

THE INVERSE THEOREM OF APPROXIMATION THEORY IN SMIRNOV–ORLICZ CLASSES

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Abstract. Let Γ be a Dini-smooth curve in the complex plane \mathbb{C} . In this study we prove inverse theorem of approximation theory by polynomials in Smirnov-Orlicz classes $E_M(G)$.

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