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String-Inspired Chern-Simons Modified Gravity In 4-Dimensions

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Abstract

Field equations of the Chern-Simons modified gravity in 4-dimensions are obtained by a truncation of the field equations of the low energy effective string models with first order corrections in the string constant included.

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1   Introduction

A Chern-Simons modified gravity in $D = 4$ dimensions proposed by Jackiw and Pi [1, 2] has attracted a lot of attention recently [3]. The Jackiw-Pi model is derived from an action that consists of the usual Einstein-Hilbert term plus a topological term with a cosmic scalar field θ appearing as a Lagrange multiplier. It was shown in the linearized approximation that one of two polarization states of the graviton is suppressed due to this modification. It was also noticed that the Kerr solution of general relativity is forbidden in this theory. This point was later analyzed in detail by Grumiller and Yunes [4]. In a more recent study, the effects of a slightly extended model on bodies orbiting the Earth were discussed by Smith et al [5]. It was suggested by Satoh, Kanno and Soda [8] that such effects should be viable in a string-inspired inflationary cosmology. An Einstein-Cartan approach leading to an interpretation with dynamical torsion was also found [6, 7]. It is our wish here to point out that the Jackiw-Pi model of Chern-Simons modified gravity and its extensions above can be accommodated within the framework of effective string field theory in $D = 4$ dimensions with first order corrections in string parameter α' taken into account. The notation and further details may be found in an older paper by one of us [9].

2   Effective String Theory Field Equations

We will start by examining the bosonic field equations arising from the action $I = \int_M L$ where M is a 4-dimensional manifold and the 4-form L is given in terms of a dilaton 0-form ϕ, and a 3-form field H. We take an action

$$L_0 = e^{-\phi} \left(R_{ab} \wedge *e^{ab} - \alpha d\phi \wedge *d\phi + \beta H \wedge *H + \lambda * 1 \right) \tag{1}$$

to the zeroth order in α'. The metric here has been scaled in accordance with a heterotic string model. But the conclusions will not be changed in principle if we adopt a scaling appropriate to different string models. We employ Lorentzian connection 1-forms $\omega^a{}_b$ in terms of which $R^a{}_b = d\omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b$. The metric tensor g on M is given in terms of an orthonormal coframe $\{e^a\}$ by $g = \eta_{ab} e^a \otimes e^b$ with $\eta_{ab} = \text{diag}(- + ++)$. The space-time orientation is fixed by the volume 4-form $*1 = e^0 \wedge e^1 \wedge e^2 \wedge e^3$. The constants α, β are order zero in α' while λ is of order α'^{-1}. We at first set the connection to be the unique metric compatible, torsion-free Levi-Civita connection. We impose

this assumption by introducing the constraint term

$$L_C = (de^a + \omega^a{}_b \wedge e^b) \wedge \lambda_a \tag{2}$$

in the action where λ_a are Lagrange multiplier 2-forms. We will comment on the possibility of having dynamical torsion later. An essential contribution at first order in α' to L comes from the Lorentz Chern-Simons form necessary for anomaly cancelation in string models. Other quadratic contributions of the curvature tensor such as the Euler-Poincaré density are also required. We will repeat our previous analysis in Ref.[9] in $D = 4$ dimensions with

$$L_1 = \eta R_{ab} \wedge R_{cd} * e^{abcd} + \mu(dH - \epsilon R_{ab} \wedge R^{ab}). \tag{3}$$

The constants η, ϵ are first order in string parameter α' and μ is a Lagrange multiplier 0-form that enforces the Bianchi condition $dH = \epsilon R_{ab} \wedge R^{ab}$. It is well-known that $R_{ab} \wedge R^{ab} = dK$ where the Chern-Simons 3-form $K = \omega_{ab} \wedge d\omega^{ab} + \frac{2}{3}\omega_{ab} \wedge \omega^a{}_c \wedge \omega^{cb}$. In fact, Euler-Poincaré density $R_{ab} \wedge R_{cd} * e^{abcd}$ is also an exact form in $D = 4$ dimensions. We keep it in the action but it doesn't give any contribution to the variational field equations.

In order to derive the effective string theory field equations, the action $I = \int_M (L_0 + L_1 + L_C)$ is going to be varied as a functional of the variables $\{e^a, \phi, H, \omega^a{}_b\}$ in a fixed local coordinate chart for M, subject to zero-torsion constraint. The Einstein field equations from the coframe e^a-variations are

$$e^{-\phi}(G_a + \alpha\tau_a[\phi] - \beta\tau_a[H] + \lambda * e_a) + D\lambda_a = 0 \tag{4}$$

where the Einstein 3-forms

$$G_a = R^{bc} \wedge *e_{abc}, \tag{5}$$

the dilaton stress-energy 3-forms

$$\tau_a[\phi] = \iota_a d\phi * d\phi + d\phi \wedge \iota_a * d\phi, \tag{6}$$

and the axion stress-energy 3-forms

$$\tau_a[H] = \iota_a H \wedge *H + H \wedge \iota_a * H. \tag{7}$$

The dilaton field equation from the ϕ-variation is

$$e^{-\phi}(R_{ab} \wedge *e^{ab} - \alpha d\phi \wedge *d\phi + \beta H \wedge *H) - 2\alpha d(e^{-\phi} * d\phi) + \lambda e^{-\phi} * 1 = 0. \tag{8}$$

H-variation gives

$$2\beta e^{-\phi} * H + d\mu = 0 \tag{9}$$

while from the μ-variation, the constraint

$$dH = \epsilon R_{ab} \wedge R^{ab} \tag{10}$$

is obtained. Since $d^2\mu = 0$, the H-field equation may be replaced by

$$\beta d(e^{-\phi} * H) = 0. \tag{11}$$

We also vary the connection 1-forms $\omega^a{}_b$ as independent variables and obtain

$$- e^{-\phi} d\phi \wedge *e_{ab} + 4\epsilon\beta e^{-\phi} R_{ab} \wedge *H = \frac{1}{2}(e_a \wedge \lambda_b - e_b \wedge \lambda_a). \tag{12}$$

These are algebraic for the multiplier 2-forms and admit the unique solution

$$\lambda_a = -2e^{-\phi}\imath_a * d\phi + 8\epsilon\beta e^{-\phi}\imath^b(R_{ba} \wedge *H) + 2\epsilon\beta e_a \wedge e^{-\phi}\imath^b\imath^c(R_{bc} \wedge *H). \tag{13}$$

Substituting this back into the Einstein field equations (4) we get

$$e^{-\phi}(G_a + \alpha\tau_a[\phi] - \beta\tau_a[H] + \lambda * e_a) - 2D(e^{-\phi}\imath_a * d\phi)$$
$$+8\epsilon\beta D(e^{-\phi}\imath^b(R_{ba} \wedge *H)) - 2\epsilon\beta e_a \wedge D(e^{-\phi}\imath^b\imath^c(R_{bc} \wedge *H)) = 0. \tag{14}$$

Taking the trace of it and comparing with (8), we see that the dilaton field equation (8) may be replaced by

$$(3 + 2\alpha)d(e^{-\phi} * d\phi) = 2\beta e^{-\phi} H \wedge *H - \lambda e^{-\phi} * 1. \tag{15}$$

The coupled system of field equations (10), (11), (14) and (15) describes an axi-dilaton gravity theory that includes Jackiw-Pi model and its extensions as special cases. In order to prove this claim, we consider a special class of solutions with constant dilaton: $\phi = \phi_0$. The reduced field equations become

$$G_a - \beta\tau_a[H] + \lambda * e_a + 8\epsilon\beta D(\imath^b(R_{ba} \wedge *H))$$
$$-2\epsilon\beta e_a \wedge D(\imath^b\imath^c(R_{bc} \wedge *H)) = 0, \tag{16}$$
$$2\beta H \wedge *H - \lambda * 1 = 0, \tag{17}$$
$$dH - \epsilon R_{ab} \wedge R^{ab} = 0, \tag{18}$$
$$\beta(d * H) = 0. \tag{19}$$

3

It is always possible to write locally in 4-dimensions,

$$* H = d\theta. \tag{20}$$

In terms of the axion 0-form θ the above reduced set of field equations reads

$$G_a - \beta \tau_a[\theta] + \lambda * e_a + 8\epsilon\beta D(\imath^b(R_{ba} \wedge d\theta))$$
$$-2\epsilon\beta e_a \wedge D(\imath^b \imath^c(R_{bc} \wedge d\theta)) = 0, \tag{21}$$
$$2\beta d\theta \wedge *d\theta + \lambda * 1 = 0, \tag{22}$$
$$\beta d * d\theta - \epsilon\beta R_{ab} \wedge R^{ab} = 0. \tag{23}$$

These are the field equations of Smith et al [5] with a judicious choice of coupling constants. We may further truncate the system by letting $\beta \to 0$ and $\lambda \to 0$ while keeping $\epsilon\beta = 1$. Then we arrive at

$$G_a + 8D(\imath^b(R_{ba} \wedge d\theta)) - 2e_a \wedge D(\imath^b \imath^c(R_{bc} \wedge d\theta)) = 0, \tag{24}$$
$$R_{ab} \wedge R^{ab} = 0 \tag{25}$$

that are precisely the field equations of Jackiw and Pi [1].

3   Concluding Comments

We first comment on variational field equations with dynamical torsion. In this case, independent $\omega^a{}_b$-variations of the action $\int_M (L_0 + L_1)$ yield the field equations

$$e^{-\phi} \left(T^a \wedge *e_{abc} - \frac{1}{2} d\phi \wedge e^a \wedge *e_{abc} + 4\epsilon\beta R_{ab} \wedge *H \right) = 0. \tag{26}$$

These are not algebraic for the connection owing to the presence of curvature forms. We thus encounter here propagating torsion. In fact, connection 1-forms with torsion are uniquely decomposed according to

$$\omega^a{}_b = \hat{\omega}^a{}_b + K^a{}_b \tag{27}$$

where $\hat{\omega}^a{}_b$ are the Levi-Civita connection 1-forms and $K^a{}_b$ are the contortion 1-forms such that $K^a{}_b \wedge e^b = T^a$. Then the corresponding curvature 2-forms are similarly decomposed as

$$R^a{}_b = \hat{R}^a{}_b + \hat{D}K^a{}_b + K^a{}_c \wedge K^c{}_b \tag{28}$$

so that (26) becomes a highly non-linear set of first order differential equations in the components of T^a's. For further details we refer to [9].

To summarize, the axi-dilaton gravity in $D = 4$ dimensions described by the field equations (10), (11), (14) and (15) includes the Chern-Simons modified gravity of Jackiw and Pi and its extensions that appeared in recent literature as special cases. Axi-dilaton gravity is a self-consistent theory of gravity whose solutions both in $D = 4$ [10] and $D \geq 5$ [11] dimensions are worthy of study in their own right from the physical view point.

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