessarily spoil the solar system precision tests and the non-minimal coupling can be considered as one of the sources for shedding light on this cosmic dark matter, and vice versa. On the other hand, there is no observational evidence on the existence of magnetic monopoles in galaxies; however, it does not exclude the possibility of their existence, and it may be explained by their scarcity. Even if magnetic monopoles are rare, they may be very heavy. Thus, they may have important effects to the dynamics of galaxies and make up the dark matter of galactic halos [58]. But it is a challenging problem to detect these rare, heavy monopoles. Future experiments such as MOEDAL [59], ANITA-II [60], AMANDA-II [61] and IceCube [62] may solve the fundamental mystery of the universe.

The author would like to thank Muzaffer Adak and the organizers of the 11th Workshop on *Quantization Dualities an Integrable Systems* for fruitful discussions and contributions. The author is partially supported by the scientific research project (BAP) 2012BSP014, Pamukkale University, Denizli, Turkey.

References

1. A.R. Prasanna, Phys. Lett. A **37**, 337 (1971).
2. G.W. Horndeski, J. Math. Phys. **17**, 1980 (1976).
3. I.T. Drummond, S.J. Hathrell, Phys. Rev. D **22**, 343 (1980).
4. H.A. Buchdahl, J. Phys. A **12**, 1037 (1979).
5. T. Dereli, G. Üçoluk, Class. Quantum Grav. **7**, 1109 (1990).
6. F. Müller-Hoissen, Class. Quantum Grav. **5**, L35 (1988).
7. A.B. Balakin, J.P.S. Lemos, Class. Quantum Grav. **22**, 1867 (2005).
8. A.B. Balakin, H. Dehnen, A.E. Zayats, Phys. Rev. D **79**, 024007 (2009).
9. A.B. Balakin, A.E. Zayats, Gravit. Cosmol. **14**, 86 (2008).
10. T. Dereli, Ö. Sert, Phys. Rev. D **83**, 065005 (2011).
11. K. Bamba, M. Sasaki, JCAP **07**, 30 (2007).
12. K. Bamba, JCAP **07**, 015 (2007).
13. Y. Liu, J. Jing, Gen. Relativ. Gravit. **44**, 7 (2012).
14. G. Lambiase, S. Mohanty, G. Scarpetta, JCAP **07**, 019 (2008).
15. S. Capozziello, S. Carloni, A. Troisi, Recent Res. Dev. Astron. Astrophys. **1**, 625 (2003).
16. S.H. Mazharimousavi, M. Halilsoy, T. Tahamtan, Eur. Phys. J. C. **72**, 1851 (2012).
17. K. Bamba, S.D. Odintsov, JCAP **04**, 024 (2008).
18. K. Bamba, S. Nojiri, S.D. Odintsov, JCAP **10**, 045 (2008).
19. M.R. Setare, J. Sadeghi, A. Banijamali, Eur. Phys. J. C **64**, 433 (2009).
20. K. Bamba, S. Nojiri, S.D. Odintsov, Phys. Rev. D **77**, 123532 (2008).
21. A. Banijamali, B. Fazlpour, Eur. Phys. J. C **71**, 1684 (2011).
22. T. Harko, Phys. Rev. D **81**, 084050 (2010).
23. T. Harko, Phys. Rev. D **81**, 044021 (2010).
24. S. Nojiri, S.D. Odintsov, Int. J. Geom. Meth. Mod. Phys. **4**, 115 (2007).
25. M.S. Turner, L.M. Widrow, Phys. Rev. D **37**, 2743 (1988).
26. F.D. Mazzitelli, F.M. Spedalieri, Phys. Rev. D **52**, 6694 (1995).
27. G. Lambiase, A.R. Prasanna, Phys. Rev. D **70**, 063502 (2004).
28. A. Raya, J.E.M. Aguilar, M. Bellini, Phys. Lett. B **638**, 314 (2006).
29. L. Campanelli, P. Cea, G.L. Fogli, L. Tedesco, Phys. Rev. D **77**, 123002 (2008).
30. K-T. Kim, P.P. Kronberg, P.E. Dewdney, T.L. Landecker, Astrophys. J. **355**, 29 (1990).
31. K-T. Kim, P.C. Tribble, P.P. Kronberg, Astrophys. J. **379**, 80 (1991).
32. T.E. Clarke, P.P. Kronberg, H. Boehringer, Astrophys. J. **547**, L111 (2001).
33. M.S. Turner, E.N. Parker, T.J. Bogdan, Phys. Rev. D **26**, 6 (1982).
34. T. Dereli, Ö. Sert, Eur. Phys. J. C **71**, 1589 (2011).
35. T. Dereli, Ö. Sert, Mod. Phys. Lett. A **26**, 1487 (2011).
36. K. Bamba, S. Nojiri, S.D. Odintsov, Phys. Rev. D **85**, 044012 (2012).
37. E. Elizalde, S.D. Odintsov, A. Romeo, Phys. Rev. D **54**, 4152 (1996).
38. K. Bamba, S. Nojiri, S.D. Odintsov, Phys. Rev. D **85**, 044012 (2012).
39. D. Grumiller, Phys. Rev. Lett. **105**, 211303 (2010) **106**, 039901(E) (2010).
40. T. Rador, S. Arapoglu, I. Semiz, arXiv:1012.4207v2 [gr-qc].
41. S. Carloni, D. Grumiller, F. Preis, Phys. Rev. D **83**, 124024 (2011).
42. G. Bertone, D. Hooper, J. Silk, Phys. Rep. **405**, 279 (2005).
43. J.D. Anderson *et al.*, Phys. Rev. Lett. **81**, 2858 (1998).
44. S. Nojiri, S.D. Odintsov, H. Stefancic, Phys. Rev. D **74**, 086009 (2006).

45. S. Capozziello, M. Francaviglia, Gen. Relativ. Gravit. **40**, 357 (2008).
46. S. Capozziello, V.F. Cardone, A. Troisi, Mon. Not. R. Astron. Soc. **375**, 1423 (2007).
47. S. Capozziello, M. De Laurentis, Ann. Phys. (Berlin) **524**, 545 (2012).
48. R. Penrose, in *Black Holes and Relativistic Stars*, edited by R.M. Wald (The University of Chicago Press, Chicago, 1998) p. 103.
49. B. Broda, Phys. Rev. Lett. **106**, 101303 (2011).
50. O. Luongo, H. Quevedo, *Toward an invariant definition of repulsive gravity*, in *The twelfth Marcel Grossmann meeting* (World Scientific, 2010) pp. 1029–1031, DOI: 10.1142/9789814374552_0122, arXiv:1005.4532 (2010).
51. M. Gasperini, Gen. Relativ. Gravit. **30**, 1703 (1998).
52. K.S. Virbhadra, G.F.R. Ellis, Phys. Rev. D **65**, 103004 (2002).
53. K.S. Virbhadra, C.R. Keeton, Phys. Rev. D **77**, 124014 (2008).
54. K.S. Virbhadra, D. Narasimha, S.M. Chitre, Astron. Astrophys. **337**, 1 (1998).
55. K.S. Virbhadra, Phys. Rev. D **79**, 083004 (2009).
56. C. Gao, F. Wu, X. Chen, Y.G. Shen, Phys. Rev. D **79**, 043511 (2009).
57. A. Aviles, L. Bonanno, O. Luongo, H. Quevedo, Phys. Rev. D **84**, 103520 (2011).
58. J. Preskill, Annu. Rev. Nucl. Part. Sci. **34**, 461 (1984).
59. See `http://moedal.web.cern.ch/content/diracs-dream-search-magnetic-monopole`.
60. M. Detrixhe *et al.*, Phys. Rev. D **83**, 023513 (2011).
61. R. Abbasi *et al.*, Eur. Phys. J. C **69**, 361 (2010).
62. See `http://icecube.wisc.edu/collaboration/governance`.