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Stress intensity factor estimation of repaired aluminum plate with bonded composite patch by combined genetic algorithms and FEM under temperature effects

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The finite element method (FEM) is used to analyze the behavior of repaired cracks in 2024-T3 aluminum with bonded patches made of unidirectional composite plates. Different plate thicknesses, number of patch layers, temperatures and crack lengths are considered in the analyses. Firstly, the K_I stress intensity factor (SIF) is calculated by the finite element method using displacement correlation technique. Secondly, non-linear estimation models are developed using genetic algorithms (GA). The developed models are validated with experimental and numerical data. A genetic algorithm patch design (GAPD) is developed to estimate the stress intensity factor without any further finite element analysis.

Keywords: Stress intensity factors, Composites patch, Adhesives, Crack, Finite element analysis, Genetic algorithms

Composite materials have been widely used in both commercial and military aerospace vehicles, aircraft and automobile structures etc. In aircraft structures, traditional repair methods such as bolt and rivets causes stress concentration problem due to drilling of additional fastener holes. Therefore, development of bonding technology in aircraft structures has been accelerated. Adhesively bonded repairs cause minimum stress concentration and alter the load path that induces efficient load transfer from cracked structure to reinforcement. Thus, the reduction of the stress intensity factor caused by bonded patch repair prevents or retard crack reinitiation or further growth[1]. Baker pioneered studies about bonded composite repairs in aircraft and marine structures. Most of his studies have been focused on Mode I conditions[2,3]. Many cracked structural components can be repaired by using bonded composite patches which, mainly by reducing the stress-intensity factor, improve the mechanical resistance and increase fatigue life[4-7]. Tsai and Shen[8] performed stress analyses for different aluminum plates, including with and without crack, crack with single-side patched composites, as well as a crack with double-side patched composites. The stress intensity factor analysis and the fatigue life calculations were performed. They found that the composite patch increases the fatigue life of the cracked plate. Aslantas *et al*.[9] calculated SIF value for repaired cracks in aluminum with bonded patches both Mode I and mixed mode conditions. Different values of patch thickness and adhesive thickness are considered. A numerical fracture mechanical analysis has been carried out for analysis of composite materials of matrix with a single fiber with a crack at the interface. Special contact elements, which have bonded feature, are used between fiber and matrix. Displacement correlation method is used to calculate opening-mode and sliding-mode stress intensity factors[10,11]. Some researchers applied fracture mechanics in different areas. Ucun e*t al*.[12] investigated the fracture path behavior of diamond segments that have been brazed on a marble cutting disc. Two dimensional linear elastic fracture mechanics principles were used to analyze propagation behavior of the crack. Stress intensity factors were calculated using displacement correlation method. It was deduced from the SEM analysis of the fractured segment surface that the fracture occurred in the diamond segment due to stress concentration near the sharp corners of the diamond particles that are embedded into the matrix.

The genetic algorithm (GA) is the one probabilistic search method which is well suited to solve large combinational design problems. It has been widely used in various engineering optimization problems such as disciplinary optimization of composite laminates, optimizing for impact loading identification of composite structures, the optimal

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design of an adhesively bonded functionally graded single or double lap joint, optimization FGM, i.e.,[13-18]. Brighenti *et al.*[19] examined the optimal shape of patch repairs for cracked plates by applying a biology-based method, known as the genetic algorithm. The optimum design procedure consists of evaluating the patch topology which maximizes or minimizes some mechanical properties of the repaired plate while keeping constant the total patched area[19]. Mathias *et al.*[20] solved a reinforcement problem using a genetic algorithm associated with a finite element code. The patch shapes, the ply orientations as well as the location of the patch are simultaneously optimized to obtain a patch which relieves a given area[20].

In this study, the finite element method is used to analyze the behavior of repaired cracks in an aluminum plate with bonded patches. Mode I condition is considered. During the numerical studies, FRANC2D/L finite element code was performed. Firstly, the K_I stress intensity factors are calculated by the finite element method using displacement correlation technique. Then an equation is constructed by using genetic algorithms for the calculation of stress intensity factor without any further finite element analysis. The non-linear form of patch design estimation was developed in order to reveal the effect of plate thicknesses, number of patch layers, temperatures and crack lengths of bonded joints on the base of GA notion. A genetic algorithm patch design estimation model was used to estimate SIF under four different parameters.

Materials and Method

Experimental study

The fiber-reinforced composite plates manufactured from unidirectional E-glass fabric having a density of 270 g/m^2 and epoxy resin by hand lay-up method. As the matrix material, an epoxy based on CY225 resin and HY225 hardener were used. The density of the laminated plate is 2.026 g/cm^3. Each composite plate's thickness is approximately 0.15 mm. The composite patch material consists of 59% fibers and 41% epoxy.

The surfaces of the specimens were prepared using the cleaning technique specified in American Society for Testing and Materials Standards (ASTM D2651-01). The specimens were then bonded with unidirectional glass fiber/epoxy composites with 2, 4, 6, and 8 layers. After the bonding process of the specimens, they were cured for 1.5 h under the curing temperature of 120^{o}C at constant pressure 3 bars. Then, specimens were cooled to a room temperature.

Test samples were prepared according to the ASTM standards. An Instron 8801 universal uni-axial testing machine, with a capacity of 50 kN, was used in all static experiments. For the determination of the mechanical properties of unidirectional glass/epoxy under static loading conditions, $[0]_6$ composite plates were used. The properties of the glass-epoxy composite plates are given in Table 1.

As can be seen from Table 1, mechanical properties of the composite materials decrease with increasing temperature. Only longitudinal properties are measured since the composite patches are bonded on plates along the loading direction. As the temperature increases, elastic modulus, Poisson's ratio and tensile strength decrease.

2024-T3 aluminum plates are widely used in aerospace industry. Test samples were prepared according to the ASTM standards and the properties of the aluminum plates are given in Table 2. Mechanical properties of aluminum plate decrease as temperatures increase, but this decrease is not as much as in the case of composite materials.

FM73 film adhesive materials are widely used in repairing aluminum plates. The property of the FM73 film adhesive is given in Table 3. Properties of adhesive materials similarly decrease with increasing temperature. The variation in shear modulus, G, with increasing temperature is higher than 60%.

In this study, 2024-T3 aluminum plates with thicknesses 2, 3.75 and 5 mm, glass/epoxy reinforced

Table 1—Mechanical properties of the composite patch material

Material	Temperature, ^{o}C	Modulus of elasticity (E_1), MPa	Poisson ratio, ν_{12}	Tensile strength (X_t), MPa
	25	44150	0,20	770
Glass-epoxy	60	39185	0,19	670
	100	36250	0,17	582

Table 2—Mechanical properties of 2024-T3

Material	Temperature ^{o}C	Modulus of elasticity (E_1), MPa	Poisson ratio, ν_{12}	Tensile strength (R_m), MPa
	25	73264	0,34	452
2024-T3	60	72100	0,33	425
	100	69850	0,32	402

Table 3—Mechanical properties for FM73

Material	Temperature, ^{o}C	Modulus of shear (G), (MPa)
	25	856
FM73	60	550
	100	380

composite patch materials with layers 2, 4, 6 and 8 ply and temperatures 22, 60 and 100°C are considered.

Numerical study

The analyzed cracked geometries are prepared according to ASTM E647 (Fig. 1)[21]. First of all, the geometry is analyzed under the unpatched condition by the FEM under linear elastic condition. Two-dimensional plane stress condition is considered. Secondly, the double patched cracked aluminum plate is considered with the same crack lengths.

The SIF values obtained by using the mesh structure shown in Fig. 2 reveals very small deviation from the values obtained (Fig. 3) from the standard equation given as Eq. (1).

$$K_I = \sigma\sqrt{\pi a}\left\{\sec\left(\frac{\pi a}{2w}\right)^{0,5}\left[1-0,025\left(\frac{a}{w}\right)^2+0,06\left(\frac{a}{w}\right)^4\right]\right\} \quad \ldots (1)$$

The plate is vertically fixed at the bottom edge and symmetric boundary condition is applied on the left edge, indicated as "S". Uniformly distributed stresses are applied on the plate.

The displacement correlation method is appropriate for numerical solution based on the finite element method. The method is one of the most popular methods used for calculating SIF by numerical techniques. After finite element or boundary element solutions are made for cracked structure, nodal displacement values of nodes *a*, *b*, *c*, and *d* are obtained[22]. The crack face displacements in opening mode are related to the stress intensity factor for Mode I fracture. By the use of the displacement of mentioned nodes, K_I is calculated by[23,24],

$$K_I = E\sqrt{\frac{\pi}{32L}}\left[4(v_a - v_c)+(v_d - v_b)\right] \quad \ldots (2)$$

where L is the element length, and v_i (i = *a*, *b*, *c* and *d*) are nodal displacement values of the nodes in vertical direction (Eq. 2).

Special quarter point elements are used at the crack tip for calculation of SIF. For the numerical solution,

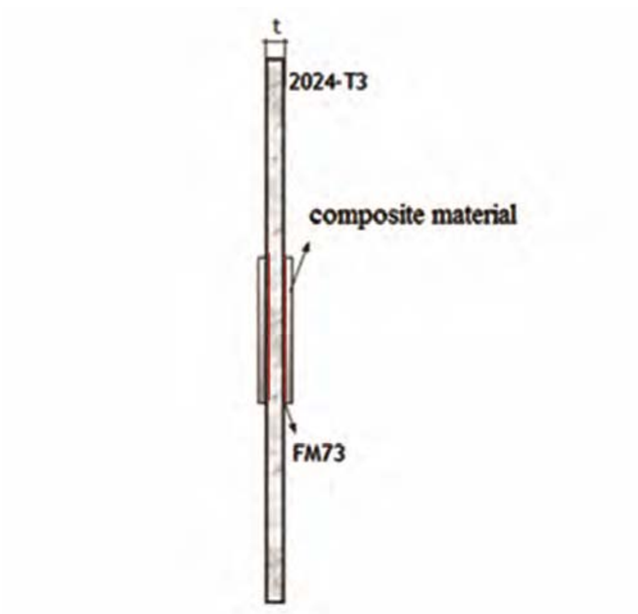

Fig. 1—Adhesive zone of aluminum plate with bonded patches

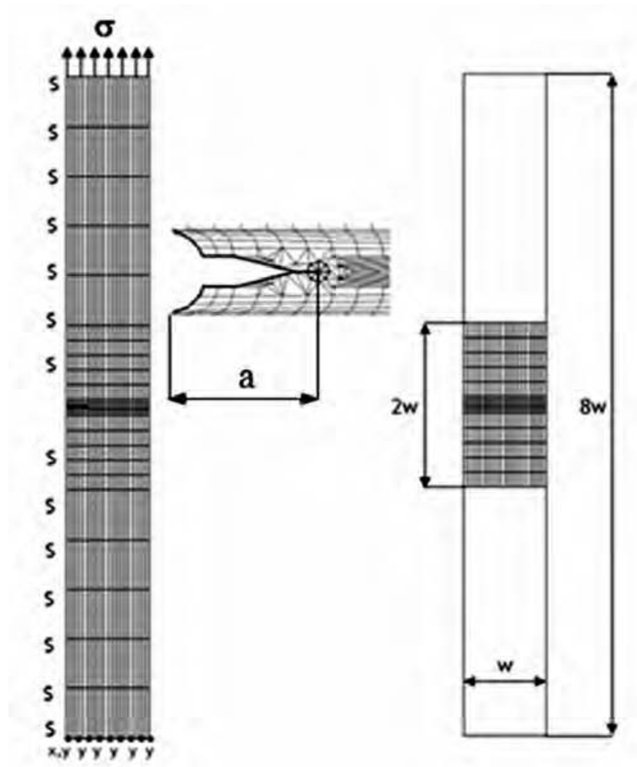

Fig. 2—The geometry of the plate according to ASTM standard

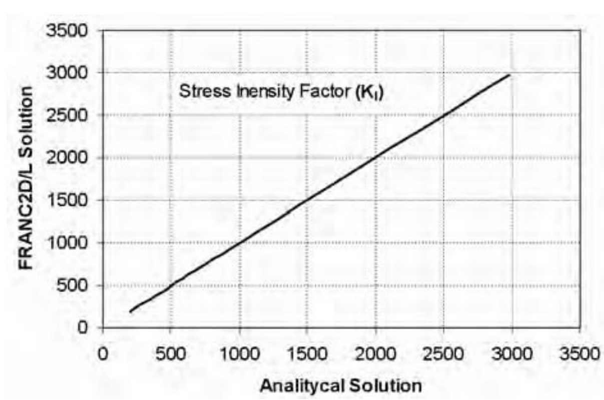

Fig. 3—A comparison of FRANC2D/L with analytical solution

two-dimensional finite element code, FRANC2D/L, is used[25]. This code is designed for fracture mechanics analysis of layered structures. Eight node isoparametric elements are used to generate the finite element mesh. Geometric dimensions of the plate with central crack and finite element mesh of the plate are given in the Fig. 2.

Genetic algorithms approach

GA is stochastic optimization method and is inspired by the evolution theory. A set of design alternatives which represent a generation in the natural analogy are allowed to reproduce and crossover among themselves, with bias allowed to the most fit members of the population. Combination of the most preferred traits of the mating members of the population results in a progeny population that is more fit than the parent population. If the measure that indicates the fitness of a generation is also the desired goal of a design process, successive generations of evolution would give rise to better values of the objective function. There are essentially three basic components and a fitness function necessary for the successful implementation of genetic algorithm. The basic operators of genetic search are selection, crossover and mutation. GA approach schema used for this study is given in Fig. 4.

GA starts with a random initial set of solutions, which is called the population. Individuals in the population are called chromosomes, which are probable solutions of the problem. Usually chromosomes are sets of binary strings. By evolving chromosomes through an iteration step, a new set of chromosomes, generation, is formed. Each generation is a combination of old and new chromosomes. This evaluation process is carried out by 3 operations; crossover, mutation and selection.

The GA approach is performed using the FEM results to determine whether a patch factor can be added to Eq. (1) so as to calculate SIF in patched plate without making any further FEM analysis. It is estimated in this study that the SIF value for the patched case can be calculated as (Eq. 3);

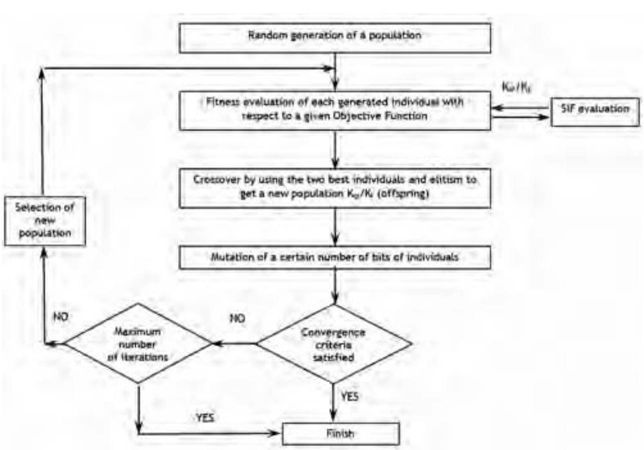

Fig. 4—GA approach schema used for this study

$$K_{Ip} = K_I * f(a/w, p, t \text{ and } T^*) \quad \dots (3)$$

where, K_{Ip}, a, w, p, t and T^* are SIF for patched condition, crack length, plate width, number of patch layers (or the thickness of the patch), plate thickness and temperatures, respectively. T^* is equal to $T/100$ (T is absolute temperature).

K_C values measured depending on plate thickness and temperature according to ASTM standard are obtained[22] and are given in Table 4.

The fitness shows the quality of design, and thus, the objective function is a sensible choice for the fitness measure. Several equations are studied for the determination of the most suitable model which are namely fully quadratic polynomial model, exponential model, exponential model with linear elements and exponential model with nonlinear elements. It is seen that the exponential model with nonlinear elements is the most suitable model for repaired cracks in an aluminum plate with bonded composite patches problem[26]. So, this model was used in the analysis.

Exponential model with nonlinear elements model of GAPD was considered to represent the repaired cracks (Eq. 4). The GAPD was written in terms of design parameters of the crack length, plate width, number of patch layers (or the thickness of the patch), plate thickness and ambient temperatures. The equation abtained is given in the Eq. (4);

$$f\left(\frac{a}{w}, t, p, T^*\right) = x_1 + x_2(t) + x_3\left(\frac{a}{w}\right)^{x_4} + \left(\frac{a}{w}.p\right)^{x_5} + \left(\frac{a}{w}.t\right)^{x_6} + \left(\frac{a}{w}.T^*\right)^{x_7} + (t.p)^{x_8} + x_9(T^*)^{x_{10}} \quad \dots (4)$$

where x_1, x_2,..., x_{10} are the corresponding design variables.

Table 4—K_C value obtained for different thicknesses and temperatures

Plate thickness, mm	K_C, MPa.mm$^{0.5}$		
	25°C	60°C	100°C
2	1301	1184	1145
3.75	1256	1217	1158
5	1123	1075	1046

Application of GAPD

The numerical data used in the genetic algorithm patch design (GAPD) are obtained by the aforementioned finite element analysis. The data supply the individual effects of the *a*/*w*, *p*, *t*, and *T** in the aluminum plate. The model is described by providing the upper and lower bounds of parameters. The GAPD model is executed by using several parameters (i.e., size of population = 150, number of generation = 230, Crossover probability = 0.8, probability of mutation = 0.02 and number of design parameters = 10).

During the estimation of GAPD parameters, numerical data were used. The estimation errors between observed and predicted values are reported. After executing the GA based on the user specified parameters, the following parameter values of GAPD were obtained by minimizing the sum of the squared error (SSE) between experimental data and estimated results (Eq. 5).

$$Min\ f(a/w, p, t, T^*) = \sum_{j}^{m} e_j \left\{ (K_{IP}/K_C)_{Numerical} - (K_{IP}/K_C)_{GA} \right\}^2 \quad \ldots (5)$$

where $(K_{Ip}/K_C)_{Numerical}$ and $(K_{Ip}/K_C)_{GA}$ are the SIF values obtained numerically and using GAPD method, respectively. *m* is the number of observations and e_j is the weighting factor. The model captures the best model by randomly selecting the set of modeling parameters and evolves them to produce the best model under minimized error.

The constants (x_1, x_2,…, x_{10}) in the proposed (Eq. 4) are obtained from a GAPD study. The final form of the equation I given as follows:

$$f(a/w, t, p, T^*) = -1.998 + 0.101(t) + 0.457 \left(\frac{a}{w}\right)^{0.258} + \left(\frac{a}{w}.p\right)^{-0.115} + \left(\frac{a}{w}.t\right)^{-0.097} + \left(\frac{a}{w}.T^*\right)^{0.217} + (t.p)^{-5.847} - 1.122(T^*)^{0.098} \quad \ldots (6)$$

The GAPD estimation model was used to estimate the SIF within the lower and upper bounds specified. The model was performed by using 1082 numerical data. Figure 5 gives the comparison of numerical and GAPD estimates (Eq. 6) of the normalized SIF as well as the coefficients of determination (R^2) for this equations. R^2 is found to be 0.991 for this problem.

Results and Discussion

The composite patch prevents an increase in SIF in a great amount as observed in previous studies[4-22-23]. The patched SIF value (K_{IP}) is given in the normalized form in the following graphs by dividing it to the unpathched SIF value (K_C) to show the considerable effect of the patch. This will be useful also in obtaining the required function of *f*(*a*/*w*, *p*, *t* and *T**). The variations of normalized SIF according to crack length, plate thickness, number of patch layers and temperature are given in Figs 5-7. As it can be seen from the figures, the patching reduces the SIF value

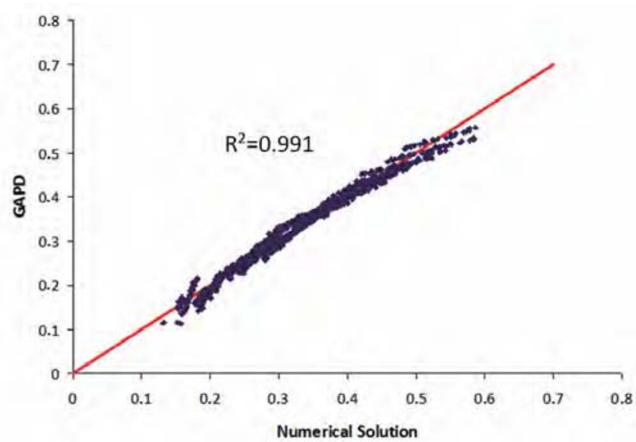

Fig. 5—Comparison of GAPD estimated and numerical results

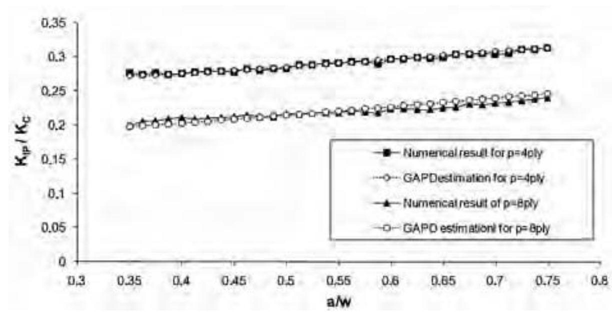

Fig. 6—Effect of number of patch layers on the SIF for *t*=3.75mm and *T*=25°C

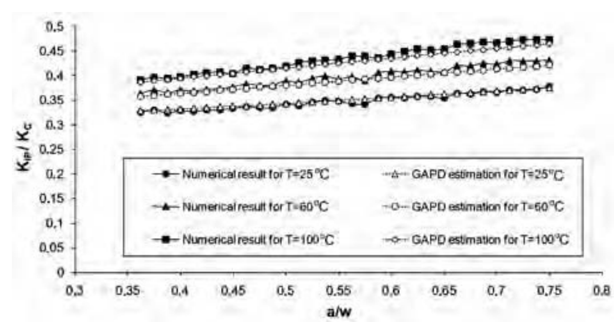

Fig. 7—Effect of temperature on the SIF for *t*=5mm and *p*=6ply

Table 5—Error rate comparison for some crack length

Crack length *a*/*w*	Plate thickness, mm	Number of patch layers, ply	Temperature, °C	Numerical K_{IP}/K_C	GAPD K_{IP}/K_C	Error, %
0.35	2	2	25	0.235	0.237	0.76
0.73	2	6	60	0.202	0.198	-1.80
0.51	3.75	2	25	0.352	0.362	2.81
0.51	5	2	25	0.458	0.463	1.09
0.40	5	2	60	0.468	0.486	3.67
0.57	3.75	4	60	0.337	0.333	-1.07
0.38	5	6	60	0.365	0.362	-0.84
0.5	3.75	6	100	0.317	0.315	-0.74
0.35	5	8	100	0.350	0.354	1.16
0.85	4	7	70	0.357	0.356	0.28
0.8	3	3	45	0.319	0.322	0.94
0.37	4.5	5	90	0.363	0.370	1.92
0.55	2.5	3	50	0.261	0.267	2.29
0.4	4	7	30	0.249	0.244	2.01
0.7	4.75	5	80	0.436	0.438	0.45

for all cases and it becomes clearer for the bigger crack length.

The effects of patch layers in SIF are shown in Fig. 6. As the number of patch layer increases, the stress intensity factor decreases. For example, while the number of patch layers increases from 4ply to 8ply, the decrease rate reaches up to 24% for a specific condition (a/w=0.4, t=3.75 and T=25°C). As it can be seen from the Fig. 6 the GAPD results are compatible with the numerical results.

Figure 7 presents the effects of temperature in stress intensity factors. As can be seen, temperature highly affects the stress intensity factor. For a/w = 0.5, t = 5 and p=6 and different temperatures, K_{IP}/K_C values are obtained as follows;

K_{IP}/K_C = 0.341 at T = 25°C, K_{IP}/K_C = 0.389 at T = 60°C and K_{IP}/K_C = 0.418 at T=100°C.

It is seen that the temperatures increase from 25°C to 100°C, the K_{IP}/K_C increases about 3%. The some results are observed in GAPD method. As indicated in Tables 1-3, mechanical properties of the patch and adhesive materials decrease with increasing temperature. For this reason, the patch is unable to serve in full capacity at elevated temperatures and the crack propagates rapidly, which results in damage at earlier phases. Under normal conditions, the composite patch prevents crack propagation and provides the resistance required for preventing the remaining cross-section to splitting into two parts. However, as temperature increases, this resistance decreases and for this reason, the patch allows the crack to propagate and the remaining cross-section cannot bear the load. Therefore, early rupture takes place.

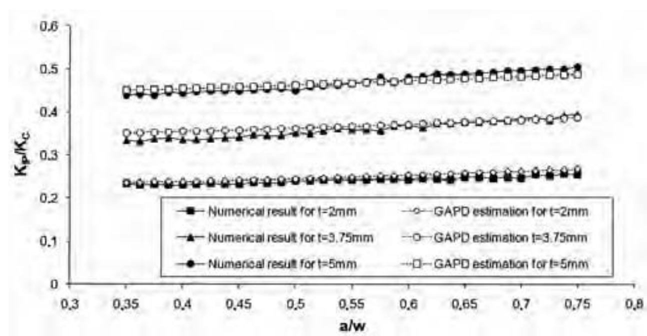

Fig. 8—Effect of plate thickness on the SIF for p=2ply and T=25°C

In Fig. 8, the effect of plate thickness can be observed clearly. As the plate thickness increases, material resistance to crack propagation decreases. For aluminum material used, the thickness value providing plane strain is 20.8 mm. Up to this thickness value, a decrease in the fracture toughness and an increase in the SIF resulting from load are the expected cases. These phonemes can be observed from Table 4. In additionally, as seen in Fig. 8, with the increasing plate thickness, crack length dependency will increase.

In Table 5, some comparisons are given to check the validity of the proposed GAPD estimation equation (Eq. 6). Some extra solutions (not included in the data set used for the construction of Eq. (6)) have been obtained by the numerical technique. These are compared with the estimated values. As can be seen from the Table 5, maximum error is lower than 4% and the average error is about 1.5%.

Conclusions

In this work, the finite element analyses are carried out to analyze the behavior of repaired cracks in an aluminum plate with bonded composite patches. The problem is considered under Mode-I condition and K_I stress intensity factors are calculated using displacement correlation method. The effects of the number of patch layers, plate thickness, temperatures and different crack lengths on the stress intensity factors are investigated. A genetic algorithm for patch design analyze is also studied. The non-linear GAPD model was used to estimate SIF. The following main conclusions may be drawn;

In Mode I loading condition, increasing rigidity decreases the stress intensity factor which means a safer design. The increasing the number of patch layers reduces the stress intensity factor. Also patching is more effective way to reduce SIF for longer crack length. For patched cases, plate thickness has more effects on SIF value than unpatched cases.

Rise in temperature is a substantial factor on the strength and fracture toughness. As the temperature increases, mechanical properties of both the aluminum plate and patching materials deteriorate and the material has a tendency to fracture as in the unpatched condition.

The GAPD model can be used as an estimation technique to predict the crack propagation for different design conditions.

The required K_I for the patched case can be obtained by substituting (a/w), (p), (t) and (T^*) into the equation obtained from this model.

Acknowledgements

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Nomenclature

K_I	=	stress intensity factor for mode I condition
K_{IP}	=	SIF for patched condition
K_C	=	fracture toughness
E	=	modulus of elasticity
ν	=	Poisson ratio
w	=	aluminum plate with
A	=	crack length
σ	=	applied stress
P	=	number of patch layers
T	=	aluminum thickness
SIF	=	stress intensity factors
GAPD	=	genetic algorithm patch design
R^2	=	coefficients of determination
L	=	element length

References

1 Seo D C & Lee J J, *Compos Struct,* 57 (2002) 323-330.
2 Baker A A, *Compos Struct,* 2 (1984) 153-181.
3 Baker A A, *Theor Appl Fract Mech*, 2 (1984) 1-16.
4 Ting T, Jones R, Chiu W K, Marshall I H & Greer J M, *Compos Struct*, 47 (1999) 737–743.
5 Belhouari M, Bouiadjra B B, Megueni A & Kaddouri K, *Compos Struct*, 65 (2004) 47–53.
6 Megueni A, Bouiadjra B B & Belhouari M, *Comput Mater Sci*, 29 (2004) 407–413.
7 Husseini-Toudeshky H, Sadeghi G & Daghyani H R, *Compos Struct*, 71 (2005) 401–406.
8 Tsai G C & Shen S B, *Compos Struct*, 64 (2004) 79-90.
9 Aslantas K, Tasgetiren S, Ergun E, & Topcu, M, *Technol Res: EJMT,* 3 (2006) 1-10.
10 Ergun E, Aslantas K & Tasgetiren S, *Mech Res Commun,* 35 (2008) 209–218.
11 Aslantas K, *Int J Solids Struct*, 40 (2003) 7475–7481.
12 Ucun I, Aslantas K & Tasgetiren S, *Fatigue Fract Eng Mater & Struct*, 31 (2008) 517–525.
13 Park C H, Lee W, Han W S & Vautrin A, *Comput Struct*, 86 (2008) 1894–1903.
14 Yong M, Falzon B G & Lannucci L, *Int J Impact Eng*, 35 (2008) 1293–1302.
15 Muc A & Gurba W, *Compos Struct*, 54 (2001) 275–281.
16 Canyurt O E, Meran C & Uslu M, *Int J Adhes Adhes,* 30 (2010) 281-287.
17 Kogiso N, Watson L T, Gurdal Z & Haftka R T, *Struct Optim,* 7 (1994) 207–218.
18 Rahul, Chakraborty D & Dutta A, *Compos Sci Technol,* 65 (2005) 2003–2013.
19 Brighenti R, Carpinteri A & Vantadori S, *Comput Methods Appl Mech Eng*, 196 (2006) 466–475.
20 Mathias J D, Balandraud X & Grediac M, *Comput Struct,* 84 (2006) 823–834.
21 ASTM E647 Standard, "*Standard Test Method for Measurement of Fatigue Crack Growth Rates*".
22 Ergun E, Taşgetiren S & Topçu M, *Comput Struct*, 92 (2010) 2622–2631.
23 Ergun E, Aslantaş K, Taşgetiren S & Topçu M, *Mater Des,* 27 (2006) 2-9.
24 Tan C L & Gao Y L, *Eng Fract Mech*, 36(6) (1990) 919-932.
25 FRANC2D/L User's Manual. Available from *www.cfg.cornell.edu*
26 Ergun E, Taşgetiren S & Topcu M, *Fatigue Fract Eng Mater Struct,* 31 (2008) 929-936.