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Haar wavelet method for solving generalized Burgers–Huxley equation

İbrahim Çelik *

Faculty of Arts and Sciences, Department of Mathematics, Pamukkale University, Denizli, Turkey

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Abstract In this paper, an efficient numerical method for the solution of nonlinear partial differential equations based on the Haar wavelets approach is proposed, and tested in the case of generalized Burgers–Huxley equation. Approximate solutions of the generalized Burgers–Huxley equation are compared with exact solutions. The proposed scheme can be used in a wide class of nonlinear reaction–diffusion equations. These calculations demonstrate that the accuracy of the Haar wavelet solutions is quite high even in the case of a small number of grid points. The present method is a very reliable, simple, small computation costs, flexible, and convenient alternative method.

1. Introduction

In this paper, following approximate solutions of the following nonlinear diffusion equation is considered:

* Tel.: +90 2582963619; fax: +90 2582963535.
E-mail address: i.celik@pau.edu.tr

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$$\frac{\partial u}{\partial t}+\alpha u^{\delta}\frac{\partial u}{\partial x}-\frac{\partial^2 u}{\partial x^2}=\beta u(1-u^{\delta})(u^{\delta}-\gamma) \quad (1)$$

by the Haar wavelet method. Where α, β, γ and δ are parameters, $\beta \geqslant 0$, $\gamma > 0$, $\gamma \in (0,1)$. Eq. (1) is a generalized Burgers–Huxley equation. When $\alpha = 0$, $\delta = 1$, Eq. (1) is reduced to the Huxley equation which describes nerve pulse propagation in nerve fibers and wall motion in liquid crystals [22,23]. When $\beta = 0$, $\delta = 1$, Eq. (1) is reduced to the Burgers equation describing the far field of wave propagation in nonlinear dissipative systems [26]. When $\alpha = 0$, $\beta = 1$, $\delta = 1$, Eq. (1) becomes the FitzHugh–Nagumo (FN) equation which is a reaction–diffusion equation used in circuit theory, biology and the area of population genetics [1]. At $\delta = 1$ and $\alpha \neq 0$, $\beta \neq 0$, Eq. (1) is turned into the Burgers–Huxley equation. This equation, which shows a prototype model for describing the interaction between reaction mechanisms, convection effects and diffusion transport, was investigated by Satsuma [21].

Various numerical techniques were used in the literature to obtain numerical solutions of the Burgers–Huxley equation. Wang et al. [24] studied the solitary wave solution of the generalized Burgers–Huxley equation while Estevez [7] presented nonclassical symmetries and the singular modified Burgers and Burgers–Huxley equation. Also Estevez and Gordoa [8] applied a complete Painleve test to the generalized Burgers–Huxley equation. In the past few years, various mathematical methods such as spectral methods [5,14,15], Adomian decomposition method [11–13], homotopy analysis method [19], the tanh-coth method [25], variational iteration method [2,3], Hopf-Cole transformation [6] and polynomial differential quadrature method [20] have been used to solve the equation.

In solving ordinary differential equations (ODEs), Chen and Hsiao [4] derived an operational matrix of integration based on the Haar wavelet method. By using the Haar wavelet method, Lepik [16], Lepik [17] solved higher order as well as nonlinear ODEs and some nonlinear evolution equations. Lepik [18] also used this method to solve Burgers and sine-Gordon equations. Hariharan et al. [10], Hariharan and Kannan [9] introduced the Haar wavelet method for solving both Fisher's and FitzHugh–Nagumo equations.

In the present paper, a new direct computational method for solving generalized Burgers–Huxley equation is introduced. This method consists of reducing the problem to a set of algebraic equation by first expanding the term, which has maximum derivative, given in the equation as Haar functions with unknown coefficients. The operational matrix of integration and product operational matrix are utilized to evaluate the coefficients of the Haar functions. Identification and optimization procedures of the solutions are greatly reduced or simplified. Since the integration of the Haar functions vector is a continuous function, the solutions obtained are continuous. This method gives us the implicit form of the approximate solutions of the problems. In this method, a few sparse matrixes can be obtained, and there are no complex integrals or methodology. Therefore, the present method is useful for obtaining the implicit form of the approximations of linear or nonlinear differential equations, and round off errors and necessity of large computer memory are significantly minimized. Therefore, this paper suggests the use of this technique for solving the generalized Burgers–Huxley equation problems. Illustrative examples are given to demonstrate the application of the proposed method.

2. The model problem

The analysis presented in this paper is based upon the generalized Burgers–Huxley equation given by Eq. (1). Behaviors of many physical systems encountered in models of reaction mechanisms, convection effects and diffusion transport give Eq. (1). The exact solution of the Eq. (1) subject to the initial condition

$$u(x,0) = \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh(a_1 x)\right]^{\frac{1}{\delta}} \tag{2}$$

was derived by Wang et al. [24] using nonlinear transformations and is given by

$$u(x,t) = \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh(a_1(x - a_2 t))\right]^{\frac{1}{\delta}}, \quad t \geqslant 0 \tag{3}$$

where

$$\begin{aligned} a_1 &= \frac{-\alpha\delta + \delta\sqrt{\alpha^2 + 4\beta(1+\delta)}}{4(1+\delta)}\gamma \\ a_2 &= \frac{\alpha\gamma}{1+\delta} - \frac{(1+\delta-\gamma)(-\alpha + \sqrt{\alpha^2 + 4\beta(1+\delta)})}{2(1+\delta)} \end{aligned} \tag{4}$$

This exact solution satisfies the following boundary conditions.

$$\begin{aligned} u(0,t) &= \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh(-a_1 a_2 t)\right]^{\frac{1}{\delta}}, \quad t \geqslant 0 \\ u(1,t) &= \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh(a_1(1 - a_2 t))\right]^{\frac{1}{\delta}}, \quad t \geqslant 0 \end{aligned} \tag{5}$$

The aim of this present study is to show that the Haar wavelet method is capable of archiving high accuracy for the problems given by the generalized Burgers–Huxley equation. The computed results are compared with the exact solutions to show the effectiveness of current method.

3. Haar wavelet method

Haar wavelet is the simplest wavelet. The Haar wavelet transform, proposed in 1909 by Alfred Haar, is the first known wavelet. Haar transform or Haar wavelet transform has been used as an earliest example for orthonormal wavelet transform with compact support. The Haar wavelet family for $x \in [0,1]$ is defined as follows:

$$h_i(x) = \begin{cases} 1 & \text{for} \quad x \in [\xi_{1,}\xi_2), \\ -1 & \text{for} \quad x \in [\xi_2, \xi_3], \\ 0 & \text{elsewhere} \end{cases} \tag{6}$$

where $\xi_1 = \frac{k}{m}, \xi_2 = \frac{k+0.5}{m}$ and $\xi_3 = \frac{k+1}{m}$. In these formulae integer $m = 2^j$, $j = 0, 1, \ldots J$ indicates the level of the wavelet; $k = 0, 1, \ldots m - 1$ is the translation parameter. Maximal level of resolution is J and 2^J is denoted as $M = 2^J$. The index i in Eq. (6) is calculated from the formula $i = m + k + 1$; in the case of minimal values $m = 1$, $k = 0$ we have $i = 2$. The maximal value of i is $i = 2M = 2^{J+1}$. It is assumed that the value $i = 1$ corresponds to the scaling function for which $h_1(x) = 1$ in $[0, 1]$.

It must be noticed that all the Haar wavelets are orthogonal to each other:

$$\int_0^1 h_i(x)h_l(x)dx = \begin{cases} 2^{-j} & i = l = 2^j + k \\ 0 & i \neq l \end{cases} \tag{7}$$

Therefore, they construct a very good transform basis. Any function $y(x)$, which is square integrable in the interval $[0, 1)$, namely $\int_0^1 y^2(x)dx$ is finite, can be expanded in a Haar series with an infinite number of terms

$$y(x) = \sum_{i=1}^{\infty} c_i h_i(x), \quad i = 2^j + k, \quad j \geqslant 0, \quad 0 \leqslant k \leqslant 2^j, \quad x \in [0, 1) \tag{8}$$

Where the Haar coefficients,

$$c_i = 2^j \int_0^1 y(x)h_i(x)dx \tag{9}$$

are determined in such a way that the integral square error

$$E = \int_0^1 \left[y(x) - \sum_{i=1}^{2M} c_i h_i(x) \right]^2 dx \tag{10}$$

is minimized.

In general, the series expansion of $y(x)$ contains infinite terms. If $y(x)$ is a piecewise constant or may be approximated as a piecewise constant during each subinterval, then $y(x)$ will be terminated at finite terms, that is

$$y(x) \cong \sum_{i=1}^{2M} c_i h_i(x) = c^T h_{2M}(x) \tag{11}$$

where the coefficient and the Haar function vectors are defined as:

$$c^T = [c_1, c_2, \ldots c_{2M}], \quad h_{2M}(x) = [h_1(x), h_2(x), \ldots h_{2M}(x)]^T$$

respectively and $x \in [0, 1)$.

The integrals of Haar function $h_i(x)$ can be evaluated as:

$$p_{i,1}(x) = \int_0^x h_i(x)dx \tag{12}$$

$$p_{i,\upsilon}(x) = \int_0^x p_{i,\upsilon-1}(x)dx, \quad \upsilon = 2, 3, \ldots \tag{13}$$

Carrying out these integrations with the aid of Eq. (6), it is found that

$$p_{i,1}(x) = \begin{cases} x - \xi_1 & \text{for} \quad x \in [\xi_1, \xi_2], \\ \xi_3 - x & \text{for} \quad x \in [\xi_2, \xi_3], \\ 0 & \text{elsewhere} \end{cases} \tag{14}$$

$$p_{i,2}(x) = \begin{cases} 0 & \text{for} \quad x \in [0, \xi_1], \\ \frac{(x-\xi_1)^2}{2} & \text{for} \quad x \in [\xi_1, \xi_2], \\ \frac{1}{4m^2} - \frac{(\xi_3-x)^2}{2} & \text{for} \quad x \in [\xi_2, \xi_3], \\ \frac{1}{4m^2} & \text{for} \quad x \in [\xi_3, 1] \end{cases} \tag{15}$$

$$p_{i,3}(x) = \begin{cases} 0 & \text{for} \quad x \in [0, \xi_1], \\ \frac{(x-\xi_1)^3}{6} & \text{for} \quad x \in [\xi_1, \xi_2], \\ \frac{x-\xi_2}{4m^2} - \frac{(\xi_3-x)^3}{6} & \text{for} \quad x \in [\xi_2, \xi_3], \\ \frac{x-\xi_2}{4m^2} & \text{for} \quad x \in [\xi_3, 1] \end{cases} \tag{16}$$

$$p_{i,4}(x) = \begin{cases} 0 & \text{for} \quad x \in [0, \xi_1], \\ \frac{(x-\xi_1)^4}{24} & \text{for} \quad x \in [\xi_1, \xi_2], \\ \frac{(x-\xi_2)^2}{8m^2} - \frac{(\xi_3-x)^4}{24} + \frac{1}{192m^4} & \text{for} \quad x \in [\xi_2, \xi_3], \\ \frac{(x-\xi_2)^2}{8m^2} + \frac{1}{192m^4} & \text{for} \quad x \in [\xi_3, 1] \end{cases} \tag{17}$$

Let us define the collocation points $x_l = (l - 0.5)/(2M)$, $l = 1,2,\ldots,2M$. By these collocation points, a discretizised form of the Haar function $h_i(x)$ can be obtained. Hence, the coefficient matrix $H(i,l) = (h_i(x_l))$, which has the dimension $2M \times 2M$, is achieved. The operational matrices of integrations Pt, which are $2M$ square matrices, are defined by the equation $Pt(i,l) = p_{i,t}(x_l)$, where t shows the order of integration.

4. Method of solution of generalized Burgers–Huxley equation

Consider the generalized Burgers–Huxley Eq. (1) with the initial condition $u(x,0) = f(x)$ and the boundary conditions $u(0,t) = g_0(t)$ and $u(1,t) = g_1(t)$.

It is assumed that $\dot{u}''(x,t)$ can be expanded in terms of Haar wavelets as

$$\dot{u}''(x,t) = \sum_{i=1}^{2M} c_i h_i(x) = c^T h_{2M}(x) \tag{18}$$

where "·" and "″" means differentiation with respect to t and x, respectively, the row vector c^T is constant in the subinterval $t \in [t_s, t_{s+1}]$.

By integrating Eq. (18) with respect to t from t_s to t and twice with respect to x from 0 to x, following equations are obtained

$$u''(x,t) = (t - t_s)\sum_{i=1}^{2M} c_i h_i(x) + u''(x, t_s) \tag{19}$$

$$u'(x,t) = (t - t_s)\sum_{i=1}^{2M} c_i p_{i,1}(x) + u'(x, t_s) - u'(0, t_s) + u'(0,t) \tag{20}$$

$$u(x,t) = (t - t_s)\sum_{i=1}^{2M} c_i p_{i,2}(x) + u(x, t_s) - u(0, t_s) - x[u'(0, t_s) - u'(0,t)] + u(0,t) \tag{21}$$

$$\dot{u}(x,t) = \sum_{i=1}^{2M} c_i p_{i,2}(x) + x\dot{u}'(0,t) + \dot{u}(0,t) \tag{22}$$

From the initial condition and boundary conditions, we have the following equation as:

$$u(x,0) = f(x), \quad u(0,t) = g_0(t), \quad u(1,t) = g_1(t), \quad u(0,t_s) = g_0(t_s), \quad u(1,t_s) = g_1(t_s), \quad \dot{u}(0,t_s) = g_0'(t_s), \quad \dot{u}(1,t_s) = g_1'(t_s)$$

At $x = 1$ in the formulae (21) and (22) and by using conditions, we have

$$u'(0,t) - u'(0,t_s) = -(t - t_s)\sum_{i=1}^{2M} c_i p_{i,2}(1) + g_1(t) - g_1(t_s) + g_0(t_s) - g_0(t) \tag{23}$$

$$\dot{u}'(0,t) = -\sum_{i=1}^{2M} c_i p_{i,2}(1) - g_0'(t) + g_1'(t) \tag{24}$$

It is obtained from Eq. (15) that

$$p_{i,2}(1) = \begin{cases} 0.5 & \text{if} \quad i = 1 \\ \frac{1}{4m^2} & \text{if} \quad i > 1 \end{cases}$$

If the Eqs. (23) and (24) are substituted into the Eqs. (19)–(21) and the results are discriticised by assuming $x \to x_l$, $t \to t_{s+1}$, we obtain

$$u''(x_l, t_{s+1}) = (t_{s+1} - t_s)\sum_{i=1}^{2M} c_i h_i(x_l) + u''(x_l, t_s)$$

$$u'(x_l, t_{s+1}) = (t_{s+1} - t_s)\sum_{i=1}^{2M} c_i p_{i,1}(x_l) - (t - t_s)\sum_{i=1}^{2M} c_i p_{i,2}(1) + u'(x_l, t_s) + g_1(t_{s+1}) - g_1(t_s) + g_0(t_s) - g_0(t_{s+1})$$

$$u(x_l, t_{s+1}) = (t_{s+1} - t_s)\sum_{i=1}^{2M} c_i p_{i,2}(x_l) + u(x_l, t_s) + g_0(t_{s+1}) - g_0(t_s) - x_l[(t_{s+1} - t_s)\sum_{i=1}^{2M} c_i p_{i,2}(1) - g_1(t_{s+1}) + g_1(t_s) - g_0(t_s) + g_0(t_{s+1})]$$

$$\dot{u}(x_l, t_{s+1}) = \sum_{i=1}^{2M} c_i p_{i,2}(x_l) - x_l\left[\sum_{i=1}^{2M} c_i p_{i,2}(1) + g_0'(t_{s+1}) - g_1'(t_{s+1})\right] + g_0'(t_{s+1})$$

We can also show matrix representation of these equations as:

$$U''_{s+1}(l) = \Delta t\, c^T H(i,l) + U_s(l)'' \tag{25}$$

$$U'_{s+1}(l) = \Delta t\, c^T [P1(i,l) - 1_l \otimes P2(i,1)] + U'_s(l) + g_1(t_{s+1}) - g_1(t_s) + g_0(t_s) - g_0(t_{s+1}) \tag{26}$$

$$U_{s+1}(l) = \Delta t\, c^T [P2(i,l) - x_l \otimes P2(i,1)] + U_s(l) + g_0(t_{s+1}) - g_0(t_s) - x_l[-g_1(t_{s+1}) + g_1(t_s) - g_0(t_s) + g_0(t_{s+1})] \tag{27}$$

$$\dot{U}_{s+1}(l) = c^T [P2(i,l) - x_l P2(i,1)] + x_l[g_0'(t_{s+1}) - g_1'(t_{s+1})] + g_0'(t_{s+1}) \tag{28}$$

where 1_l is a unit function, which is shown as $1_l = e(x_l) = 1$, $P2(i,1)$ is a 2M-dimensional column vector and $\otimes$ is a Kronecker product.

In the following the scheme

$$\dot{U}_{s+1}(l) = U''_s(l) - \alpha U^\delta_s(l) U'_s + \beta U_s(l)(1 - U^\delta_s)(U^\delta_s - \gamma) \tag{29}$$

which leads us from the time layer t_s to t_{s+1} is used.

Substituting Eqs. (25)–(28) into the Eq. (29), we have

$$c^T [P2(i,l) - x_l P2(i,1)] = U''_s(l) - \alpha U^\delta_s(l) U'_s + \beta U_s(l)(1 - U^\delta_s)(U^\delta_s - \gamma) - x_l[g_0'(t_{s+1}) - g_1'(t_{s+1})] - g_0'(t_{s+1}). \tag{30}$$

From Eq. (30) the wavelet coefficients c^T can be successively calculated. This solution process is started with

$$U_0(l) = f(x_l)$$
$$U'_0(l) = f'(x_l)$$
$$U''_0(l) = f''(x_l).$$

The solution of the resulting algebraic linear system of equations, which was constructed by applying the grid points, was obtained by the Maple software. Hence we can obtain the implicit form of the approximate solution of the generalized Burgers–Huxley equation as a Haar series.

5. Numerical results

Haar wavelet method is applied to solve the generalized Burgers–Huxley equation

$$\frac{\partial u}{\partial t} + \alpha u^{\delta} \frac{\partial u}{\partial x} - \frac{\partial^2 u}{\partial x^2} = \beta u(1 - u^{\delta})(u^{\delta} - \gamma)$$

with the initial condition

$$u(x,0) = \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh(a_1 x)\right]^{\frac{1}{\delta}}$$

and boundary conditions

$$u(0,t) = \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh(-a_1 a_2 t)\right]^{\frac{1}{\delta}}, u(1,t) = \left[\frac{\gamma}{2} + \frac{\gamma}{2}\tanh(a_1(1 - a_2 t))\right]^{\frac{1}{\delta}}, t \geqslant 0$$

for various values of α, β, γ and δ. Comparisons of the computed results with exact solutions and approximate results, which is given in the Javidi and Golbabai [15] and Sari and Guraslan [20], showed that the method has the capability of solving the generalized Burgers–Huxley equation and also gives highly accurate solutions with minimal computational effort for both time and space.

All of the examples, given in the following, J is taken as 3 or M is taken as 8 and Δt is taken as 0.0001.

Table 1 The absolute errors for various values of δ and x with $\alpha = 1$, $\beta = 1$, $\gamma = 10^{-3}$ and $t = 0.8$.

x	$\delta = 1$	$\delta = 2$	$\delta = 4$	$\delta = 8$
0.03125	5.595048500e−9	2.60170e−7	1.83160e−6	5.10040e−6
0.09375	1.585754820e−8	7.37310e−7	5.18900e−6	1.40414e−5
0.15625	2.464804780e−8	1.14654e−6	8.05670e−6	2.17646e−5
0.21875	3.197174720e−8	1.48748e−6	1.04554e−5	2.83351e−5
0.28125	3.783204670e−8	1.76007e−6	1.23644e−5	3.34685e−5
0.34375	4.222624610e−8	1.96449e−6	1.37580e−5	3.73524e−5
0.40625	4.515634540e−8	2.10047e−6	1.47775e−5	3.99420e−5
0.46875	4.662174470e−8	2.17497e−6	1.51994e−5	4.12351e−5
0.53125	4.662274380e−8	2.17497e−6	1.52546e−5	4.12347e−5
0.59375	4.515724300e−8	2.10054e−6	1.47283e−5	3.99412e−5
0.65625	4.222824200e−8	1.96433e−6	1.38299e−5	3.73509e−5
0.71875	3.783454090e−8	1.76033e−6	1.23582e−5	3.34663e−5
0.78125	3.197443970e−8	1.48745e−6	1.04418e−5	2.83371e−5
0.84375	2.465083840e−8	1.14629e−6	8.04390e−6	2.17548e−5
0.90625	1.586053710e−8	7.37610e−7	5.17310e−6	1.40463e−5
0.96875	5.596235600e−9	2.60370e−7	1.82170e−6	5.1034e−6

Example 1. This present method is applied to Eq. (1) for $\alpha = 1, \beta = 1, \gamma = 0.001$ and the absolute errors for various values of δ are given in Table 1. When the exact and the current results are compared, the results are very accurate as indicated in the table.

Example 2. This present method is applied to Eq. (1) for $\alpha = 0.1$, $\beta = 0.001$, $\gamma = 0.0001$ and the absolute errors for various values of δ are given in Table 2. Comparison between the exact and the current results indicated that the results are very accurate.

Table 2 The absolute errors for various values of δ and x with $\alpha = 0.1$, $\beta = 10^{-3}$, $\gamma = 10^{-4}$ and $t = 1$.

x	$\delta = 1$	$\delta = 2$	$\delta = 4$	$\delta = 8$
0.03125	0	9.999995623e−13	9.999986950e−12	0
0.09375	9.999996436e−15	9.999995623e−13	1.000000083e−11	1.000000083e−10
0.15625	0	9.999995623e−13	0	0
0.21875	1.000000321e−14	0	0	0
0.28125	0	9.999995623e−13	1.000000083e−11	0
0.34375	0	1.000000430e−12	1.000000083e−11	0
0.40625	1.000000321e−14	0	1.000000083e−11	1.000000083e−10
0.46875	0	0	1.000000083e−11	0
0.53125	0	1.000000430e−12	0	0
0.59375	1.000000321e−14	1.000000430e−12	0	9.999995276e−11
0.65625	0	9.999995623e−13	1.000000083e−11	0
0.71875	9.999996436e−15	9.999995623e−13	1.000000083e−11	0
0.78125	0	0	1.000000083e−11	0
0.84375	1.000000321e−14	9.999995623e−13	0	0
0.90625	0	0	0	0
0.96875	9.999996436e−15	0	9.999986950e−12	0

Table 3 The absolute errors for various values of δ and x with $\alpha = -0.1$, $\beta = 0.1$, $\gamma = 10^{-3}$ and $t = 0.9$.

x	$\delta = 1$	$\delta = 2$	$\delta = 4$	$\delta = 8$
0.03125	8.160000000e−10	3.373000000e−8	2.083000000e−7	5.822000000e−7
0.09375	2.353300000e−9	1.009300000e−7	6.480000000e−7	1.697400000e−6
0.15625	3.671100000e−9	1.585800000e−7	1.024800000e−6	2.653700000e−6
0.21875	4.768200000e−9	2.066800000e−7	1.338800000e−6	3.450300000e−6
0.28125	5.646400000e−9	2.451600000e−7	1.590100000e−6	4.087700000e−6
0.34375	6.305200000e−9	2.739900000e−7	1.778300000e−6	4.565500000e−6
0.40625	6.743800000e−9	2.932800000e−7	1.904100000e−6	4.884800000e−6
0.46875	6.962800000e−9	3.029700000e−7	1.966300000e−6	5.044500000e−6
0.53125	6.963000000e−9	3.031300000e−7	1.967900000e−6	5.051500000e−6
0.59375	6.743700000e−9	2.933900000e−7	1.905400000e−6	4.890800000e−6
0.65625	6.305400000e−9	2.741000000e−7	1.779300000e−6	4.570700000e−6
0.71875	5.646300000e−9	2.452300000e−7	1.591000000e−6	4.092000000e−6
0.78125	4.768200000e−9	2.067400000e−7	1.339300000e−6	3.453200000e−6
0.84375	3.671100000e−9	1.586300000e−7	1.025300000e−6	2.655700000e−6
0.90625	2.353500000e−9	1.009900000e−7	6.482000000e−7	1.698600000e−6
0.96875	8.160000000e−10	3.375000000e−8	2.085000000e−7	5.827000000e−7

Example 3. The method presented in this paper is also applied to the Eq. (1) for $\alpha = -0.1, \beta = 0.1, \gamma = 0.001$, for $\alpha = 1, \delta = 1, \gamma = 0.001$ and for $\alpha = 5, \beta = 10, \delta = 2$. The absolute errors for various values of δ, β and γ are given in Tables 3–5, respectively. Very accurate results can be seen in the tables when the exact and the current results are compared.

Example 4. Solutions of Eq. (1) at $t = 1,2,\ldots,10$ for $\alpha = 5, \beta = 10, \gamma = 0.001, \delta = 2$ have been obtained, and results are shown in Fig. 1. For $t = 10$, graphical presentation of the absolute error is depicted in Fig. 2.These tables and figures demonstrate that the accuracy of the Haar wavelet solutions is quite high even in the case of a small number of grid points.

Table 4 The absolute errors for various values of β and x with $\alpha = 1, \delta = 1, \gamma = 0.001$ and $t = 0.9$.

x	$\beta = 1$	$\beta = 10$	$\beta = 50$	$\beta = 100$
0.03125	5.582547900e−9	6.798773400e−8	3.60059750e−7	7.29591030e−7
0.09375	1.584504740e−8	1.909932160e−7	1.01050738e−6	2.04759917e−6
0.15625	2.463554700e−8	2.964153830e−7	1.56803746e−6	3.17739969e−6
0.21875	3.195924640e−8	3.842702350e−7	2.03265728e−6	4.11893700e−6
0.28125	3.781954570e−8	4.545315720e−7	2.40437064e−6	4.87220466e−6
0.34375	4.221374500e−8	5.072288940e−7	2.68315874e−6	5.43717907e−6
0.40625	4.514384420e−8	5.423647020e−7	2.86903276e−6	5.81384324e−6
0.46875	4.660924340e−8	5.599371920e−7	2.96198002e−6	6.00219464e−6
0.53125	4.661024240e−8	5.599423680e−7	2.96200600e−6	6.00222726e−6
0.59375	4.514474140e−8	5.423813270e−7	2.86910920e−6	5.81393500e−6
0.65625	4.221574020e−8	5.072559700e−7	2.68328143e−6	5.43732034e−6
0.71875	3.782203900e−8	4.545663960e−7	2.40451488e−6	4.87237838e−6
0.78125	3.196193760e−8	3.843046050e−7	2.03281662e−6	4.11913152e−6
0.84375	2.463833620e−8	2.964501980e−7	1.56818819e−6	3.17757672e−6
0.90625	1.584803470e−8	1.910167720e−7	1.01062166e−6	2.04773970e−6
0.96875	5.583733000e−9	6.799693000e−8	3.60105340e−7	7.29635830e−7

Table 5 The absolute errors for various values of γ and x with $\alpha = 5, \beta = 10, \delta = 2$ and $t = 0.9$.

x	$\gamma = 10^{-2}$	$\gamma = 10^{-3}$	$\gamma = 10^{-4}$	$\gamma = 10^{-5}$
0.03125	7.409190e−5	2.39188e−6	7.45010e−8	3.13900e−9
0.09375	2.0808842e−4	6.69752e−6	2.11236e−7	7.85800e−9
0.15625	3.2311679e−4	1.039120e−5	3.28945e−7	8.38700e−9
0.21875	4.1912177e−4	1.347084e−5	4.26520e−7	1.32090e−8
0.28125	4.9605331e−4	1.593606e−5	5.04569e−7	1.71850e−8
0.34375	5.5386716e−4	1.778511e−5	5.63132e−7	1.73240e−8
0.40625	5.9252067e−4	1.901796e−5	6.02174e−7	1.80870e−8
0.46875	6.1199834e−4	1.963472e−5	6.26828e−7	1.93560e−8
0.53125	6.1224540e−4	1.963558e−5	6.24923e−7	1.93580e−8
0.59375	5.9324207e−4	1.902067e−5	6.04050e−7	1.80580e−8
0.65625	5.5497941e−4	1.778920e−5	5.62524e−7	1.73260e−8
0.71875	4.9746632e−4	1.593980e−5	5.03402e−7	1.72340e−8
0.78125	4.2063949e−4	1.347557e−5	4.23075e−7	1.32170e−8
0.84375	3.2454397e−4	1.039612e−5	3.26235e−7	8.39500e−9
0.90625	2.0920196e−4	6.70132e−6	2.06215e−7	7.94600e−9
0.96875	7.453704e−5	2.39340e−6	7.26800e−8	3.13500e−9

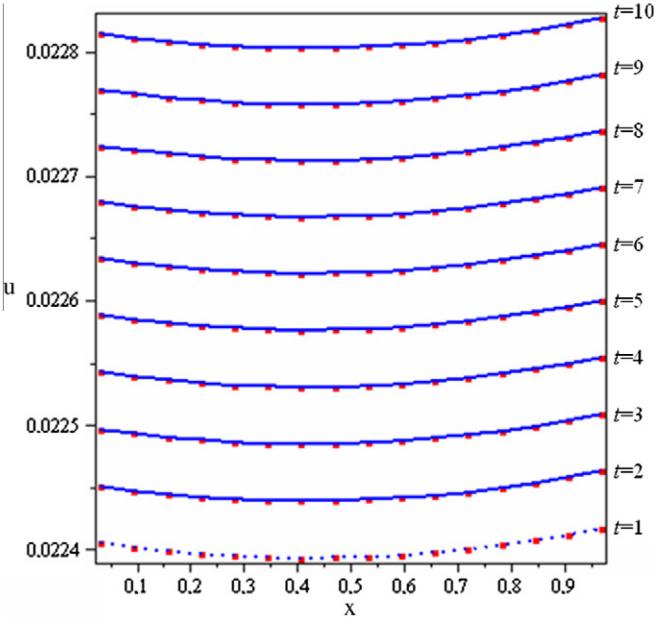

Figure 1 Solutions of Eq. (1) at different times for $\alpha = 5$, $\beta = 10$, $\gamma = 0.001$, $\delta = 2$.

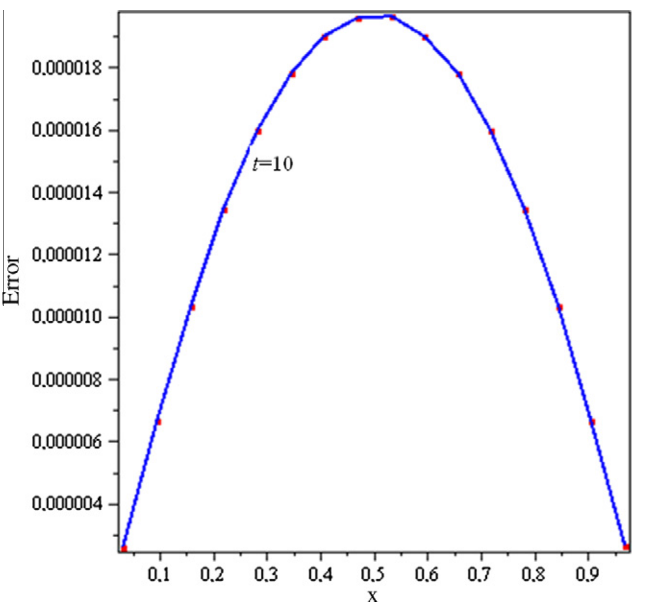

Figure 2 The absolute error for $\alpha = 5$, $\beta = 10$, $\gamma = 0.001$, $\delta = 2$ and $t = 10$.

6. Conclusion

In this paper, Haar wavelet approach is proposed for the generalized Burgers–Huxley equation. Approximate solutions of the generalized Burgers–Huxley equation, obtained by computer simulation, are compared with exact solutions. Comparisons of the absolute errors of our methods with absolute errors given in Javidi and Golbabai [15] and Sari and Guraslan [20] show that our method is efficient method. These calculations demonstrate that the accuracy of the Haar wavelet solutions is quite high even in the case of a small number of grid points. In this method, there are no complex integrals or methodology except a few construction of spars transform matrix. Applications of this method are very simple, and also it gives the implicit form of the approximate solutions of the problems. These are the main advantages of the method. This method is also very convenient for solving the boundary value problems, since the boundary conditions in the solution are taken care of automatically. Hence, the present method is a very reliable, simple, fast, minimal computation costs, flexible, and convenient alternative method.

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